Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/1.1.2.8-P-x-c-x-^m-a+b-x^2-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [174]. This is test number [24].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (174)	% 0.00 (0)
Mathematica	% 100.00 (174)	% 0.00 (0)
Maple	% 97.70 (170)	% 2.30 (4)
Maxima	% 97.70 (170)	% 2.30 (4)
Fricas	% 90.80 (158)	% 9.20 (16)
Sympy	% 80.46 (140)	% 19.54 (34)
Giac	% 97.70 (170)	% 2.30 (4)
Mupad	% 74.14 (129)	% 25.86 (45)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

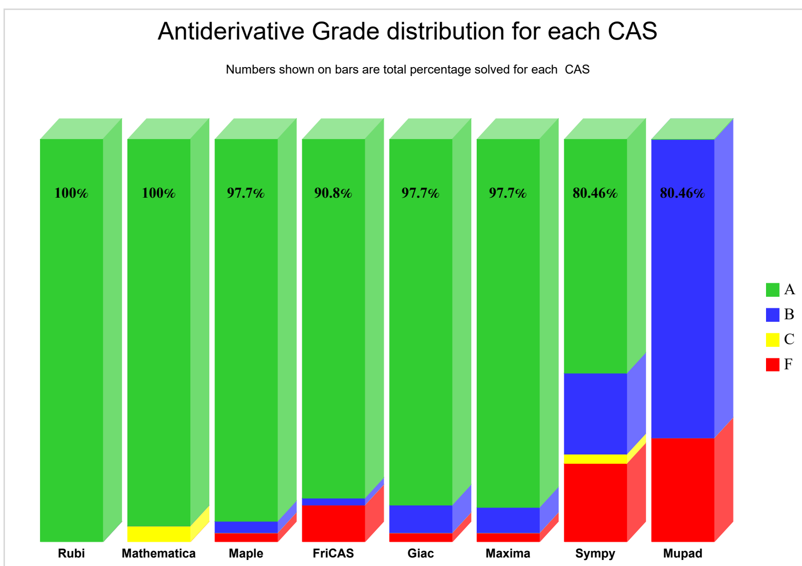
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

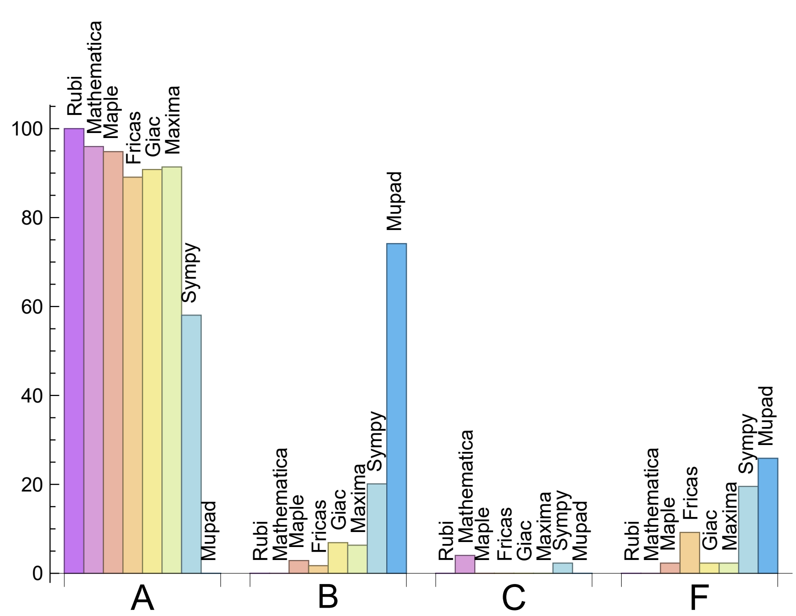
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	95.98	0.00	4.02	0.00
Maple	94.83	2.87	0.00	2.30
Maxima	91.38	6.32	0.00	2.30
Fricas	89.08	1.72	0.00	9.20
Sympy	58.05	20.11	2.30	19.54
Giac	90.80	6.90	0.00	2.30
Mupad	0.00	74.14	0.00	25.86

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	4	100.00 %	0.00 %	0.00 %
Maxima	4	100.00 %	0.00 %	0.00 %
Fricas	16	25.00 %	0.00 %	75.00 %
Sympy	34	0.00 %	100.00 %	0.00 %
Giac	4	100.00 %	0.00 %	0.00 %
Mupad	45	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

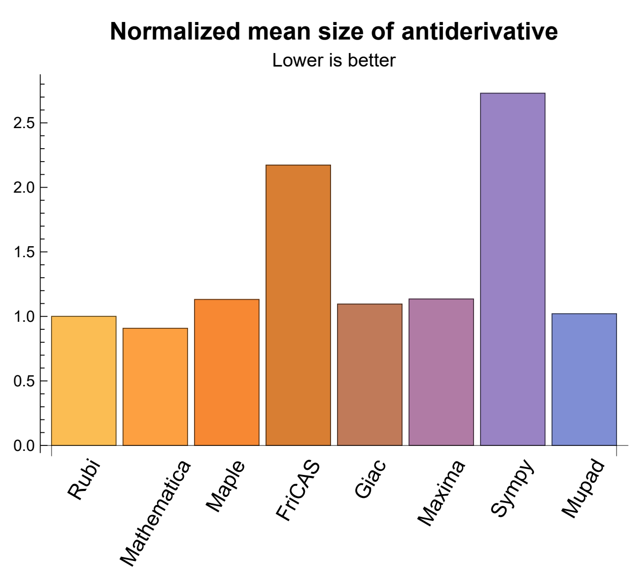
1.3 Performance

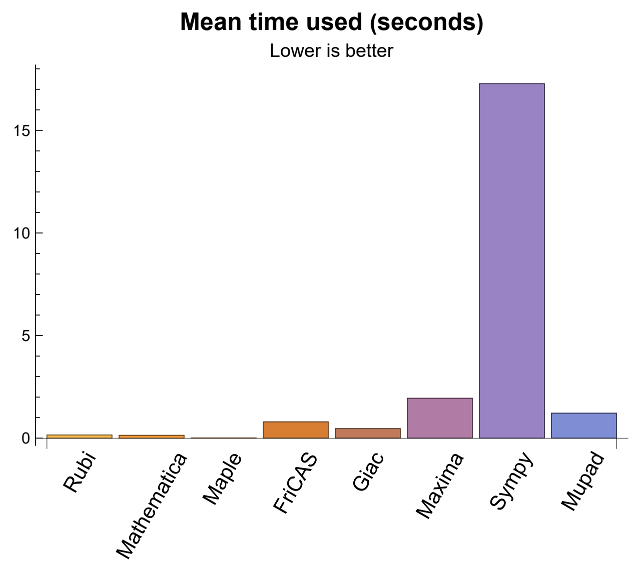
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	129.82	1.00	121.00	1.00
Mathematica	0.14	113.94	0.91	105.50	0.90
Maple	0.01	154.84	1.13	133.50	1.10
Maxima	1.95	164.95	1.14	122.00	0.99
Fricas	0.79	293.15	2.17	242.50	2.14
Sympy	17.28	349.33	2.73	202.50	1.75
Giac	0.46	151.56	1.10	125.50	0.99
Mupad	1.22	129.83	1.02	108.00	0.98

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

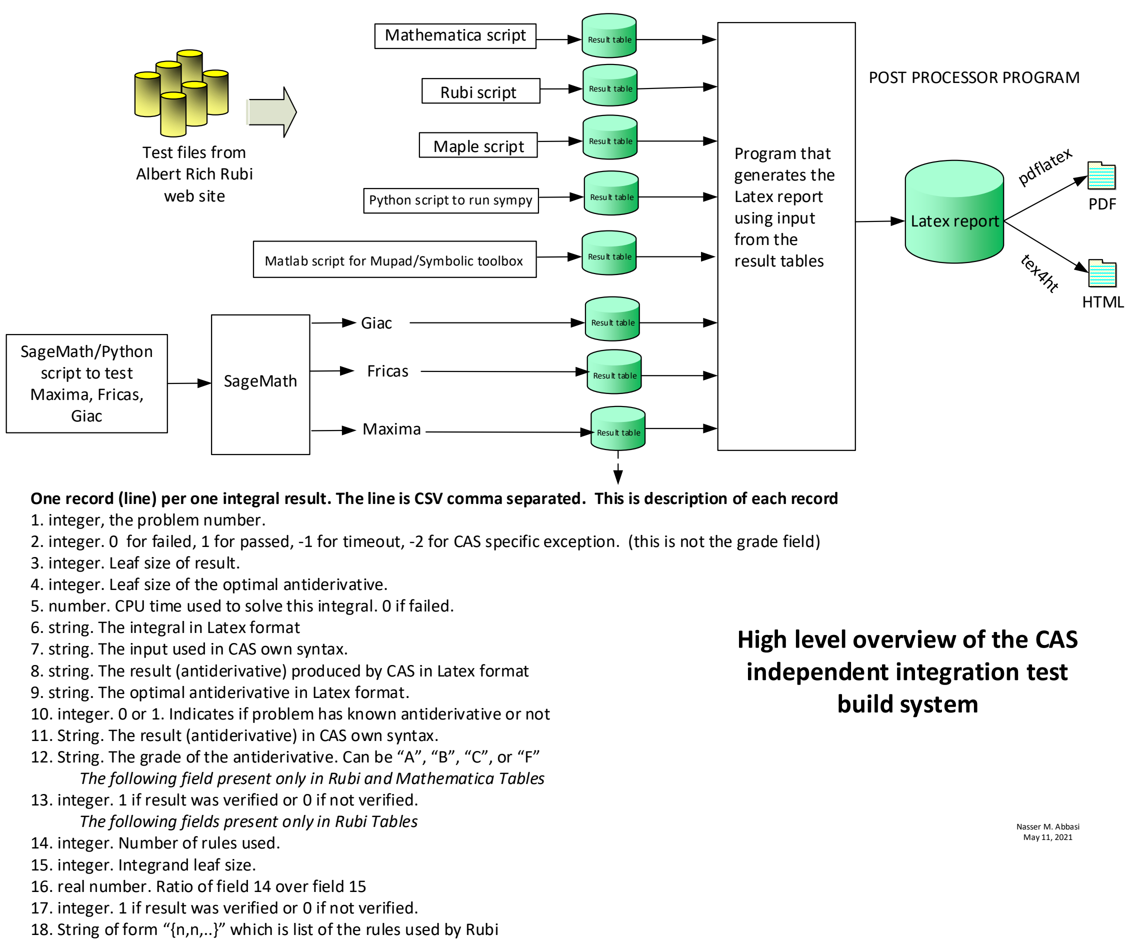
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { }

C grade: { 13, 14, 20, 21, 148, 149, 150 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174 }

B grade: { 48, 161, 162, 172, 173 }

C grade: { }

F grade: { 58, 59, 60, 61 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 164, 165, 166, 167, 168, 169, 170, 171 }

B grade: { 47, 48, 49, 159, 160, 161, 162, 163, 172, 173, 174 }

C grade: { }

F grade: { 58, 59, 60, 61 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 66, 67, 68, 69, 74, 75, 76, 77, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { 107, 108, 109 }

C grade: { }

F grade: { 58, 59, 60, 61, 62, 63, 64, 65, 70, 71, 72, 73, 78, 79, 80, 81 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 35, 36, 38, 43, 44, 45, 46, 53, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 105, 106, 110, 111, 112, 113, 114, 116, 117, 120, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 138, 139, 140, 143, 144, 145, 146, 147, 153, 154, 155, 156, 169, 170, 171 }

B grade: { 25, 33, 34, 37, 39, 40, 41, 42, 52, 54, 55, 56, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 102, 103, 104, 115, 118, 119, 121, 122, 131, 151, 152, 157, 158 }

C grade: { 58, 59, 60, 61 }

F grade: { 47, 48, 49, 50, 51, 57, 91, 92, 93, 99, 100, 101, 107, 108, 109, 132, 141, 142, 148, 149, 150, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 174 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 169, 170, 171, 172, 173, 174 }

B grade: { 7, 14, 28, 35, 45, 150, 156, 157, 158, 166, 167, 168 }

C grade: { }

F grade: { 58, 59, 60, 61 }

2.1.8 Mupad

A grade: { }

B grade: { 4, 5, 6, 7, 11, 12, 13, 14, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 90, 92, 93, 96, 98, 100, 101, 102, 104, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 155, 156, 157, 158, 166, 167, 169, 170, 171 }

C grade: { }

F grade: { 1, 2, 3, 8, 9, 10, 15, 16, 17, 22, 29, 36, 47, 48, 58, 59, 60, 61, 86, 87, 88, 89, 91, 94, 95, 97, 99, 103, 105, 107, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 168, 172, 173, 174 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	107	115	107	206	192	93	-1
normalized size	1	1.00	0.84	0.91	0.84	1.62	1.51	0.73	-0.01
time (sec)	N/A	0.082	0.231	0.010	1.407	0.939	16.250	0.509	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	93	94	86	175	165	81	-1
normalized size	1	1.00	0.89	0.90	0.83	1.68	1.59	0.78	-0.01
time (sec)	N/A	0.049	0.192	0.007	1.363	0.632	10.139	0.471	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	75	67	157	124	68	-1
normalized size	1	1.00	1.08	0.94	0.84	1.96	1.55	0.85	-0.01
time (sec)	N/A	0.026	0.152	0.005	1.319	0.575	14.405	0.440	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	53	45	128	70	55	52
normalized size	1	1.00	1.00	0.79	0.67	1.91	1.04	0.82	0.78
time (sec)	N/A	0.019	0.057	0.006	1.318	1.199	6.510	0.414	1.162

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	100	78	59	341	107	78	68
normalized size	1	1.00	1.27	0.99	0.75	4.32	1.35	0.99	0.86
time (sec)	N/A	0.061	0.253	0.006	1.360	0.990	8.800	0.452	1.236

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	99	97	59	333	124	102	89
normalized size	1	1.00	1.32	1.29	0.79	4.44	1.65	1.36	1.19
time (sec)	N/A	0.059	0.178	0.010	1.386	1.380	11.972	0.474	1.688

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	108	121	83	377	107	163	94
normalized size	1	1.00	1.35	1.51	1.04	4.71	1.34	2.04	1.18
time (sec)	N/A	0.060	0.093	0.007	1.362	0.957	5.499	0.482	1.795

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	126	134	126	254	318	115	-1
normalized size	1	1.00	0.84	0.89	0.84	1.69	2.12	0.77	-0.01
time (sec)	N/A	0.095	0.258	0.012	1.390	0.951	20.789	0.440	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	113	113	105	223	287	103	-1
normalized size	1	1.00	0.89	0.89	0.83	1.76	2.26	0.81	-0.01
time (sec)	N/A	0.062	0.240	0.009	1.374	0.857	20.411	0.506	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	107	94	86	205	223	89	-1
normalized size	1	1.00	1.04	0.91	0.83	1.99	2.17	0.86	-0.01
time (sec)	N/A	0.033	0.194	0.006	1.351	0.841	22.469	0.473	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	69	61	176	219	76	54
normalized size	1	1.00	1.01	0.79	0.70	2.02	2.52	0.87	0.62
time (sec)	N/A	0.026	0.078	0.005	1.412	1.021	12.912	0.440	1.182

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	118	107	88	439	218	100	83
normalized size	1	1.00	1.11	1.01	0.83	4.14	2.06	0.94	0.78
time (sec)	N/A	0.094	0.306	0.009	1.319	0.979	35.492	0.539	1.312

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	105	126	88	411	184	124	86
normalized size	1	1.00	0.97	1.17	0.81	3.81	1.70	1.15	0.80
time (sec)	N/A	0.089	0.184	0.009	1.386	0.676	13.290	0.602	1.877

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	90	150	112	425	182	191	91
normalized size	1	1.00	0.81	1.35	1.01	3.83	1.64	1.72	0.82
time (sec)	N/A	0.085	0.063	0.010	1.279	0.944	15.566	0.550	2.120

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	145	153	145	302	469	140	-1
normalized size	1	1.00	0.84	0.88	0.84	1.75	2.71	0.81	-0.01
time (sec)	N/A	0.105	0.402	0.010	1.358	1.026	37.442	0.529	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	131	132	124	271	442	128	-1
normalized size	1	1.00	0.87	0.88	0.83	1.81	2.95	0.85	-0.01
time (sec)	N/A	0.069	0.406	0.007	1.373	0.935	61.217	0.460	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	112	113	105	253	354	114	-1
normalized size	1	1.00	0.89	0.90	0.83	2.01	2.81	0.90	-0.01
time (sec)	N/A	0.044	0.550	0.007	1.444	0.954	26.540	0.499	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	108	85	77	224	348	101	54
normalized size	1	1.00	1.01	0.79	0.72	2.09	3.25	0.94	0.50
time (sec)	N/A	0.037	0.088	0.005	1.397	0.863	26.046	0.608	1.163

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	139	138	119	539	323	125	101
normalized size	1	1.00	1.05	1.05	0.90	4.08	2.45	0.95	0.77
time (sec)	N/A	0.159	0.361	0.007	1.378	0.636	40.753	0.585	1.247

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	117	158	120	519	318	150	104
normalized size	1	1.00	0.86	1.16	0.88	3.82	2.34	1.10	0.76
time (sec)	N/A	0.132	0.232	0.010	1.403	1.018	18.911	0.499	2.164

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	92	181	143	535	279	219	111
normalized size	1	1.00	0.65	1.28	1.01	3.79	1.98	1.55	0.79
time (sec)	N/A	0.117	0.035	0.012	1.382	0.891	12.983	0.569	2.591

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	76	96	88	158	150	74	-1
normalized size	1	1.00	0.73	0.92	0.85	1.52	1.44	0.71	-0.01
time (sec)	N/A	0.077	0.051	0.010	1.361	0.672	7.912	0.498	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	75	67	127	94	61	93
normalized size	1	1.00	0.79	0.93	0.83	1.57	1.16	0.75	1.15
time (sec)	N/A	0.042	0.060	0.008	1.293	0.906	6.228	0.524	1.467

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	55	47	109	70	50	82
normalized size	1	1.00	1.02	0.98	0.84	1.95	1.25	0.89	1.46
time (sec)	N/A	0.023	0.042	0.004	1.326	1.194	6.255	0.479	1.237

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	29	92	102	39	36
normalized size	1	1.00	1.07	0.86	0.67	2.14	2.37	0.91	0.84
time (sec)	N/A	0.015	0.059	0.006	1.355	0.934	2.641	0.547	1.144

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	52	33	273	99	58	42
normalized size	1	1.00	1.00	0.98	0.62	5.15	1.87	1.09	0.79
time (sec)	N/A	0.040	0.020	0.007	1.377	0.668	5.141	0.499	1.299

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	49	37	101	41	65	39
normalized size	1	1.00	1.00	1.04	0.79	2.15	0.87	1.38	0.83
time (sec)	N/A	0.033	0.039	0.007	1.306	0.954	2.799	0.511	1.201

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	63	68	56	123	66	146	58
normalized size	1	1.00	0.88	0.94	0.78	1.71	0.92	2.03	0.81
time (sec)	N/A	0.053	0.136	0.011	1.315	0.832	7.056	0.431	1.350

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	72	93	85	197	117	70	-1
normalized size	1	1.00	0.89	1.15	1.05	2.43	1.44	0.86	-0.01
time (sec)	N/A	0.043	0.063	0.010	1.352	1.004	10.335	0.512	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	72	64	164	83	58	61
normalized size	1	1.00	1.02	1.09	0.97	2.48	1.26	0.88	0.92
time (sec)	N/A	0.036	0.059	0.007	1.296	0.982	16.608	0.547	1.342

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	64	54	46	147	66	48	53
normalized size	1	1.00	1.33	1.12	0.96	3.06	1.38	1.00	1.10
time (sec)	N/A	0.020	0.081	0.007	1.309	1.418	15.185	0.498	1.055

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	26	31	35	46	23	24
normalized size	1	1.00	0.96	0.93	1.11	1.25	1.64	0.82	0.86
time (sec)	N/A	0.007	0.032	0.003	1.366	0.782	10.223	0.443	0.909

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	60	48	146	206	59	50
normalized size	1	1.00	1.00	1.28	1.02	3.11	4.38	1.26	1.06
time (sec)	N/A	0.038	0.048	0.009	1.299	1.158	11.294	0.445	1.291

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	72	80	68	169	235	96	70
normalized size	1	1.00	1.03	1.14	0.97	2.41	3.36	1.37	1.00
time (sec)	N/A	0.057	0.044	0.009	1.319	1.152	15.826	0.502	1.448

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	101	89	211	124	171	94
normalized size	1	1.00	0.79	1.06	0.94	2.22	1.31	1.80	0.99
time (sec)	N/A	0.078	0.223	0.010	1.353	1.229	10.795	0.497	1.592

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	69	91	102	239	400	70	-1
normalized size	1	1.00	0.87	1.15	1.29	3.03	5.06	0.89	-0.01
time (sec)	N/A	0.042	0.124	0.010	1.409	1.056	18.323	0.482	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	41	70	63	141	36	51
normalized size	1	1.00	0.83	0.77	1.32	1.19	2.66	0.68	0.96
time (sec)	N/A	0.022	0.055	0.006	1.360	0.917	17.325	0.482	0.967

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	47	32	29	51	49	95	26	34
normalized size	1	0.94	0.64	0.58	1.02	0.98	1.90	0.52	0.68
time (sec)	N/A	0.014	0.023	0.003	1.311	0.660	13.983	0.518	0.924

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	48	62	146	37	41
normalized size	1	1.00	0.84	0.78	0.94	1.22	2.86	0.73	0.80
time (sec)	N/A	0.010	0.047	0.004	1.367	0.608	13.203	0.478	0.929

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	69	92	80	239	840	82	80
normalized size	1	1.00	0.91	1.21	1.05	3.14	11.05	1.08	1.05
time (sec)	N/A	0.063	0.098	0.008	1.367	0.666	25.923	0.540	1.375

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	95	112	100	264	910	119	96
normalized size	1	1.00	0.91	1.08	0.96	2.54	8.75	1.14	0.92
time (sec)	N/A	0.088	0.073	0.010	1.342	0.742	24.183	0.553	1.579

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	106	134	122	307	1034	197	123
normalized size	1	1.00	0.82	1.04	0.95	2.38	8.02	1.53	0.95
time (sec)	N/A	0.119	0.215	0.011	1.342	0.726	24.756	0.534	1.621

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	29	28	31	24	19	20
normalized size	1	1.00	0.89	1.07	1.04	1.15	0.89	0.70	0.74
time (sec)	N/A	0.008	0.030	0.008	2.969	0.594	0.240	0.471	0.040

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	29	28	31	24	19	20
normalized size	1	1.00	0.89	1.07	1.04	1.15	0.89	0.70	0.74
time (sec)	N/A	0.017	0.020	0.005	2.889	0.618	0.294	0.417	0.033

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	33	15	22	26	27	30	14
normalized size	1	1.00	1.94	0.88	1.29	1.53	1.59	1.76	0.82
time (sec)	N/A	0.006	0.009	0.003	2.876	0.617	0.186	0.410	0.919

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.003	0.005	0.003	2.972	0.691	0.153	0.458	0.039

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	165	265	435	522	0	204	-1
normalized size	1	1.00	0.77	1.24	2.04	2.45	0.00	0.96	-0.00
time (sec)	N/A	0.324	0.649	0.048	1.680	1.106	0.000	0.539	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	147	277	447	467	0	138	-1
normalized size	1	1.00	0.98	1.85	2.98	3.11	0.00	0.92	-0.01
time (sec)	N/A	0.165	0.408	0.013	1.635	0.885	0.000	0.527	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	89	95	240	137	0	112	196
normalized size	1	1.00	0.67	0.72	1.82	1.04	0.00	0.85	1.48
time (sec)	N/A	0.166	0.114	0.005	1.443	1.026	0.000	0.624	1.274

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	78	76	253	122	0	81	186
normalized size	1	1.00	0.52	0.51	1.70	0.82	0.00	0.54	1.25
time (sec)	N/A	0.181	0.104	0.008	1.418	0.593	0.000	0.580	1.192

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	84	85	179	131	0	95	133
normalized size	1	1.00	0.60	0.61	1.29	0.94	0.00	0.68	0.96
time (sec)	N/A	0.151	0.126	0.008	1.380	0.634	0.000	0.586	1.138

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	87	88	197	134	904	94	133
normalized size	1	1.00	0.63	0.63	1.42	0.96	6.50	0.68	0.96
time (sec)	N/A	0.131	0.142	0.008	1.420	0.514	118.646	0.518	1.094

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	75	73	123	119	796	82	99
normalized size	1	1.00	0.63	0.61	1.03	1.00	6.69	0.69	0.83
time (sec)	N/A	0.088	0.088	0.005	1.341	0.692	85.296	0.604	1.049

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	92	96	153	137	1880	112	115
normalized size	1	1.00	0.72	0.76	1.20	1.08	14.80	0.88	0.91
time (sec)	N/A	0.069	0.086	0.007	1.334	0.710	94.218	0.504	1.033

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	120	169	157	465	6613	152	159
normalized size	1	1.00	0.87	1.22	1.14	3.37	47.92	1.10	1.15
time (sec)	N/A	0.162	0.229	0.012	1.448	0.768	107.414	0.573	1.620

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	158	240	228	525	6922	239	225
normalized size	1	1.00	0.84	1.28	1.21	2.79	36.82	1.27	1.20
time (sec)	N/A	0.381	0.217	0.012	1.412	0.755	165.702	0.511	2.095

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	178	288	265	688	0	325	279
normalized size	1	1.00	0.81	1.32	1.21	3.14	0.00	1.48	1.27
time (sec)	N/A	0.480	0.534	0.015	1.495	0.932	0.000	0.484	2.518

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	0	0	0	97	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	2.16	0.00	-0.02
time (sec)	N/A	0.017	0.009	0.071	0.000	0.690	1.970	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	0	0	0	192	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	2.11	0.00	-0.01
time (sec)	N/A	0.043	0.041	0.067	0.000	0.696	6.404	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	56	0	0	0	204	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	2.68	0.00	-0.01
time (sec)	N/A	0.039	0.072	0.067	0.000	0.656	6.475	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	99	0	0	0	298	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	2.46	0.00	-0.01
time (sec)	N/A	0.122	0.095	0.066	0.000	0.633	7.602	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	0	60	57	57
normalized size	1	1.00	1.00	0.83	0.82	0.00	0.92	0.88	0.88
time (sec)	N/A	0.074	0.031	0.003	1.341	0.000	0.101	0.369	1.205

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	0	60	57	57
normalized size	1	1.00	1.00	0.83	0.82	0.00	0.92	0.88	0.88
time (sec)	N/A	0.076	0.016	0.002	1.325	0.000	0.091	0.379	1.184

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	0	60	57	57
normalized size	1	1.00	1.00	0.83	0.82	0.00	0.92	0.88	0.88
time (sec)	N/A	0.061	0.010	0.003	1.312	0.000	0.136	0.318	1.185

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	0	56	54	54
normalized size	1	1.00	1.00	0.85	0.83	0.00	0.93	0.90	0.90
time (sec)	N/A	0.041	0.009	0.000	1.322	0.000	0.121	0.361	1.164

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	48	48	54	53	52
normalized size	1	1.00	1.00	0.95	0.86	0.86	0.96	0.95	0.93
time (sec)	N/A	0.040	0.016	0.003	1.354	0.722	0.325	0.410	1.170

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	50	48	55	49	50	49
normalized size	1	1.00	1.00	0.93	0.89	1.02	0.91	0.93	0.91
time (sec)	N/A	0.048	0.055	0.006	1.343	0.727	0.280	0.386	1.145

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	51	48	48	55	51	48	47
normalized size	1	1.00	0.94	0.89	0.89	1.02	0.94	0.89	0.87
time (sec)	N/A	0.048	0.037	0.007	1.316	0.625	0.507	0.375	1.138

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	55	51	49	55	54	50	50
normalized size	1	1.00	1.02	0.94	0.91	1.02	1.00	0.93	0.93
time (sec)	N/A	0.049	0.037	0.004	1.328	0.592	1.010	0.432	1.148

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	98	102	101	0	110	105	108
normalized size	1	1.00	0.90	0.94	0.93	0.00	1.01	0.96	0.99
time (sec)	N/A	0.124	0.057	0.001	1.365	0.000	0.135	0.350	1.130

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	92	102	101	0	110	105	108
normalized size	1	1.00	0.84	0.94	0.93	0.00	1.01	0.96	0.99
time (sec)	N/A	0.112	0.070	0.001	1.305	0.000	0.142	0.369	1.111

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	92	102	101	0	110	105	107
normalized size	1	1.00	0.88	0.98	0.97	0.00	1.06	1.01	1.03
time (sec)	N/A	0.074	0.044	0.002	1.389	0.000	0.088	0.360	1.111

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	88	99	98	0	107	102	105
normalized size	1	1.00	0.89	1.00	0.99	0.00	1.08	1.03	1.06
time (sec)	N/A	0.072	0.056	0.002	1.348	0.000	0.089	0.379	1.106

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	88	100	96	96	104	100	103
normalized size	1	1.00	0.96	1.09	1.04	1.04	1.13	1.09	1.12
time (sec)	N/A	0.069	0.079	0.003	1.384	0.531	0.318	0.464	1.106

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	88	98	96	103	99	98	92
normalized size	1	1.00	0.98	1.09	1.07	1.14	1.10	1.09	1.02
time (sec)	N/A	0.080	0.060	0.007	1.315	0.481	0.351	0.378	1.107

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	87	97	96	103	100	97	103
normalized size	1	1.00	0.89	0.99	0.98	1.05	1.02	0.99	1.05
time (sec)	N/A	0.086	0.044	0.007	1.325	0.496	0.578	0.335	1.106

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	83	97	97	103	100	97	106
normalized size	1	1.00	0.85	0.99	0.99	1.05	1.02	0.99	1.08
time (sec)	N/A	0.086	0.056	0.007	1.370	0.743	1.456	0.430	1.275

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	149	150	145	0	163	153	153
normalized size	1	1.00	1.00	1.01	0.97	0.00	1.09	1.03	1.03
time (sec)	N/A	0.186	0.026	0.002	1.328	0.000	0.167	0.459	1.299

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	125	150	145	0	165	153	153
normalized size	1	1.00	0.84	1.01	0.97	0.00	1.11	1.03	1.03
time (sec)	N/A	0.141	0.065	0.000	1.361	0.000	0.138	0.461	1.275

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	124	150	145	0	163	153	153
normalized size	1	1.00	0.90	1.09	1.05	0.00	1.18	1.11	1.11
time (sec)	N/A	0.095	0.063	0.002	1.336	0.000	0.136	0.362	1.278

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	121	147	142	0	158	149	149
normalized size	1	1.00	0.91	1.11	1.07	0.00	1.19	1.12	1.12
time (sec)	N/A	0.093	0.050	0.001	1.341	0.000	0.135	0.401	1.259

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	121	148	140	140	158	148	147
normalized size	1	1.00	0.94	1.15	1.09	1.09	1.22	1.15	1.14
time (sec)	N/A	0.090	0.067	0.004	1.321	0.710	0.399	0.327	1.264

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	123	145	139	147	150	145	121
normalized size	1	1.00	0.99	1.17	1.12	1.19	1.21	1.17	0.98
time (sec)	N/A	0.109	0.081	0.007	1.348	0.635	0.456	0.406	1.184

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	124	144	139	147	151	144	143
normalized size	1	1.00	0.92	1.07	1.03	1.09	1.12	1.07	1.06
time (sec)	N/A	0.112	0.062	0.007	1.357	0.541	0.674	0.409	1.257

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	124	146	142	147	155	146	148
normalized size	1	1.00	0.89	1.05	1.02	1.06	1.12	1.05	1.06
time (sec)	N/A	0.114	0.050	0.009	1.317	0.817	1.086	0.396	1.368

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	130	176	145	332	316	161	-1
normalized size	1	1.00	0.86	1.17	0.96	2.20	2.09	1.07	-0.01
time (sec)	N/A	0.145	0.082	0.007	2.962	0.707	1.434	0.431	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	114	152	127	270	274	137	-1
normalized size	1	1.00	0.88	1.17	0.98	2.08	2.11	1.05	-0.01
time (sec)	N/A	0.124	0.104	0.006	2.932	0.613	1.349	0.350	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	128	98	238	245	112	-1
normalized size	1	1.00	0.86	1.15	0.88	2.14	2.21	1.01	-0.01
time (sec)	N/A	0.113	0.057	0.005	3.002	0.688	1.647	0.399	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	81	106	82	180	211	88	-1
normalized size	1	1.00	0.88	1.15	0.89	1.96	2.29	0.96	-0.01
time (sec)	N/A	0.085	0.071	0.006	3.007	0.715	0.988	0.423	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	83	64	157	219	66	79
normalized size	1	1.00	0.93	1.14	0.88	2.15	3.00	0.90	1.08
time (sec)	N/A	0.065	0.043	0.003	2.929	0.681	0.881	0.460	1.425

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	73	80	65	158	0	66	-1
normalized size	1	1.00	1.01	1.11	0.90	2.19	0.00	0.92	-0.01
time (sec)	N/A	0.098	0.056	0.007	2.931	0.680	0.000	0.451	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	83	67	165	0	68	78
normalized size	1	1.00	0.99	1.09	0.88	2.17	0.00	0.89	1.03
time (sec)	N/A	0.098	0.048	0.008	3.001	0.820	0.000	0.383	1.212

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	102	76	205	0	80	97
normalized size	1	1.00	0.91	1.11	0.83	2.23	0.00	0.87	1.05
time (sec)	N/A	0.109	0.084	0.008	3.052	0.561	0.000	0.386	1.304

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	139	201	150	468	335	159	-1
normalized size	1	1.00	0.79	1.14	0.85	2.66	1.90	0.90	-0.01
time (sec)	N/A	0.268	0.134	0.011	2.835	0.583	4.769	0.432	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	128	177	127	372	289	131	-1
normalized size	1	1.00	0.83	1.15	0.82	2.42	1.88	0.85	-0.01
time (sec)	N/A	0.241	0.083	0.010	2.999	0.695	3.868	0.377	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	100	154	108	357	284	111	152
normalized size	1	1.00	0.75	1.15	0.81	2.66	2.12	0.83	1.13
time (sec)	N/A	0.226	0.082	0.010	2.963	0.621	4.611	0.390	1.292

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	92	127	84	287	212	81	-1
normalized size	1	1.00	0.91	1.26	0.83	2.84	2.10	0.80	-0.01
time (sec)	N/A	0.118	0.051	0.010	3.016	0.604	5.763	0.431	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	97	89	257	233	88	110
normalized size	1	1.00	0.89	1.04	0.96	2.76	2.51	0.95	1.18
time (sec)	N/A	0.065	0.091	0.008	2.945	0.688	3.094	0.375	1.316

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	125	87	296	0	93	-1
normalized size	1	1.00	0.89	1.32	0.92	3.12	0.00	0.98	-0.01
time (sec)	N/A	0.122	0.076	0.013	2.914	0.761	0.000	0.348	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	136	105	336	0	103	133
normalized size	1	1.00	1.00	1.24	0.95	3.05	0.00	0.94	1.21
time (sec)	N/A	0.142	0.074	0.014	2.985	0.678	0.000	0.342	1.410

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	112	169	117	441	0	126	158
normalized size	1	1.00	0.83	1.25	0.87	3.27	0.00	0.93	1.17
time (sec)	N/A	0.204	0.103	0.016	2.945	0.821	0.000	0.422	1.348

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	139	235	165	574	357	157	232
normalized size	1	1.00	0.75	1.27	0.89	3.10	1.93	0.85	1.25
time (sec)	N/A	0.338	0.117	0.013	3.075	0.762	29.587	0.410	1.556

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	126	206	136	480	282	122	-1
normalized size	1	1.00	0.81	1.33	0.88	3.10	1.82	0.79	-0.01
time (sec)	N/A	0.232	0.076	0.012	2.984	0.741	29.728	0.385	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	122	133	146	447	304	128	195
normalized size	1	1.00	0.90	0.98	1.07	3.29	2.24	0.94	1.43
time (sec)	N/A	0.158	0.095	0.010	2.977	0.763	20.010	0.422	1.391

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	99	110	111	357	178	97	-1
normalized size	1	1.00	0.83	0.92	0.93	3.00	1.50	0.82	-0.01
time (sec)	N/A	0.114	0.134	0.010	2.993	0.731	16.372	0.391	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	104	111	122	346	184	106	163
normalized size	1	1.00	0.90	0.96	1.05	2.98	1.59	0.91	1.41
time (sec)	N/A	0.068	0.114	0.009	2.998	0.727	11.272	0.386	1.327

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	117	184	133	488	0	128	-1
normalized size	1	1.00	0.90	1.42	1.02	3.75	0.00	0.98	-0.01
time (sec)	N/A	0.135	0.109	0.015	2.904	0.813	0.000	0.479	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	141	195	152	524	0	141	202
normalized size	1	1.00	0.98	1.35	1.06	3.64	0.00	0.98	1.40
time (sec)	N/A	0.228	0.099	0.016	2.986	0.698	0.000	0.490	1.400

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	147	250	172	696	0	162	229
normalized size	1	1.00	0.84	1.44	0.99	4.00	0.00	0.93	1.32
time (sec)	N/A	0.311	0.163	0.018	2.991	1.021	0.000	0.380	1.456

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	24	15	25	20
normalized size	1	1.00	1.00	0.95	0.90	1.20	0.75	1.25	1.00
time (sec)	N/A	0.023	0.007	0.010	1.339	0.638	0.174	0.317	0.912

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	15	22	14	22	17	14
normalized size	1	1.00	0.78	0.65	0.96	0.61	0.96	0.74	0.61
time (sec)	N/A	0.021	0.009	0.006	1.344	0.691	0.694	0.401	0.102

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	20	20	20	20	20
normalized size	1	1.00	1.00	0.84	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.024	0.013	0.004	2.920	0.627	0.141	0.355	0.038

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	22	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.73	0.80	0.80
time (sec)	N/A	0.025	0.007	0.003	2.899	0.560	0.111	0.389	0.035

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	210	278	213	452	384	250	289
normalized size	1	1.00	1.00	1.32	1.01	2.15	1.83	1.19	1.38
time (sec)	N/A	0.161	0.156	0.005	2.897	0.565	1.652	0.457	0.929

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	162	230	172	368	337	200	243
normalized size	1	1.00	0.94	1.34	1.00	2.14	1.96	1.16	1.41
time (sec)	N/A	0.123	0.121	0.006	2.930	0.592	1.321	0.368	0.944

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	128	182	133	286	185	152	193
normalized size	1	1.00	0.94	1.34	0.98	2.10	1.36	1.12	1.42
time (sec)	N/A	0.105	0.097	0.003	2.964	0.686	1.128	0.416	0.912

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	98	135	94	236	160	106	96
normalized size	1	1.00	0.98	1.35	0.94	2.36	1.60	1.06	0.96
time (sec)	N/A	0.062	0.083	0.003	2.972	0.724	1.149	0.448	0.936

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	114	80	211	150	86	76
normalized size	1	1.00	0.99	1.36	0.95	2.51	1.79	1.02	0.90
time (sec)	N/A	0.094	0.066	0.007	2.929	0.516	1.638	0.337	1.067

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	115	79	216	151	81	80
normalized size	1	1.00	1.01	1.40	0.96	2.63	1.84	0.99	0.98
time (sec)	N/A	0.087	0.087	0.009	2.938	0.677	2.266	0.355	0.111

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	103	142	97	246	167	105	94
normalized size	1	1.00	0.99	1.37	0.93	2.37	1.61	1.01	0.90
time (sec)	N/A	0.102	0.088	0.007	3.090	0.779	6.728	0.367	1.203

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	139	190	134	292	301	151	127
normalized size	1	1.00	1.01	1.39	0.98	2.13	2.20	1.10	0.93
time (sec)	N/A	0.130	0.124	0.010	3.031	0.648	21.646	0.455	0.981

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	174	238	175	374	354	201	161
normalized size	1	1.00	0.99	1.36	1.00	2.14	2.02	1.15	0.92
time (sec)	N/A	0.146	0.144	0.010	3.006	0.749	32.721	0.423	1.018

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	211	286	214	458	398	249	197
normalized size	1	1.00	1.00	1.36	1.01	2.17	1.89	1.18	0.93
time (sec)	N/A	0.175	0.171	0.010	2.952	0.668	84.142	0.356	0.988

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	227	309	227	572	444	252	413
normalized size	1	1.00	0.95	1.29	0.95	2.38	1.85	1.05	1.72
time (sec)	N/A	0.294	0.128	0.013	3.007	0.571	3.070	0.387	0.099

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	187	258	183	478	257	201	288
normalized size	1	1.00	0.93	1.28	0.91	2.37	1.27	1.00	1.43
time (sec)	N/A	0.235	0.104	0.012	3.006	0.705	4.762	0.417	0.966

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	148	212	140	418	221	152	153
normalized size	1	1.00	0.91	1.30	0.86	2.56	1.36	0.93	0.94
time (sec)	N/A	0.229	0.089	0.012	2.957	0.645	3.036	0.373	1.002

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	122	177	117	364	201	126	113
normalized size	1	1.00	1.03	1.50	0.99	3.08	1.70	1.07	0.96
time (sec)	N/A	0.120	0.098	0.011	2.967	0.654	2.877	0.403	0.100

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	115	165	117	354	197	122	112
normalized size	1	1.00	1.03	1.47	1.04	3.16	1.76	1.09	1.00
time (sec)	N/A	0.132	0.070	0.013	2.992	0.614	9.465	0.360	0.996

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	125	182	130	378	212	123	119
normalized size	1	1.00	1.03	1.50	1.07	3.12	1.75	1.02	0.98
time (sec)	N/A	0.158	0.079	0.015	3.023	0.746	25.992	0.483	0.131

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	151	219	151	438	226	151	145
normalized size	1	1.00	0.99	1.44	0.99	2.88	1.49	0.99	0.95
time (sec)	N/A	0.213	0.091	0.015	2.989	0.713	32.024	0.490	0.999

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	190	268	194	488	394	201	181
normalized size	1	1.00	1.01	1.42	1.03	2.58	2.08	1.06	0.96
time (sec)	N/A	0.293	0.109	0.019	3.047	0.619	99.022	0.433	0.987

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	230	318	238	582	0	252	219
normalized size	1	1.00	1.00	1.38	1.03	2.53	0.00	1.10	0.95
time (sec)	N/A	0.376	0.124	0.016	2.985	0.668	0.000	0.378	1.010

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	272	394	281	762	503	301	506
normalized size	1	1.00	0.95	1.37	0.98	2.66	1.75	1.05	1.76
time (sec)	N/A	0.492	0.186	0.016	3.104	0.582	16.433	0.400	0.999

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	232	343	237	668	316	250	348
normalized size	1	1.00	0.94	1.39	0.96	2.70	1.28	1.01	1.41
time (sec)	N/A	0.411	0.152	0.015	2.991	0.644	18.216	0.470	0.107

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	176	294	193	614	280	200	206
normalized size	1	1.00	0.85	1.42	0.93	2.97	1.35	0.97	1.00
time (sec)	N/A	0.334	0.177	0.014	3.038	0.505	17.840	0.509	0.954

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	156	259	169	555	260	173	163
normalized size	1	1.00	0.93	1.55	1.01	3.32	1.56	1.04	0.98
time (sec)	N/A	0.262	0.134	0.012	3.021	0.742	13.069	0.448	1.018

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	141	234	154	504	243	149	148
normalized size	1	1.00	0.96	1.59	1.05	3.43	1.65	1.01	1.01
time (sec)	N/A	0.150	0.122	0.010	2.996	0.808	10.095	0.457	1.047

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	155	237	161	517	250	153	149
normalized size	1	1.00	1.01	1.55	1.05	3.38	1.63	1.00	0.97
time (sec)	N/A	0.176	0.132	0.013	2.992	0.632	26.563	0.449	1.094

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	169	264	181	570	270	170	166
normalized size	1	1.00	1.01	1.57	1.08	3.39	1.61	1.01	0.99
time (sec)	N/A	0.242	0.159	0.015	3.073	0.795	70.490	0.449	1.027

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	196	300	202	628	284	198	192
normalized size	1	1.00	1.00	1.53	1.03	3.20	1.45	1.01	0.98
time (sec)	N/A	0.350	0.124	0.018	2.978	0.567	150.751	0.439	1.043

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	234	351	247	678	0	250	230
normalized size	1	1.00	1.00	1.50	1.06	2.90	0.00	1.07	0.98
time (sec)	N/A	0.486	0.147	0.019	3.026	0.853	0.000	0.466	1.046

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	276	401	291	772	0	301	268
normalized size	1	1.00	1.00	1.45	1.05	2.79	0.00	1.09	0.97
time (sec)	N/A	0.603	0.158	0.022	3.090	0.510	0.000	0.403	1.070

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	158	193	347	177	442	264	186
normalized size	1	1.00	0.74	0.90	1.62	0.83	2.07	1.23	0.87
time (sec)	N/A	0.252	0.174	0.008	1.401	0.716	4.676	0.449	1.191

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	122	145	263	134	340	197	146
normalized size	1	1.00	0.73	0.87	1.57	0.80	2.04	1.18	0.87
time (sec)	N/A	0.194	0.123	0.008	1.449	0.659	2.905	0.508	1.107

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	99	180	94	238	130	103
normalized size	1	1.00	0.74	0.82	1.49	0.78	1.97	1.07	0.85
time (sec)	N/A	0.133	0.085	0.004	1.349	0.644	2.189	0.407	1.064

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	86	134	122	205	102	127	99
normalized size	1	1.00	0.83	1.30	1.18	1.99	0.99	1.23	0.96
time (sec)	N/A	0.140	0.130	0.009	1.346	0.709	37.870	0.398	1.810

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	131	127	104	210	138	114	99
normalized size	1	1.00	1.31	1.27	1.04	2.10	1.38	1.14	0.99
time (sec)	N/A	0.203	0.412	0.010	1.327	0.687	127.426	0.554	1.947

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	141	162	128	221	0	141	133
normalized size	1	1.00	1.24	1.42	1.12	1.94	0.00	1.24	1.17
time (sec)	N/A	0.233	0.374	0.011	1.379	0.868	0.000	0.397	2.191

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	162	238	193	261	0	232	199
normalized size	1	1.00	1.11	1.63	1.32	1.79	0.00	1.59	1.36
time (sec)	N/A	0.276	1.024	0.013	1.388	0.583	0.000	0.391	2.543

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	140	320	275	341	0	361	277
normalized size	1	1.00	0.72	1.64	1.41	1.75	0.00	1.85	1.42
time (sec)	N/A	0.350	0.336	0.016	1.346	0.997	0.000	0.397	2.913

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	184	368	339	414	586	224	-1
normalized size	1	1.00	0.75	1.50	1.38	1.69	2.39	0.91	-0.00
time (sec)	N/A	0.258	0.238	0.024	1.459	0.989	42.122	0.527	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	149	284	255	329	444	175	-1
normalized size	1	1.00	0.77	1.46	1.31	1.70	2.29	0.90	-0.01
time (sec)	N/A	0.208	0.167	0.010	1.387	0.790	32.562	0.537	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	118	203	174	250	362	129	-1
normalized size	1	1.00	0.81	1.40	1.20	1.72	2.50	0.89	-0.01
time (sec)	N/A	0.119	0.114	0.007	1.350	0.704	13.711	0.564	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	103	140	118	216	250	121	-1
normalized size	1	1.00	0.88	1.20	1.01	1.85	2.14	1.03	-0.01
time (sec)	N/A	0.136	0.118	0.010	1.259	0.614	9.052	0.522	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	93	117	102	210	197	176	143
normalized size	1	1.00	0.85	1.06	0.93	1.91	1.79	1.60	1.30
time (sec)	N/A	0.127	0.113	0.011	1.333	0.790	4.729	0.559	2.199

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	95	136	128	221	456	324	105
normalized size	1	1.00	0.81	1.15	1.08	1.87	3.86	2.75	0.89
time (sec)	N/A	0.133	0.113	0.011	1.327	0.788	6.207	0.597	1.724

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	103	111	193	100	891	554	124
normalized size	1	1.00	0.74	0.79	1.38	0.71	6.36	3.96	0.89
time (sec)	N/A	0.184	0.081	0.006	1.403	0.806	6.712	0.571	1.280

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	134	157	275	141	1642	667	171
normalized size	1	1.00	0.71	0.83	1.46	0.75	8.69	3.53	0.90
time (sec)	N/A	0.254	0.096	0.007	1.386	1.180	7.689	0.603	1.282

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	273	517	1221	987	0	342	-1
normalized size	1	1.00	0.72	1.36	3.20	2.59	0.00	0.90	-0.00
time (sec)	N/A	0.662	0.576	0.296	1.888	1.133	0.000	0.642	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	229	460	986	816	0	265	-1
normalized size	1	1.00	0.82	1.65	3.53	2.92	0.00	0.95	-0.00
time (sec)	N/A	0.455	0.479	0.011	1.787	0.942	0.000	0.593	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	194	405	753	653	0	203	-1
normalized size	1	1.00	0.92	1.93	3.59	3.11	0.00	0.97	-0.00
time (sec)	N/A	0.387	0.497	0.013	1.686	0.956	0.000	0.598	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	192	168	363	533	491	0	160	-1
normalized size	1	1.07	0.94	2.03	2.98	2.74	0.00	0.89	-0.01
time (sec)	N/A	0.314	0.456	0.010	1.661	0.760	0.000	0.550	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	98	109	335	141	0	131	-1
normalized size	1	1.00	0.73	0.81	2.50	1.05	0.00	0.98	-0.01
time (sec)	N/A	0.210	0.107	0.006	1.423	0.710	0.000	0.534	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	179	133	157	313	182	0	211	-1
normalized size	1	0.97	0.72	0.85	1.69	0.98	0.00	1.14	-0.01
time (sec)	N/A	0.246	0.184	0.008	1.471	0.943	0.000	0.575	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	165	205	337	225	0	349	-1
normalized size	1	1.00	0.68	0.85	1.39	0.93	0.00	1.44	-0.00
time (sec)	N/A	0.322	0.134	0.010	1.468	1.000	0.000	0.557	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	275	202	253	398	270	0	592	405
normalized size	1	0.98	0.72	0.90	1.42	0.96	0.00	2.11	1.44
time (sec)	N/A	0.429	0.158	0.009	1.545	1.208	0.000	0.632	2.397

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	234	301	489	311	0	938	421
normalized size	1	1.00	0.70	0.90	1.46	0.93	0.00	2.81	1.26
time (sec)	N/A	0.480	0.161	0.010	1.518	1.900	0.000	0.712	2.836

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	380	270	349	579	354	0	1162	-1
normalized size	1	0.97	0.69	0.89	1.48	0.90	0.00	2.96	-0.00
time (sec)	N/A	0.546	0.180	0.010	1.574	2.308	0.000	0.736	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	158	193	347	177	442	264	186
normalized size	1	1.00	0.74	0.90	1.62	0.83	2.07	1.23	0.87
time (sec)	N/A	0.222	0.185	0.007	1.435	0.547	9.372	0.454	1.203

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	122	145	263	134	340	197	146
normalized size	1	1.00	0.73	0.87	1.57	0.80	2.04	1.18	0.87
time (sec)	N/A	0.175	0.131	0.007	1.367	0.674	5.409	0.498	1.140

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	99	180	94	238	130	103
normalized size	1	1.00	0.74	0.82	1.49	0.78	1.97	1.07	0.85
time (sec)	N/A	0.150	0.090	0.006	1.355	0.670	3.406	0.398	1.084

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	257	221	478	826	705	0	224	-1
normalized size	1	0.98	0.85	1.83	3.16	2.70	0.00	0.86	-0.00
time (sec)	N/A	0.716	0.590	0.011	1.747	0.919	0.000	0.559	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	250	197	427	597	567	0	204	-1
normalized size	1	1.17	0.92	2.00	2.79	2.65	0.00	0.95	-0.00
time (sec)	N/A	0.410	0.514	0.009	1.743	1.175	0.000	0.590	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	138	166	421	187	0	220	-1
normalized size	1	1.00	0.72	0.86	2.18	0.97	0.00	1.14	-0.01
time (sec)	N/A	0.341	0.189	0.007	1.540	1.023	0.000	0.602	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [13] had the largest ratio of [.4000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	20	0.250
2	A	5	5	1.00	20	0.250
3	A	4	4	1.00	18	0.222
4	A	4	4	1.00	17	0.235
5	A	7	7	1.00	20	0.350
6	A	7	7	1.00	20	0.350
7	A	7	7	1.00	20	0.350
8	A	7	5	1.00	20	0.250
9	A	6	5	1.00	20	0.250
10	A	5	4	1.00	18	0.222
11	A	5	4	1.00	17	0.235
12	A	8	7	1.00	20	0.350
13	A	8	8	1.00	20	0.400
14	A	8	7	1.00	20	0.350
15	A	8	5	1.00	20	0.250
16	A	7	5	1.00	20	0.250
17	A	6	4	1.00	18	0.222
18	A	6	4	1.00	17	0.235
19	A	9	7	1.00	20	0.350
20	A	9	8	1.00	20	0.400

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	9	8	1.00	20	0.400
22	A	5	4	1.00	20	0.200
23	A	4	4	1.00	20	0.200
24	A	3	3	1.00	18	0.167
25	A	3	3	1.00	17	0.176
26	A	6	6	1.00	20	0.300
27	A	4	4	1.00	20	0.200
28	A	5	5	1.00	20	0.250
29	A	4	4	1.00	20	0.200
30	A	4	4	1.00	20	0.200
31	A	3	3	1.00	18	0.167
32	A	1	1	1.00	17	0.059
33	A	5	5	1.00	20	0.250
34	A	5	5	1.00	20	0.250
35	A	6	6	1.00	20	0.300
36	A	4	4	1.00	20	0.200
37	A	2	2	1.00	20	0.100
38	A	2	2	0.94	18	0.111
39	A	2	2	1.00	17	0.118
40	A	6	5	1.00	20	0.250
41	A	6	5	1.00	20	0.250
42	A	7	6	1.00	20	0.300
43	A	2	2	1.00	18	0.111
44	A	3	3	1.00	19	0.158
45	A	2	2	1.00	13	0.154
46	A	2	2	1.00	13	0.154
47	A	7	5	1.00	25	0.200
48	A	6	5	1.00	25	0.200
49	A	5	4	1.00	25	0.160
50	A	4	4	1.00	25	0.160
51	A	4	4	1.00	25	0.160
52	A	4	4	1.00	25	0.160
53	A	4	4	1.00	23	0.174
54	A	5	4	1.00	22	0.182
55	A	8	6	1.00	25	0.240
56	A	8	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	9	6	1.00	25	0.240
58	A	2	2	1.00	16	0.125
59	A	3	2	1.00	20	0.100
60	A	2	2	1.00	22	0.091
61	A	5	3	1.00	25	0.120
62	A	2	1	1.00	26	0.038
63	A	2	1	1.00	26	0.038
64	A	2	1	1.00	24	0.042
65	A	2	1	1.00	23	0.043
66	A	2	1	1.00	26	0.038
67	A	2	1	1.00	26	0.038
68	A	2	1	1.00	26	0.038
69	A	2	1	1.00	26	0.038
70	A	2	1	1.00	28	0.036
71	A	2	1	1.00	28	0.036
72	A	3	2	1.00	26	0.077
73	A	3	2	1.00	25	0.080
74	A	3	2	1.00	28	0.071
75	A	3	2	1.00	28	0.071
76	A	2	1	1.00	28	0.036
77	A	2	1	1.00	28	0.036
78	A	2	1	1.00	28	0.036
79	A	2	1	1.00	28	0.036
80	A	3	2	1.00	26	0.077
81	A	3	2	1.00	25	0.080
82	A	3	2	1.00	28	0.071
83	A	3	2	1.00	28	0.071
84	A	2	1	1.00	28	0.036
85	A	2	1	1.00	28	0.036
86	A	5	4	1.00	28	0.143
87	A	5	4	1.00	28	0.143
88	A	5	4	1.00	28	0.143
89	A	5	4	1.00	26	0.154
90	A	5	4	1.00	25	0.160
91	A	5	4	1.00	28	0.143
92	A	5	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	5	4	1.00	28	0.143
94	A	6	5	1.00	28	0.179
95	A	6	5	1.00	28	0.179
96	A	6	5	1.00	28	0.179
97	A	6	5	1.00	26	0.192
98	A	4	4	1.00	25	0.160
99	A	6	5	1.00	28	0.179
100	A	6	5	1.00	28	0.179
101	A	6	5	1.00	28	0.179
102	A	7	5	1.00	28	0.179
103	A	6	5	1.00	28	0.179
104	A	5	4	1.00	28	0.143
105	A	4	4	1.00	26	0.154
106	A	3	3	1.00	25	0.120
107	A	7	6	1.00	28	0.214
108	A	7	5	1.00	28	0.179
109	A	7	5	1.00	28	0.179
110	A	4	3	1.00	17	0.176
111	A	4	3	1.00	17	0.176
112	A	4	4	1.00	21	0.190
113	A	6	5	1.00	15	0.333
114	A	3	2	1.00	30	0.067
115	A	3	2	1.00	30	0.067
116	A	3	2	1.00	30	0.067
117	A	3	2	1.00	27	0.074
118	A	3	2	1.00	30	0.067
119	A	3	2	1.00	30	0.067
120	A	3	2	1.00	30	0.067
121	A	3	2	1.00	30	0.067
122	A	3	2	1.00	30	0.067
123	A	3	2	1.00	30	0.067
124	A	5	4	1.00	30	0.133
125	A	5	4	1.00	30	0.133
126	A	5	4	1.00	30	0.133
127	A	4	3	1.00	27	0.111
128	A	4	3	1.00	30	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	4	3	1.00	30	0.100
130	A	4	3	1.00	30	0.100
131	A	4	3	1.00	30	0.100
132	A	4	3	1.00	30	0.100
133	A	6	5	1.00	30	0.167
134	A	6	5	1.00	30	0.167
135	A	6	5	1.00	30	0.167
136	A	6	5	1.00	30	0.167
137	A	4	4	1.00	27	0.148
138	A	4	4	1.00	30	0.133
139	A	5	4	1.00	30	0.133
140	A	5	3	1.00	30	0.100
141	A	5	3	1.00	30	0.100
142	A	5	3	1.00	30	0.100
143	A	3	2	1.00	32	0.062
144	A	3	2	1.00	32	0.062
145	A	3	2	1.00	30	0.067
146	A	5	4	1.00	32	0.125
147	A	6	5	1.00	32	0.156
148	A	6	6	1.00	32	0.188
149	A	6	6	1.00	32	0.188
150	A	7	7	1.00	32	0.219
151	A	7	6	1.00	32	0.188
152	A	6	6	1.00	32	0.188
153	A	5	5	1.00	29	0.172
154	A	6	6	1.00	32	0.188
155	A	6	6	1.00	32	0.188
156	A	6	6	1.00	32	0.188
157	A	5	3	1.00	32	0.094
158	A	6	4	1.00	32	0.125
159	A	11	9	1.00	32	0.281
160	A	10	9	1.00	32	0.281
161	A	9	9	1.00	32	0.281
162	A	8	8	1.07	32	0.250
163	A	5	4	1.00	29	0.138
164	A	6	5	0.97	32	0.156

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	7	5	1.00	32	0.156
166	A	8	4	0.98	32	0.125
167	A	9	5	1.00	32	0.156
168	A	10	5	0.97	32	0.156
169	A	4	3	1.00	33	0.091
170	A	4	3	1.00	33	0.091
171	A	4	3	1.00	31	0.097
172	A	10	9	0.98	37	0.243
173	A	6	5	1.17	34	0.147
174	A	6	4	1.00	37	0.108

Chapter 3

Listing of integrals

3.1 $\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=127

$$\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}-\frac{a\left(a+bx^2\right)^{3/2}(16A+15Bx)}{120b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

```
[Out] 1/5*A*x^2*(b*x^2+a)^(3/2)/b+1/6*B*x^3*(b*x^2+a)^(3/2)/b-1/120*a*(15*B*x+16*
A)*(b*x^2+a)^(3/2)/b^2+1/16*a^3*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2
)+1/16*a^2*B*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {833, 780, 195, 217, 206}

$$\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}-\frac{a\left(a+bx^2\right)^{3/2}(16A+15Bx)}{120b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*B*x*Sqrt[a + b*x^2])/(16*b^2) + (A*x^2*(a + b*x^2)^(3/2))/(5*b) + (B*x
^3*(a + b*x^2)^(3/2))/(6*b) - (a*(16*A + 15*B*x)*(a + b*x^2)^(3/2))/(120*b^
2) + (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3(A+Bx)\sqrt{a+bx^2}\,dx &= \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}+\frac{\int x^2(-3aB+6Abx)\sqrt{a+bx^2}\,dx}{6b}\\
&= \frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}+\frac{\int x(-12aAb-15abBx)\sqrt{a+bx^2}\,dx}{30b^2}\\
&= \frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{a(16A+15Bx)\left(a+bx^2\right)^{3/2}}{120b^2}+\frac{\left(a^2B\right)\int\sqrt{a+}}{8b^2}\\
&= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{a(16A+15Bx)\left(a+bx^2\right)^{3/}}{120b^2}\\
&= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{a(16A+15Bx)\left(a+bx^2\right)^{3/}}{120b^2}\\
&= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{a(16A+15Bx)\left(a+bx^2\right)^{3/}}{120b^2}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 107, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\frac{15a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}\left(-a^2(32A+15Bx)+2abx^2(8A+5Bx)+8b^2x^4(6A+5Bx)\right)\right)}{240b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(8*b^2*x^4*(6*A + 5*B*x) + 2*a*b*x^2*(8*A + 5*B*x
) - a^2*(32*A + 15*B*x)) + (15*a^(5/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt
[1 + (b*x^2)/a]))/(240*b^(5/2))
```

fricas [A] time = 0.94, size = 206, normalized size = 1.62

$$\left[\frac{15\,Ba^3\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(40\,Bb^3x^5+48\,Ab^3x^4+10\,Bab^2x^3+16\,Aab^2x^2-15\,Ba^2bx-\right.}{480\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*x^3 + 16*A*a*b^2*x^2 - 15*B*a^2
*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3, -1/240*(15*B*a^3*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*x^3
+ 16*A*a*b^2*x^2 - 15*B*a^2*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.51, size = 93, normalized size = 0.73

$$-\frac{Ba^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}+\frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\,(5\,Bx+6\,A)x+\frac{5\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)x-\frac{15\,Ba^2}{b^2}\right)x-\frac{32\,A\ldots}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/240*sqrt(b*x
^2 + a)*((2*((4*(5*B*x + 6*A)*x + 5*B*a/b)*x + 8*A*a/b)*x - 15*B*a^2/b^2)*x
 - 32*A*a^2/b^2)
```

maple [A] time = 0.01, size = 115, normalized size = 0.91

$$\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B\,x^3}{6b}+\frac{B\,a^3\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{5}{2}}}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}A\,x^2}{5b}+\frac{\sqrt{b\,x^2+a}\;B\,a^2x}{16b^2}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Bax}{8b^2}-\frac{2\left(b\,x^2+\ldots\right.}{15\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*B*x^3*(b*x^2+a)^(3/2)/b-1/8*B*a/b^2*x*(b*x^2+a)^(3/2)+1/16*a^2*B*x*(b*x
^2+a)^(1/2)/b^2+1/16*B*a^3/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/5*A*x^2*
(b*x^2+a)^(3/2)/b-2/15*A*a/b^2*(b*x^2+a)^(3/2)
```

maxima [A] time = 1.41, size = 107, normalized size = 0.84

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx^3}{6\,b}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ax^2}{5\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bax}{8\,b^2}+\frac{\sqrt{bx^2+a}\,Ba^2x}{16\,b^2}+\frac{Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}-\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Aa}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(3/2)*B*x^3/b + 1/5*(b*x^2 + a)^(3/2)*A*x^2/b - 1/8*(b*x^2
+ a)^(3/2)*B*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*B*a^2*x/b^2 + 1/16*B*a^3*arcsin
h(b*x/sqrt(a*b))/b^(5/2) - 2/15*(b*x^2 + a)^(3/2)*A*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\sqrt{b\,x^2+a}\;\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] int(x^3*(a + b*x^2)^(1/2)*(A + B*x), x)
```

sympy [A] time = 16.25, size = 192, normalized size = 1.51

$$A\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)-\frac{Ba^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{5B\sqrt{a}\,x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^3\operatorname{asinh}\big(}{16b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(
15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) - B*a**
(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - B*a**(3/2)*x**3/(48*b*sqrt(1 + b*x**
2/a)) + 5*B*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + B*a**3*asinh(sqrt(b)*x/s
qrt(a))/(16*b**(5/2)) + B*b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.2 $\int x^2(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=104

$$-\frac{a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}-\frac{\left(a+bx^2\right)^{3/2}(8aB-15Abx)}{60b^2}-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}$$

```
[Out] 1/5*B*x^2*(b*x^2+a)^(3/2)/b-1/60*(-15*A*b*x+8*B*a)*(b*x^2+a)^(3/2)/b^2-1/8*
a^2*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/8*a*A*x*(b*x^2+a)^(1/2)/
b
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {833, 780, 195, 217, 206}

$$-\frac{a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}-\frac{\left(a+bx^2\right)^{3/2}(8aB-15Abx)}{60b^2}-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -(a*A*x*Sqrt[a + b*x^2])/(8*b) + (B*x^2*(a + b*x^2)^(3/2))/(5*b) - ((8*a*B
- 15*A*b*x)*(a + b*x^2)^(3/2))/(60*b^2) - (a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(8*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
```

```
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx = \frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{\int x(-2aB+5Abx)\sqrt{a+bx^2}\,dx}{5b}$$
$$= \frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{(aA)\int\sqrt{a+bx^2}\,dx}{4b}$$
$$= -\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{\left(a^2A\right)\int\frac{1}{\sqrt{a+bx^2}}}{8b}$$
$$= -\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{\left(a^2A\right)\text{Subst}\Big(}{}$$
$$= -\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{a^2A\tanh^{-1}\Big(}{8b^{3/2}}$$

Mathematica [A] time = 0.19, size = 93, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(-\frac{15a^{3/2}A\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}-16a^2B+abx(15A+8Bx)+6b^2x^3(5A+4Bx)\right)}{120b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^2*B + 6*b^2*x^3*(5*A + 4*B*x) + a*b*x*(15*A + 8*B*x
) - (15*a^(3/2)*A*Sqrt[b]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]
))/(120*b^2)
```

fricas [A] time = 0.63, size = 175, normalized size = 1.68

$$\left[\frac{15\,Aa^2\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(24\,Bb^2x^4+30\,Ab^2x^3+8\,Babx^2+15\,Aabx-16\,Ba^2\right)\sqrt{bx^2+}}{240\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*A*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqrt(
b*x^2 + a))/b^2, 1/120*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) + (24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqr
t(b*x^2 + a))/b^2]
```

giac [A] time = 0.47, size = 81, normalized size = 0.78

$$\frac{Aa^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}+\frac{1}{120}\,\sqrt{bx^2+a}\left(\left(2\left(3\,(4\,Bx+5\,A)x+\frac{4\,Ba}{b}\right)x+\frac{15\,Aa}{b}\right)x-\frac{16\,Ba^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/120*sqrt(b*x^2
 + a)*((2*(3*(4*B*x + 5*A)*x + 4*B*a/b)*x + 15*A*a/b)*x - 16*B*a^2/b^2)
```

maple [A] time = 0.01, size = 94, normalized size = 0.90

$$-\frac{A a^{2} \ln \left(\sqrt{b}\, x+\sqrt{b\,x^{2}+a}\right)}{8 b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{2}+a}\; A a x}{8 b}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} B\,x^{2}}{5 b}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} A x}{4 b}-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{2}} B a}{15 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*B*x^2*(b*x^2+a)^(3/2)/b-2/15*B*a/b^2*(b*x^2+a)^(3/2)+1/4*A*x*(b*x^2+a)^
(3/2)/b-1/8*a*A*x*(b*x^2+a)^(1/2)/b-1/8*A*a^2/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2))
```

maxima [A] time = 1.36, size = 86, normalized size = 0.83

$$\frac{\left(bx^{2}+a\right)^{\frac{3}{2}} Bx^{2}}{5\,b}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}} Ax}{4\,b}-\frac{\sqrt{bx^{2}+a}\,Aax}{8\,b}-\frac{Aa^{2} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}-\frac{2\left(bx^{2}+a\right)^{\frac{3}{2}} Ba}{15\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^2 + a)^(3/2)*B*x^2/b + 1/4*(b*x^2 + a)^(3/2)*A*x/b - 1/8*sqrt(b*x^
2 + a)*A*a*x/b - 1/8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/15*(b*x^2 + a
)^(3/2)*B*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\,\sqrt{b\,x^{2}+a}\;\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(1/2)*(A + B*x), x)
```

sympy [A] time = 10.14, size = 165, normalized size = 1.59

$$\frac{Aa^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3A\sqrt{a}\,x^{3}}{8\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Aa^{2}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Abx^{5}}{4\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}+B\left(\begin{cases}-\frac{2a^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{ax^{2}\sqrt{a+bx^{2}}}{15b}+\frac{x^{4}\sqrt{a+bx^{2}}}{5} & \text{for } b \\ \frac{\sqrt{a}\,x^{4}}{4} & \text{other}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*x**3/(8*sqrt(1 + b*x**2
/a)) - A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + A*b*x**5/(4*sqrt(a)*s
qrt(1 + b*x**2/a)) + B*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x*
*2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x
**4/4, True))
```

3.3 $\int x(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(4A+3Bx)}{12b}-\frac{aBx\sqrt{a+bx^2}}{8b}$$

```
[Out] 1/12*(3*B*x+4*A)*(b*x^2+a)^(3/2)/b-1/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))/b^(3/2)-1/8*a*B*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {780, 195, 217, 206}

$$-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(4A+3Bx)}{12b}-\frac{aBx\sqrt{a+bx^2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -(a*B*x*Sqrt[a + b*x^2])/(8*b) + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/(12*b) -
 (a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rubi steps

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx = \frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b} - \frac{(aB)\int\sqrt{a+bx^2}\,dx}{4b}$$

$$= -\frac{aBx\sqrt{a+bx^2}}{8b} + \frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b} - \frac{\left(a^2B\right)\int\frac{1}{\sqrt{a+bx^2}}\,dx}{8b}$$

$$= -\frac{aBx\sqrt{a+bx^2}}{8b} + \frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b} - \frac{\left(a^2B\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b}$$

$$= -\frac{aBx\sqrt{a+bx^2}}{8b} + \frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b} - \frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 1.08

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\left(8aA+3aBx+8Abx^2+6bBx^3\right)-\frac{3a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{24b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(8*a*A + 3*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) - (3*a^
(3/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(24*b^(3/2))
```

fricas [A] time = 0.58, size = 157, normalized size = 1.96

$$\left[\frac{3\,Ba^2\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(6\,Bb^2x^3+8\,Ab^2x^2+3\,Babx+8\,Aab\right)\sqrt{bx^2+a}}{48\,b^2}, \frac{3\,Ba^2\sqrt{-b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/48*(3*B*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(6*B*b^2*x^3 + 8*A*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2, 1/2
4*(3*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (6*B*b^2*x^3 + 8*A
*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.44, size = 68, normalized size = 0.85

$$\frac{Ba^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}+\frac{1}{24}\,\sqrt{bx^2+a}\left(\left(2\,(3\,Bx+4\,A)x+\frac{3\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/24*sqrt(b*x^2
+ a)*((2*(3*B*x + 4*A)*x + 3*B*a/b)*x + 8*A*a/b)
```

maple [A] time = 0.00, size = 75, normalized size = 0.94

$$-\frac{B\,a^2\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}\;Bax}{8b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Bx}{4b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}A}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x*(b*x^2+a)^(3/2)/b-1/8*a*B*x*(b*x^2+a)^(1/2)/b-1/8*B*a^2/b^(3/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))+1/3*A*(b*x^2+a)^(3/2)/b
```

maxima [A] time = 1.32, size = 67, normalized size = 0.84

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx}{4\,b}-\frac{\sqrt{bx^2+a}\,Bax}{8\,b}-\frac{Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*B*x/b - 1/8*sqrt(b*x^2 + a)*B*a*x/b - 1/8*B*a^2*arcsi
nh(b*x/sqrt(a*b))/b^(3/2) + 1/3*(b*x^2 + a)^(3/2)*A/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{b\,x^2+a}\,\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] int(x*(a + b*x^2)^(1/2)*(A + B*x), x)
```

sympy [A] time = 14.41, size = 124, normalized size = 1.55

$$A\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}\,x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Bbx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
+ B*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*x**3/(8*sqrt(1 + b*x*
*2/a)) - B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + B*b*x**5/(4*sqrt(a)
*sqrt(1 + b*x**2/a))
```

3.4 $\int (A + Bx)\sqrt{a + bx^2}\, dx$

Optimal. Leaf size=67

$$\frac{1}{2}Ax\sqrt{a + bx^2} + \frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} + \frac{B\left(a + bx^2\right)^{3/2}}{3b}$$

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b+1/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+
1/2*A*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {641, 195, 217, 206}

$$\frac{1}{2}Ax\sqrt{a + bx^2} + \frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} + \frac{B\left(a + bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (A*x*Sqrt[a + b*x^2])/2 + (B*(a + b*x^2)^(3/2))/(3*b) + (a*A*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (A+Bx)\sqrt{a+bx^2}\,dx &= \frac{B\left(a+bx^2\right)^{3/2}}{3b}+A\int \sqrt{a+bx^2}\,dx\\ &= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{1}{2}(aA)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{1}{2}(aA)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\,(2aB+bx(3A+2Bx))+3aA\sqrt{b}\,\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(2*a*B + b*x*(3*A + 2*B*x)) + 3*a*A*Sqrt[b]*Log[b*x + Sqrt
[b]*Sqrt[a + b*x^2]])/(6*b)
```

fricas [A] time = 1.20, size = 128, normalized size = 1.91

$$\left[\frac{3\,Aa\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,Bbx^2+3\,Abx+2\,Ba\right)\sqrt{bx^2+a}}{12\,b},-\frac{3\,Aa\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a))/b, -1/6*(3*A*a*sqrt(-b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a
))/b]
```

giac [A] time = 0.41, size = 55, normalized size = 0.82

$$-\frac{Aa\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}+\frac{1}{6}\,\sqrt{bx^2+a}\left((2\,Bx+3\,A)x+\frac{2\,Ba}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*A*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/6*sqrt(b*x^2 +
a)*((2*B*x + 3*A)*x + 2*B*a/b)
```

maple [A] time = 0.01, size = 53, normalized size = 0.79

$$\frac{Aa\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\;Ax}{2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b+1/2*A*x*(b*x^2+a)^(1/2)+1/2*A*a/b^(1/2)*ln(b^(1/2)*
x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 45, normalized size = 0.67

$$\frac{1}{2}\sqrt{bx^2+a}\,Ax + \frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}B}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*A*x + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/3*(b*x
^2 + a)^(3/2)*B/b
```

mupad [B] time = 1.16, size = 52, normalized size = 0.78

$$\frac{B\left(b\,x^2+a\right)^{3/2}}{3\,b} + \frac{A\,x\,\sqrt{b\,x^2+a}}{2} + \frac{A\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] (B*(a + b*x^2)^(3/2))/(3*b) + (A*x*(a + b*x^2)^(1/2))/2 + (A*a*log(b^(1/2)*
x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

sympy [A] time = 6.51, size = 70, normalized size = 1.04

$$\frac{A\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{Aa\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}} + B\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + A*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
 + B*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True
))
```

3.5 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}\sqrt{a+bx^2}\,(2A+Bx)-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] -A*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*a*B*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))/b^(1/2)+1/2*(B*x+2*A)*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {815, 844, 217, 206, 266, 63, 208}

$$\frac{1}{2}\sqrt{a+bx^2}\,(2A+Bx)-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/2 + (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/(2*Sqrt[b]) - Sqrt[a]*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
```

```
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx &= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{\int \frac{2aAb+abBx}{x\sqrt{a+bx^2}}\,dx}{2b}\\ &= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+(aA)\int \frac{1}{x\sqrt{a+bx^2}}\,dx+\frac{1}{2}(aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{1}{2}(aA)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)+\frac{1}{2}(aB)\,\text{Subst}\left(\int\right.\\ &= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{(aA)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b}\right.}{b}\\ &= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.25, size = 100, normalized size = 1.27

$$\frac{1}{2}\left(\frac{a^{3/2}B\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{a+bx^2}}+\sqrt{a+bx^2}\,(2A+Bx)-2\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2] + (a^(3/2)*B*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt
[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[a + b*x^2]) - 2*Sqrt[a]*A*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/2
```

fricas [A] time = 0.99, size = 341, normalized size = 4.32

$$\left[\frac{Ba\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,A\sqrt{a}\,b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(Bbx+2\,Ab)\sqrt{bx^2+a}}{4\,b},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*A*sqr
t(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B*b*x + 2*A
*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) - A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (B*b
*x + 2*A*b)*sqrt(b*x^2 + a))/b, 1/4*(4*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - 2*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*b*x
+ 2*A*b)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.45, size = 78, normalized size = 0.99

$$\frac{2\,Aa\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{Ba\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}+\frac{1}{2}\,\sqrt{bx^2+a}\,(Bx+2\,A)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 1/2*B*a*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/2*sqrt(b*x^2 + a)*(B*x + 2
*A)
```

maple [A] time = 0.01, size = 78, normalized size = 0.99

$$-A\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\frac{Ba\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\,Bx}{2}+\sqrt{b\,x^2+a}\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x,x)
```

```
[Out] 1/2*B*x*(b*x^2+a)^(1/2)+1/2*B*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-A*a^(
1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+A*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.36, size = 59, normalized size = 0.75

$$\frac{1}{2}\,\sqrt{bx^2+a}\,Bx+\frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}-A\sqrt{a}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\sqrt{bx^2+a}\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x + 1/2*B*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*sqrt(a
)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)*A
```

mupad [B] time = 1.24, size = 68, normalized size = 0.86

$$A\,\sqrt{b\,x^2+a}-A\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+\frac{B\,x\,\sqrt{b\,x^2+a}}{2}+\frac{B\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x,x)
```

```
[Out] A*(a + b*x^2)^(1/2) - A*a^(1/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + (B*x*(a
+ b*x^2)^(1/2))/2 + (B*a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

<u>**sympy**</u> **[A]** time = 8.80, size = 107, normalized size = 1.35

$$-A\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Aa}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{A\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}+\frac{B\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)
) + A*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + B
*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

3.6 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{a+bx^2}\,(A-Bx)}{x}+A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -B*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+A*arctanh(x*b^(1/2)/(b*x^2+a)^(
1/2))*b^(1/2)-(-B*x+A)*(b*x^2+a)^(1/2)/x
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {813, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a+bx^2}\,(A-Bx)}{x}+A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] -(((A - B*x)*Sqrt[a + b*x^2])/x) + A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
```

```
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = -\frac{(A-Bx)\sqrt{a+bx^2}}{x} - \frac{1}{2}\int \frac{-2aB-2Abx}{x\sqrt{a+bx^2}}\,dx$$
$$= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + (Ab)\int \frac{1}{\sqrt{a+bx^2}}\,dx + (aB)\int \frac{1}{x\sqrt{a+bx^2}}\,dx$$
$$= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + (Ab)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) + \frac{1}{2}(aB)\,\text{Subst}\Big(\int$$
$$= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right) + \frac{(aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{b}$$
$$= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right) - \sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.18, size = 99, normalized size = 1.32

$$\frac{\sqrt{a+bx^2}\,(Bx-A)}{x} + \frac{\sqrt{a}\,A\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a+bx^2}} - \sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] ((-A + B*x)*Sqrt[a + b*x^2])/x + (Sqrt[a]*A*Sqrt[b]*Sqrt[1 + (b*x^2)/a]*Arc
Sinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a + b*x^2] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]]
```

fricas [A] time = 1.38, size = 333, normalized size = 4.44

$$\left[\frac{A\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+B\sqrt{a}\,x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}\,(Bx-A)}{2\,x}, -\frac{2\,A\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + B*sqrt(
a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(b*x^2 + a
)*(B*x - A))/x, -1/2*(2*A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - B
*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(b*x
^2 + a)*(B*x - A))/x, 1/2*(2*B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a))
+ A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b*x^
2 + a)*(B*x - A))/x, -(A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - B*
sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*(B*x - A))/x]
```

giac [A] time = 0.47, size = 102, normalized size = 1.36

$$\frac{2\,Ba\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-A\sqrt{b}\,\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)+\sqrt{bx^2+a}\,B+\frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - A*sqrt(b)*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + sqrt(b*x^2 + a)*B + 2*A*a*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.01, size = 97, normalized size = 1.29

$$A\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)-B\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\frac{\sqrt{b\,x^2+a}\,Abx}{a}+\sqrt{b\,x^2+a}\,B-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] -A/a/x*(b*x^2+a)^(3/2)+A/a*b*x*(b*x^2+a)^(1/2)+A*b^(1/2)*ln(b^(1/2)*x+(b*x^
2+a)^(1/2))-B*a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+B*(b*x^2+a)^(1/
2)
```

maxima [A] time = 1.39, size = 59, normalized size = 0.79

$$A\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-B\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\sqrt{bx^2+a}\,B-\frac{\sqrt{bx^2+a}\,A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] A*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - B*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x)))
+ sqrt(b*x^2 + a)*B - sqrt(b*x^2 + a)*A/x
```

mupad [B] time = 1.69, size = 89, normalized size = 1.19

$$B\,\sqrt{b\,x^2+a}-\frac{A\,\sqrt{b\,x^2+a}}{x}-B\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)-\frac{A\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x^2,x)
```

```
[Out] B*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(1/2))/x - B*a^(1/2)*atanh((a + b*x^2)
^(1/2)/a^(1/2)) - (A*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(1/2)
*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

sympy [A] time = 11.97, size = 124, normalized size = 1.65

$$-\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+A\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Abx}{\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}-B\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Ba}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{B\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A*
b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + B
*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

3.7 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{a+bx^2}\,(A+2Bx)}{2x^2}-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -1/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+B*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))*b^(1/2)-1/2*(2*B*x+A)*(b*x^2+a)^(1/2)/x^2
```

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = $\{811, 844, 217, 206, 266, 63, 208\}$

$$-\frac{\sqrt{a+bx^2}\,(A+2Bx)}{2x^2}-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] -((A + 2*B*x)*Sqrt[a + b*x^2])/(2*x^2) + Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]] - (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
```

```
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx &= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}-\frac{\int \frac{-2aAb-4abBx}{x\sqrt{a+bx^2}}\,dx}{4a}\\ &= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\frac{1}{2}(Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx+(bB)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\frac{1}{4}(Ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)+(bB)\,\text{Subst}\left(\int \frac{}{1}\right.\\ &= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{1}{2}A\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\right.\\ &= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 108, normalized size = 1.35

$$-\frac{\sqrt{a+bx^2}\left(a\sqrt{\frac{bx^2}{a}+1}\,(A+2Bx)+Abx^2\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)-2\sqrt{a}\sqrt{b}\,Bx^2\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{2ax^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] -1/2*(Sqrt[a + b*x^2]*(a*(A + 2*B*x)*Sqrt[1 + (b*x^2)/a] - 2*Sqrt[a]*Sqrt[b
]*B*x^2*ArcSinh[(Sqrt[b]*x)/Sqrt[a]] + A*b*x^2*ArcTanh[Sqrt[1 + (b*x^2)/a]]
))/(a*x^2*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 0.96, size = 377, normalized size = 4.71

$$\left[\frac{2\,Ba\sqrt{b}\,x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+A\sqrt{a}\,bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Bax+Aa)\sqrt{bx^2+}}{4\,ax^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + A
*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*B
*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), -1/4*(4*B*a*sqrt(-b)*x^2*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(a) + 2*a)/x^2) + 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), 1/2*(A*sqrt
(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*x^2*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^
2), -1/2*(2*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(-a
)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*x^2 + a))
/(a*x^2)]
```

giac [B] time = 0.48, size = 163, normalized size = 2.04

$$\frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-B\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*sqrt(b)*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b
 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2
+ a))*A*a*b - 2*B*a^2*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

maple [A] time = 0.01, size = 121, normalized size = 1.51

$$-\frac{Ab\ln\left(\frac{2a+2\sqrt{bx^2+a}\,\sqrt{a}}{x}\right)}{2\sqrt{a}}+B\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)+\frac{\sqrt{bx^2+a}\,Bbx}{a}+\frac{\sqrt{bx^2+a}\,Ab}{2a}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}B}{ax}-\frac{\left(bx^2+a\right.}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2*A/a/x^2*(b*x^2+a)^(3/2)-1/2*A/a^(1/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)+1/2*A/a*b*(b*x^2+a)^(1/2)-B/a/x*(b*x^2+a)^(3/2)+B/a*b*x*(b*x^2+a)^(
1/2)+B*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.36, size = 83, normalized size = 1.04

$$B\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{Ab\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,\sqrt{a}}+\frac{\sqrt{bx^2+a}\,Ab}{2\,a}-\frac{\sqrt{bx^2+a}\,B}{x}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] B*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sq
rt(a) + 1/2*sqrt(b*x^2 + a)*A*b/a - sqrt(b*x^2 + a)*B/x - 1/2*(b*x^2 + a)^(
3/2)*A/(a*x^2)
```

mupad [B] time = 1.79, size = 94, normalized size = 1.18

$$-\frac{A\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{B\,\sqrt{b\,x^2+a}}{x}-\frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}-\frac{B\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x^3,x)
```

```
[Out] - (A*(a + b*x^2)^(1/2))/(2*x^2) - (B*(a + b*x^2)^(1/2))/x - (A*b*atanh((a +
 b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2)) - (B*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2
))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

sympy [A] time = 5.50, size = 107, normalized size = 1.34

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}-\frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Bbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*s
qrt(a)) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sqrt
(a)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

3.8 $\int x^3(A + Bx)\left(a + bx^2\right)^{3/2} dx$

Optimal. Leaf size=150

$$\frac{3a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}} + \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2} - \frac{a\left(a+bx^2\right)^{5/2}(32A+35Bx)}{560b^2} + \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b} + \frac{Bx}{}$$

```
[Out] 1/64*a^2*B*x*(b*x^2+a)^(3/2)/b^2+1/7*A*x^2*(b*x^2+a)^(5/2)/b+1/8*B*x^3*(b*x
^2+a)^(5/2)/b-1/560*a*(35*B*x+32*A)*(b*x^2+a)^(5/2)/b^2+3/128*a^4*B*arctanh
(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+3/128*a^3*B*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {833, 780, 195, 217, 206}

$$\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2} + \frac{3a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}} - \frac{a\left(a+bx^2\right)^{5/2}(32A+35Bx)}{560b^2} + \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b} + \frac{Bx}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a^3*B*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*B*x*(a + b*x^2)^(3/2))/(64*b^2
) + (A*x^2*(a + b*x^2)^(5/2))/(7*b) + (B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a*
(32*A + 35*B*x)*(a + b*x^2)^(5/2))/(560*b^2) + (3*a^4*B*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
```

```
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}dx = \frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}+\frac{\int x^2(-3aB+8Abx)\left(a+bx^2\right)^{3/2}dx}{8b}$$
$$=\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}+\frac{\int x(-16aAb-21abBx)\left(a+bx^2\right)^{3/2}d}{56b^2}$$
$$=\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2}+\frac{\left(a^2B\right)\int}{}$$
$$=\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{a(32A+35Bx)\left(a\right.}{560b^2}$$
$$=\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}$$
$$=\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}$$
$$=\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}$$

Mathematica [A] time = 0.26, size = 126, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\frac{105a^{7/2}B\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}\left(-a^3(256A+105Bx)+2a^2bx^2(64A+35Bx)+8ab^2x^4(128A+105Bx)+8\right.\right.}{4480b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(80*b^3*x^6*(8*A + 7*B*x) + 2*a^2*b*x^2*(64*A + 3
5*B*x) + 8*a*b^2*x^4*(128*A + 105*B*x) - a^3*(256*A + 105*B*x)) + (105*a^(7
/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(4480*b^(5/2))
```

fricas [A] time = 0.95, size = 254, normalized size = 1.69

$$\left[\frac{105\,Ba^4\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(560\,Bb^4x^7+640\,Ab^4x^6+840\,Bab^3x^5+1024\,Aab^3x^4+7\right.}{8960\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8960*(105*B*a^4*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^4 + 7
0*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sqrt(b*x
^2 + a))/b^3, -1/4480*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) - (560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^4 + 7
```

```
0*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sqrt(b*x
^2 + a))/b^3]
```

giac [A] time = 0.44, size = 115, normalized size = 0.77

$$-\frac{3\,Ba^4\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}-\frac{1}{4480}\,\sqrt{bx^2+a}\left(\frac{256\,Aa^3}{b^2}+\left(\frac{105\,Ba^3}{b^2}-2\left(\frac{64\,Aa^2}{b}+\left(\frac{35\,Ba^2}{b}+4\,(128\,Aa+5\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/4480*sqrt(b
*x^2 + a)*(256*A*a^3/b^2 + (105*B*a^3/b^2 - 2*(64*A*a^2/b + (35*B*a^2/b + 4
*(128*A*a + 5*(21*B*a + 2*(7*B*b*x + 8*A*b)*x)*x)*x)*x)*x)*x)
```

maple [A] time = 0.01, size = 134, normalized size = 0.89

$$\frac{3B\,a^4\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{128b^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^2+a}\;B\,a^3x}{128b^2}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}B\,x^3}{8b}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}A\,x^2}{7b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B\,a^2x}{64b^2}-\frac{\left(b\,x^2+\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/8*B*x^3*(b*x^2+a)^(5/2)/b-1/16*B*a/b^2*x*(b*x^2+a)^(5/2)+1/64*a^2*B*x*(b*
x^2+a)^(3/2)/b^2+3/128*a^3*B*x*(b*x^2+a)^(1/2)/b^2+3/128*B*a^4/b^(5/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))+1/7*A*x^2*(b*x^2+a)^(5/2)/b-2/35*A*a/b^2*(b*x^2+a
)^(5/2)
```

maxima [A] time = 1.39, size = 126, normalized size = 0.84

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bx^3}{8\,b}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ax^2}{7\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bax}{16\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ba^2x}{64\,b^2}+\frac{3\,\sqrt{bx^2+a}\,Ba^3x}{128\,b^2}+\frac{3\,Ba^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{2\,\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*B*x^3/b + 1/7*(b*x^2 + a)^(5/2)*A*x^2/b - 1/16*(b*x^2
 + a)^(5/2)*B*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*B*a^2*x/b^2 + 3/128*sqrt(b*x
^2 + a)*B*a^3*x/b^2 + 3/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/35*(b*
x^2 + a)^(5/2)*A*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] int(x^3*(a + b*x^2)^(3/2)*(A + B*x), x)
```

sympy [A] time = 20.79, size = 318, normalized size = 2.12

$$Aa\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+Ab\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7}\\ \frac{\sqrt{a}\,x^6}{6}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)
/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + A*b
*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**
2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, N
e(b, 0)), (sqrt(a)*x**6/6, True)) - 3*B*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**
2/a)) - B*a**(5/2)*x**3/(128*b*sqrt(1 + b*x**2/a)) + 13*B*a**(3/2)*x**5/(64
*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*b*x**7/(16*sqrt(1 + b*x**2/a)) + 3*B*a**
4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) + B*b**2*x**9/(8*sqrt(a)*sqrt(1 +
 b*x**2/a))
```

3.9 $\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{a^3A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{\left(a+bx^2\right)^{5/2}(12aB-35Abx)}{210b^2}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}$$

```
[Out] -1/24*a*A*x*(b*x^2+a)^(3/2)/b+1/7*B*x^2*(b*x^2+a)^(5/2)/b-1/210*(-35*A*b*x+
12*B*a)*(b*x^2+a)^(5/2)/b^2-1/16*a^3*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b
^(3/2)-1/16*a^2*A*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {833, 780, 195, 217, 206}

$$-\frac{a^3A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{\left(a+bx^2\right)^{5/2}(12aB-35Abx)}{210b^2}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^2*A*x*Sqrt[a + b*x^2])/(16*b) - (a*A*x*(a + b*x^2)^(3/2))/(24*b) + (B*x
^2*(a + b*x^2)^(5/2))/(7*b) - ((12*a*B - 35*A*b*x)*(a + b*x^2)^(5/2))/(210*
b^2) - (a^3*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
```

```
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b} + \frac{\int x(-2aB+7Abx)\left(a+bx^2\right)^{3/2}\,dx}{7b}$$

$$= \frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b} - \frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2} - \frac{(aA)\int\left(a+bx^2\right)^{3/2}\,dx}{6b}$$

$$= -\frac{aAx\left(a+bx^2\right)^{3/2}}{24b} + \frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b} - \frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2} - \frac{\left(a^2A\right.}{}$$

$$= -\frac{a^2Ax\sqrt{a+bx^2}}{16b} - \frac{aAx\left(a+bx^2\right)^{3/2}}{24b} + \frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b} - \frac{(12aB-35Abx)\left(a\right.}{210b^2}$$

$$= -\frac{a^2Ax\sqrt{a+bx^2}}{16b} - \frac{aAx\left(a+bx^2\right)^{3/2}}{24b} + \frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b} - \frac{(12aB-35Abx)\left(a\right.}{210b^2}$$

$$= -\frac{a^2Ax\sqrt{a+bx^2}}{16b} - \frac{aAx\left(a+bx^2\right)^{3/2}}{24b} + \frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b} - \frac{(12aB-35Abx)\left(a\right.}{210b^2}$$

Mathematica [A] time = 0.24, size = 113, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(-\frac{105a^{5/2}A\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}} - 96a^3B + 3a^2bx(35A+16Bx) + 2ab^2x^3(245A+192Bx) + 40b^3x^5(7A+6B\right.}{1680b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-96*a^3*B + 40*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(35*A +
16*B*x) + 2*a*b^2*x^3*(245*A + 192*B*x) - (105*a^(5/2)*A*Sqrt[b]*ArcSinh[(S
qrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(1680*b^2)
```

fricas [A] time = 0.86, size = 223, normalized size = 1.76

$$\left[\frac{105\,Aa^3\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(240\,Bb^3x^6+280\,Ab^3x^5+384\,Bab^2x^4+490\,Aab^2x^3+4\right.}{3360\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*A*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(240*B*b^3*x^6 + 280*A*b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 + 48
*B*a^2*b*x^2 + 105*A*a^2*b*x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2, 1/1680*(105*
A*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (240*B*b^3*x^6 + 280*A*
b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 + 48*B*a^2*b*x^2 + 105*A*a^2*b*
x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.51, size = 103, normalized size = 0.81

$$\frac{Aa^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}} - \frac{1}{1680}\,\sqrt{bx^2+a}\left(\frac{96\,Ba^3}{b^2} - \left(\frac{105\,Aa^2}{b} + 2\left(\frac{24\,Ba^2}{b} + (245\,Aa + 4\,(48\,Ba + 5\,(6\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/1680*sqrt(b*x
^2 + a)*(96*B*a^3/b^2 - (105*A*a^2/b + 2*(24*B*a^2/b + (245*A*a + 4*(48*B*a
 + 5*(6*B*b*x + 7*A*b)*x)*x)*x)*x)*x)
```

maple [A] time = 0.01, size = 113, normalized size = 0.89

$$-\frac{A\,a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{2}+a}\,A\,a^{2}x}{16b}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Aax}{24b}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}B\,x^{2}}{7b}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Ax}{6b}-\frac{2\left(b\,x^{2}+a\right.}{35b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/7*B*x^2*(b*x^2+a)^(5/2)/b-2/35*B*a/b^2*(b*x^2+a)^(5/2)+1/6*A*x*(b*x^2+a)^
(5/2)/b-1/24*a*A*x*(b*x^2+a)^(3/2)/b-1/16*a^2*A*x*(b*x^2+a)^(1/2)/b-1/16*A*
a^3/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.37, size = 105, normalized size = 0.83

$$\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bx^{2}}{7\,b}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ax}{6\,b}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Aax}{24\,b}-\frac{\sqrt{bx^{2}+a}\,Aa^{2}x}{16\,b}-\frac{Aa^{3}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}-\frac{2\left(bx^{2}+a\right)^{\frac{5}{2}}Ba}{35\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(5/2)*B*x^2/b + 1/6*(b*x^2 + a)^(5/2)*A*x/b - 1/24*(b*x^2 +
 a)^(3/2)*A*a*x/b - 1/16*sqrt(b*x^2 + a)*A*a^2*x/b - 1/16*A*a^3*arcsinh(b*x
/sqrt(a*b))/b^(3/2) - 2/35*(b*x^2 + a)^(5/2)*B*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\left(b\,x^{2}+a\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(3/2)*(A + B*x), x)
```

sympy [A] time = 20.41, size = 287, normalized size = 2.26

$$\frac{Aa^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{17Aa^{\frac{3}{2}}x^{3}}{48\sqrt{1+\frac{bx^{2}}{a}}}+\frac{11A\sqrt{a}\,bx^{5}}{24\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Aa^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Ab^{2}x^{7}}{6\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}+Ba\left(\begin{cases}-\frac{2a^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{ax^{2}\sqrt{a+bx^{2}}}{15b}+\frac{x^{4}}{} \\ \frac{\sqrt{a}\,x^{4}}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*A*a**(3/2)*x**3/(48*sqrt(1 + b*
x**2/a)) + 11*A*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - A*a**3*asinh(sqrt(
b)*x/sqrt(a))/(16*b**(3/2)) + A*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) +
B*a*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)
/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*b
*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**
2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, N
e(b, 0)), (sqrt(a)*x**6/6, True))
```

3.10 $\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Bx\sqrt{a+bx^2}}{16b}+\frac{\left(a+bx^2\right)^{5/2}(6A+5Bx)}{30b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}$$

```
[Out] -1/24*a*B*x*(b*x^2+a)^(3/2)/b+1/30*(5*B*x+6*A)*(b*x^2+a)^(5/2)/b-1/16*a^3*B
*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/16*a^2*B*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {780, 195, 217, 206}

$$-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Bx\sqrt{a+bx^2}}{16b}+\frac{\left(a+bx^2\right)^{5/2}(6A+5Bx)}{30b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^2*B*x*Sqrt[a + b*x^2])/(16*b) - (a*B*x*(a + b*x^2)^(3/2))/(24*b) + ((6*
A + 5*B*x)*(a + b*x^2)^(5/2))/(30*b) - (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(16*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule steps

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}dx = \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b} - \frac{(aB)\int \left(a+bx^2\right)^{3/2}dx}{6b}$$
$$= -\frac{aBx\left(a+bx^2\right)^{3/2}}{24b} + \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b} - \frac{\left(a^2B\right)\int \sqrt{a+bx^2}\,dx}{8b}$$
$$= -\frac{a^2Bx\sqrt{a+bx^2}}{16b} - \frac{aBx\left(a+bx^2\right)^{3/2}}{24b} + \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b} - \frac{\left(a^3B\right)\int \frac{1}{\sqrt{a+bx^2}}}{16b}$$
$$= -\frac{a^2Bx\sqrt{a+bx^2}}{16b} - \frac{aBx\left(a+bx^2\right)^{3/2}}{24b} + \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b} - \frac{\left(a^3B\right)\text{Subst}\left(\int\right.}{}$$
$$= -\frac{a^2Bx\sqrt{a+bx^2}}{16b} - \frac{aBx\left(a+bx^2\right)^{3/2}}{24b} + \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b} - \frac{a^3B\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.}{16b^{3/2}}$$

Mathematica [A] time = 0.19, size = 107, normalized size = 1.04

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\left(3a^2(16A+5Bx)+2abx^2(48A+35Bx)+8b^2x^4(6A+5Bx)\right)-\frac{15a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{240b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(8*b^2*x^4*(6*A + 5*B*x) + 3*a^2*(16*A + 5*B*x) +
 2*a*b*x^2*(48*A + 35*B*x)) - (15*a^(5/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/S
qrt[1 + (b*x^2)/a]))/(240*b^(3/2))
```

fricas [A] time = 0.84, size = 205, normalized size = 1.99

$$\left[\frac{15\,Ba^3\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(40\,Bb^3x^5+48\,Ab^3x^4+70\,Bab^2x^3+96\,Aab^2x^2+15\,Ba^2bx+\right.}{480\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x^3 + 96*A*a*b^2*x^2 + 15*B*a^2
*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2, 1/240*(15*B*a^3*sqrt(-b)*arctan(sq
rt(-b)*x/sqrt(b*x^2 + a)) + (40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x^3 +
 96*A*a*b^2*x^2 + 15*B*a^2*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.47, size = 89, normalized size = 0.86

$$\frac{Ba^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}+\frac{1}{240}\,\sqrt{bx^2+a}\left(\frac{48\,Aa^2}{b}+\left(\frac{15\,Ba^2}{b}+2\,(48\,Aa+(35\,Ba+4\,(5\,Bbx+6\,Ab)x)x)x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/240*sqrt(b*x^
2 + a)*(48*A*a^2/b + (15*B*a^2/b + 2*(48*A*a + (35*B*a + 4*(5*B*b*x + 6*A*b
)*x)*x)*x)*x)
```

maple [A] time = 0.01, size = 94, normalized size = 0.91

$$-\frac{B\,a^{3} \ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16 b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{2}+a}\;B\,a^{2} x}{16 b}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} B a x}{24 b}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} B x}{6 b}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} A}{5 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*B*x*(b*x^2+a)^(5/2)/b-1/24*a*B*x*(b*x^2+a)^(3/2)/b-1/16*a^2*B*x*(b*x^2+
a)^(1/2)/b-1/16*B*a^3/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/5*A/b*(b*x^2+
a)^(5/2)
```

maxima [A] time = 1.35, size = 86, normalized size = 0.83

$$\frac{\left(bx^{2}+a\right)^{\frac{5}{2}} Bx}{6\,b}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}} Bax}{24\,b}-\frac{\sqrt{bx^{2}+a}\,Ba^{2} x}{16\,b}-\frac{Ba^{3} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}} A}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*B*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16*sqrt(b*
x^2 + a)*B*a^2*x/b - 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/5*(b*x^2
 + a)^(5/2)*A/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(b\,x^{2}+a\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] int(x*(a + b*x^2)^(3/2)*(A + B*x), x)
```

sympy [A] time = 22.47, size = 223, normalized size = 2.17

$$Aa\left(\begin{cases}\frac{\sqrt{a}\,x^{2}}{2} & \text{for}\: b=0\\\frac{\left(a+bx^{2}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Ab\left(\begin{cases}-\frac{2a^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{ax^{2}\sqrt{a+bx^{2}}}{15b}+\frac{x^{4}\sqrt{a+bx^{2}}}{5} & \text{for}\: b\neq 0\\\frac{\sqrt{a}\,x^{4}}{4} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{17Ba}{48\sqrt{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)
) + A*b*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x
**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) +
 B*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*B*a**(3/2)*x**3/(48*sqrt(1 + b
*x**2/a)) + 11*B*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - B*a**3*asinh(sqrt
(b)*x/sqrt(a))/(16*b**(3/2)) + B*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.11 $\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=87

$$\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

```
[Out] 1/4*A*x*(b*x^2+a)^(3/2)+1/5*B*(b*x^2+a)^(5/2)/b+3/8*a^2*A*arctanh(x*b^(1/2)
/(b*x^2+a)^(1/2))/b^(1/2)+3/8*a*A*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*A*x*Sqrt[a + b*x^2])/8 + (A*x*(a + b*x^2)^(3/2))/4 + (B*(a + b*x^2)^(5
/2))/(5*b) + (3*a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{B\left(a+bx^2\right)^{5/2}}{5b} + A\int \left(a+bx^2\right)^{3/2}\,dx$$
$$= \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{1}{4}(3aA)\int \sqrt{a+bx^2}\,dx$$
$$= \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{1}{8}\left(3a^2A\right)\int \frac{1}{\sqrt{a+bx^2}}\,d$$
$$= \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{1}{8}\left(3a^2A\right)\text{Subst}\left(\int \frac{1}{1-\right.$$
$$= \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 1.01

$$\frac{\sqrt{a+bx^2}\left(8a^2B+abx(25A+16Bx)+2b^2x^3(5A+4Bx)\right)+15a^2A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*B + 2*b^2*x^3*(5*A + 4*B*x) + a*b*x*(25*A + 16*B*x)
) + 15*a^2*A*Sqrt[b]*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(40*b)
```

fricas [A] time = 1.02, size = 176, normalized size = 2.02

$$\left[\frac{15\,Aa^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,Aabx+8\,Ba^2\right)\sqrt{bx^2}}{80\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/80*(15*A*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(b*
x^2 + a))/b, -1/40*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(b*x
^2 + a))/b]
```

giac [A] time = 0.44, size = 76, normalized size = 0.87

$$-\frac{3\,Aa^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}+\frac{1}{40}\,\sqrt{bx^2+a}\left(\frac{8\,Ba^2}{b}+(25\,Aa+2\,(8\,Ba+(4\,Bbx+5\,Ab)x)x)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/40*sqrt(b*x^2
 + a)*(8*B*a^2/b + (25*A*a + 2*(8*B*a + (4*B*b*x + 5*A*b)*x)*x)*x)
```

maple [A] time = 0.00, size = 69, normalized size = 0.79

$$\frac{3A\,a^2\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}+\frac{3\sqrt{b\,x^2+a}\;Aax}{8}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Ax}{4}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}B}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/5*B*(b*x^2+a)^(5/2)/b+1/4*A*x*(b*x^2+a)^(3/2)+3/8*a*A*x*(b*x^2+a)^(1/2)+3
/8*A*a^2/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.41, size = 61, normalized size = 0.70

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ax+\frac{3}{8}\sqrt{bx^2+a}\,Aax+\frac{3\,Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 3/8*A*a^2*arcsinh(b
*x/sqrt(a*b))/sqrt(b) + 1/5*(b*x^2 + a)^(5/2)*B/b
```

mupad [B] time = 1.18, size = 54, normalized size = 0.62

$$\frac{B\left(b\,x^2+a\right)^{5/2}}{5\,b}+\frac{A\,x\left(b\,x^2+a\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] (B*(a + b*x^2)^(5/2))/(5*b) + (A*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2],
 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

sympy [A] time = 12.91, size = 219, normalized size = 2.52

$$\frac{Aa^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}\,bx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Ab^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Bb$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + A*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) + 3
*A*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*A*a**2*asinh(sqrt(b)*x/sqrt(a)
)/(8*sqrt(b)) + A*b**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a*Piecewise(
(sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + B*b*Piecew
ise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) +
x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

3.12 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=106

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{8}a\sqrt{a+bx^2}\,(8A+3Bx)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

```
[Out] 1/12*(3*B*x+4*A)*(b*x^2+a)^(3/2)-a^(3/2)*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))
+3/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+1/8*a*(3*B*x+8*A)*(b*
x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {815, 844, 217, 206, 266, 63, 208}

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{8}a\sqrt{a+bx^2}\,(8A+3Bx)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] (a*(8*A + 3*B*x)*Sqrt[a + b*x^2])/8 + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/12
+ (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b]) - a^(3/2)*A*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{\int \frac{(4aAb+3abBx)\sqrt{a+bx^2}}{x}\,dx}{4b}$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{\int \frac{8a^2Ab^2+3a^2b^2Bx}{x\sqrt{a+bx^2}}\,dx}{8b^2}$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \left(a^2A\right)\int \frac{1}{x\sqrt{a+bx^2}}\,dx +$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{2}\left(a^2A\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+}}\right.$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} +$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} -$$

Mathematica [A] time = 0.31, size = 118, normalized size = 1.11

$$\frac{1}{24}\left(-24a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{9a^{5/2}B\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{a+bx^2}} + \sqrt{a+bx^2}\left(32aA+15aBx+8Abx^2+6bBx\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(32*a*A + 15*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) + (9*a^(5/2)*B
*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[a + b*x^2]
) - 24*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/24
```

fricas [A] time = 0.98, size = 439, normalized size = 4.14

$$\left[\frac{9\,Ba^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+24\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(6\,Bb^2x^3+8\,Ab^2x^2+15\,B\right.}{48\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 24
*A*a^(3/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*B*b
^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, -1/24*(9*
B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 12*A*a^(3/2)*b*log(-(b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (6*B*b^2*x^3 + 8*A*b^2*x^2 +
15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, 1/48*(48*A*sqrt(-a)*a*b*arctan(s
qrt(-a)/sqrt(b*x^2 + a)) + 9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) + 2*(6*B*b^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqr
t(b*x^2 + a))/b, -1/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - 24*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*x^3 + 8*A*
b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.54, size = 100, normalized size = 0.94

$$\frac{2\,Aa^2\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\,Ba^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}+\frac{1}{24}\,\sqrt{bx^2+a}\,(32\,Aa+(15\,Ba+2\,(3\,Bbx+4\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/8*B*a^
2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/24*sqrt(b*x^2 + a)*(32
*A*a + (15*B*a + 2*(3*B*b*x + 4*A*b)*x)*x)
```

maple [A] time = 0.01, size = 107, normalized size = 1.01

$$-A\,a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\frac{3B\,a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}+\frac{3\sqrt{b\,x^2+a}\,Bax}{8}+\sqrt{b\,x^2+a}\,Aa+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x,x)
```

```
[Out] 1/4*(b*x^2+a)^(3/2)*B*x+3/8*(b*x^2+a)^(1/2)*B*a*x+3/8*B*a^2/b^(1/2)*ln(b^(1
/2)*x+(b*x^2+a)^(1/2))+1/3*A*(b*x^2+a)^(3/2)-A*a^(3/2)*ln((2*a+2*(b*x^2+a)^
(1/2)*a^(1/2))/x)+A*(b*x^2+a)^(1/2)*a
```

maxima [A] time = 1.32, size = 88, normalized size = 0.83

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bx+\frac{3}{8}\sqrt{bx^2+a}\,Bax+\frac{3\,Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}-Aa^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}A+\sqrt{bx^2+a}\,Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*B*x + 3/8*sqrt(b*x^2 + a)*B*a*x + 3/8*B*a^2*arcsinh(b
*x/sqrt(a*b))/sqrt(b) - A*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^
2 + a)^(3/2)*A + sqrt(b*x^2 + a)*A*a
```

mupad [B] time = 1.31, size = 83, normalized size = 0.78

$$\frac{A\left(b\,x^2+a\right)^{3/2}}{3}-A\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+A\,a\,\sqrt{b\,x^2+a}+\frac{B\,x\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x,x)
```

```
[Out] (A*(a + b*x^2)^(3/2))/3 - A*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + A*a*
(a + b*x^2)^(1/2) + (B*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*
x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

sympy [A] time = 35.49, size = 218, normalized size = 2.06

$$-Aa^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Aa^{2}}{\sqrt{b}\,x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Aa\sqrt{b}\,x}{\sqrt{\frac{a}{bx^{2}}+1}}+Ab\left(\begin{cases}\frac{\sqrt{a}\,x^{2}}{2} & \text{for } b=0\\ \frac{\left(a+bx^{2}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{3}{2}}x\sqrt{1+\frac{bx^{2}}{a}}}{2}+\frac{Ba^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^{2}}{a}}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x,x)
```

```
[Out] -A*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**2/(sqrt(b)*x*sqrt(a/(b*x**2)
+ 1)) + A*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + A*b*Piecewise((sqrt(a)*x**2/2,
 Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + B*a**(3/2)*x*sqrt(1 + b*x*
*2/a)/2 + B*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*b*x**3/(8*sqrt(
1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + B*b**2*x**
5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.13 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=108

$$a^{3/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}\,(2aB+3Abx)+\frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -1/3*(-B*x+3*A)*(b*x^2+a)^(3/2)/x-a^(3/2)*B*arctanh((b*x^2+a)^(1/2)/a^(1/2)
)+3/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)+1/2*(3*A*b*x+2*B*a)*(b
*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {813, 815, 844, 217, 206, 266, 63, 208}

$$a^{3/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}\,(2aB+3Abx)+\frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] ((2*a*B + 3*A*b*x)*Sqrt[a + b*x^2])/2 - ((3*A - B*x)*(a + b*x^2)^(3/2))/(3*
x) + (3*a*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - a^(3/2)*B*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = -\frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{(-2aB-6Abx)\sqrt{a+bx^2}}{x}\,dx$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} - \frac{\int \frac{-4a^2bB-6aAb^2x}{x\sqrt{a+bx^2}}\,dx}{4b}$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{1}{2}(3aAb)\int \frac{1}{\sqrt{a+bx^2}}\,dx + ($$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{1}{2}(3aAb)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\right.$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Mathematica [C] time = 0.18, size = 105, normalized size = 0.97

$$-a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{a^2A\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{a+bx^2}} + \frac{1}{3}B\sqrt{a+bx^2}\left(4a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] (B*Sqrt[a + b*x^2]*(4*a + b*x^2))/3 - a^(3/2)*B*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]] - (a^2*A*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*
x^2)/a)])/(x*Sqrt[a + b*x^2])
```

fricas [A] time = 0.68, size = 411, normalized size = 3.81

$$\left[\frac{9\,Aa\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+6\,Ba^{\frac{3}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,Bbx^3+3\,Abx^2+8\,B\right.}{12\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 6*
B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*b*
x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*A*a*sqrt(-b)
*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*
sqrt(b*x^2 + a))/x, 1/12*(12*B*sqrt(-a)*a*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)
) + 9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*
B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*A*a*sqrt
(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 6*B*sqrt(-a)*a*x*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 +
 a))/x]
```

giac [A] time = 0.60, size = 124, normalized size = 1.15

$$\frac{2\,Ba^2\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3}{2}\,Aa\sqrt{b}\,\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)+\frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{6}\,\sqrt{bx^2+a}\,(8\,B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*A*a*
sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^2*sqrt(b)/((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a) + 1/6*sqrt(b*x^2 + a)*(8*B*a + (2*B*b*x + 3*A*b
)*x)
```

maple [A] time = 0.01, size = 126, normalized size = 1.17

$$\frac{3Aa\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2}-B\,a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\frac{3\sqrt{b\,x^2+a}\,Abx}{2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Abx}{a}+\sqrt{b\,x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x^2,x)
```

```
[Out] -(b*x^2+a)^(5/2)*A/a/x+(b*x^2+a)^(3/2)*A/a*b*x+3/2*(b*x^2+a)^(1/2)*A*b*x+3/
2*A*a*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/3*B*(b*x^2+a)^(3/2)-B*a^(3/2)
*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+B*(b*x^2+a)^(1/2)*a
```

maxima [A] time = 1.39, size = 88, normalized size = 0.81

$$\frac{3}{2}\,\sqrt{bx^2+a}\,Abx+\frac{3}{2}\,Aa\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-Ba^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{3}\,\left(bx^2+a\right)^{\frac{3}{2}}B+\sqrt{bx^2+a}\,Ba-\frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*A*b*x + 3/2*A*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - B*a^(3
/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*B + sqrt(b*x^2 +
a)*B*a - (b*x^2 + a)^(3/2)*A/x
```

mupad [B] time = 1.88, size = 86, normalized size = 0.80

$$\frac{B\left(b\,x^2+a\right)^{3/2}}{3}-B\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+B\,a\,\sqrt{b\,x^2+a}-\frac{A\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x^2,x)
```

```
[Out] (B*(a + b*x^2)^(3/2))/3 - B*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + B*a*
(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x
^2)/a))/(x*((b*x^2)/a + 1)^(3/2))
```

sympy [A] time = 13.29, size = 184, normalized size = 1.70

$$-\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{A\sqrt{a}\,bx\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{A\sqrt{a}\,bx}{\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}-Ba^{\frac{3}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Ba^2}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ba\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+ \cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] -A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b*x*sqrt(1 + b*x**2/a)/2 - A
*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*A*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2
- B*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2/(sqrt(b)*x*sqrt(a/(b*x**2)
 + 1)) + B*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*b*Piecewise((sqrt(a)*x**2/2
, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
```

3.14 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=111

$$-\frac{\left(a+bx^2\right)^{3/2}(A-Bx)}{2x^2}-\frac{3\sqrt{a+bx^2}\,(aB-Abx)}{2x}-\frac{3}{2}\sqrt{a}\,Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -1/2*(-B*x+A)*(b*x^2+a)^(3/2)/x^2-3/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))*
a^(1/2)+3/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)-3/2*(-A*b*x+B*a)
*(b*x^2+a)^(1/2)/x
```

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {813, 844, 217, 206, 266, 63, 208}

$$-\frac{\left(a+bx^2\right)^{3/2}(A-Bx)}{2x^2}-\frac{3\sqrt{a+bx^2}\,(aB-Abx)}{2x}-\frac{3}{2}\sqrt{a}\,Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] (-3*(a*B - A*b*x)*Sqrt[a + b*x^2])/(2*x) - ((A - B*x)*(a + b*x^2)^(3/2))/(2
*x^2) + (3*a*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - (3*Sqrt[a]
*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx &= -\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}-\frac{3}{8}\int \frac{(-4aB-4Abx)\sqrt{a+bx^2}}{x^2}\,dx\\
&= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x}-\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{3}{16}\int \frac{8aAb+8abBx}{x\sqrt{a+bx^2}}\,dx\\
&= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x}-\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{1}{2}(3aAb)\int \frac{1}{x\sqrt{a+bx^2}}\,dx+\frac{1}{2}\\
&= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x}-\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{1}{4}(3aAb)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.\\
&= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x}-\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\\
&= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x}-\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\end{aligned}$$

Mathematica [C] time = 0.06, size = 90, normalized size = 0.81

$$\frac{Ab\left(a+bx^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^2}{a}+1\right)}{5a^2}-\frac{aB\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] -((a*B*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^2)/a)])/(x
*Sqrt[1 + (b*x^2)/a])) + (A*b*(a + b*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7
/2, 1 + (b*x^2)/a])/(5*a^2)
```

fricas [A] time = 0.94, size = 425, normalized size = 3.83

$$\left[\frac{3\,Ba\sqrt{b}\,x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+3\,A\sqrt{a}\,bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(Bbx^3+2\,Abx^2-2\,Ba\right.}{4\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 3
*A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B
*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2, -1/4*(6*B*a*sqrt(
-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(a)*b*x^2*log(-(b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x
- A*a)*sqrt(b*x^2 + a))/x^2, 1/4*(6*A*sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b
*x^2 + a)) + 3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) + 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2, -1/2*(3
*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(-a)*b*x^2*a
rctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqr
t(b*x^2 + a))/x^2]
```

giac [B] time = 0.55, size = 191, normalized size = 1.72

$$\frac{3\,Aab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3}{2}\,Ba\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)+\frac{1}{2}\,(Bbx+2\,Ab)\sqrt{bx^2+a}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 3*A*a*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*B*a*
sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/2*(B*b*x + 2*A*b)*sqrt(b
*x^2 + a) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a*b + 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*B*a^2*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b - 2*B*a^3
*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

maple [A] time = 0.01, size = 150, normalized size = 1.35

$$-\frac{3A\sqrt{a}\,b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2}+\frac{3Ba\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2}+\frac{3\sqrt{b\,x^2+a}\,Bbx}{2}+\frac{3\sqrt{b\,x^2+a}\,Ab}{2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x^3,x)
```

```
[Out] -1/2*A/a/x^2*(b*x^2+a)^(5/2)+1/2*A/a*b*(b*x^2+a)^(3/2)-3/2*A*a^(1/2)*b*ln((
2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/2*A*b*(b*x^2+a)^(1/2)-B/a/x*(b*x^2+a)^(
5/2)+B/a*b*x*(b*x^2+a)^(3/2)+3/2*B*b*x*(b*x^2+a)^(1/2)+3/2*B*a*b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.28, size = 112, normalized size = 1.01

$$\frac{3}{2}\sqrt{bx^2+a}\,Bbx+\frac{3}{2}\,Ba\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{3}{2}\,A\sqrt{a}\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{3}{2}\sqrt{bx^2+a}\,Ab+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ab}{2\,a}-\frac{\left(bx^2+\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*B*b*x + 3/2*B*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 3/2*A*
sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*A*b + 1/2*(b*
x^2 + a)^(3/2)*A*b/a - (b*x^2 + a)^(3/2)*B/x - 1/2*(b*x^2 + a)^(5/2)*A/(a*x
^2)
```

mupad [B] time = 2.12, size = 91, normalized size = 0.82

$$A\,b\,\sqrt{b\,x^2+a}-\frac{A\,a\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{3\,A\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}-\frac{B\,{\left(b\,x^2+a\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x^3,x)
```

```
[Out] A*b*(a + b*x^2)^(1/2) - (A*a*(a + b*x^2)^(1/2))/(2*x^2) - (3*A*a^(1/2)*b*at
anh((a + b*x^2)^(1/2)/a^(1/2)))/2 - (B*(a + b*x^2)^(3/2)*hypergeom([-3/2, -
1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a + 1)^(3/2))
```

sympy [A] time = 15.57, size = 182, normalized size = 1.64

$$-\frac{3A\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{Aa\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}-\frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}\,bx\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{B\sqrt{a}\,bx}{\sqrt{1+\frac{bx^2}{a}}}+3B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] -3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/(b*x**2) +
 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*x/sqrt(a/(b*x
**2) + 1) - B*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(a)*b*x*sqrt(1 + b*x*
*2/a)/2 - B*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*B*a*sqrt(b)*asinh(sqrt(b)*x/
sqrt(a))/2
```

3.15 $\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=173

$$\frac{3a^5B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}-\frac{a\left(a+bx^2\right)^{7/2}(160A+189Bx}{5040b^2}$$

```
[Out] 1/128*a^3*B*x*(b*x^2+a)^(3/2)/b^2+1/160*a^2*B*x*(b*x^2+a)^(5/2)/b^2+1/9*A*x
^2*(b*x^2+a)^(7/2)/b+1/10*B*x^3*(b*x^2+a)^(7/2)/b-1/5040*a*(189*B*x+160*A)*
(b*x^2+a)^(7/2)/b^2+3/256*a^5*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+
3/256*a^4*B*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] time = 0.10, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {833, 780, 195, 217, 206}

$$\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{3a^5B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}-\frac{a\left(a+bx^2\right)^{7/2}(160A+189Bx}{5040b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (3*a^4*B*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*B*x*(a + b*x^2)^(3/2))/(128*b^
2) + (a^2*B*x*(a + b*x^2)^(5/2))/(160*b^2) + (A*x^2*(a + b*x^2)^(7/2))/(9*b
) + (B*x^3*(a + b*x^2)^(7/2))/(10*b) - (a*(160*A + 189*B*x)*(a + b*x^2)^(7/
2))/(5040*b^2) + (3*a^5*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2
))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}dx = \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}+\frac{\int x^2(-3aB+10Abx)\left(a+bx^2\right)^{5/2}dx}{10b}$$

$$= \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}+\frac{\int x(-20aAb-27abBx)\left(a+bx^2\right)^{5/2}dx}{90b^2}$$

$$= \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2}+\frac{\left(3a^2B\right)\cdots}{\cdots}$$

$$= \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{a(160A+189Bx)\left(a\cdots\right.}{5040b^2}$$

$$= \frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\cdots$$

$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\cdots$$

$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\cdots$$

$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\cdots$$

Mathematica [A] time = 0.40, size = 145, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\frac{945a^{9/2}B\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}\left(-5a^4(512A+189Bx)+10a^3bx^2(128A+63Bx)+24a^2b^2x^4(800A+651Bx)\cdots\right.\right.}{80640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(896*b^4*x^8*(10*A + 9*B*x) + 10*a^3*b*x^2*(128*A
 + 63*B*x) - 5*a^4*(512*A + 189*B*x) + 24*a^2*b^2*x^4*(800*A + 651*B*x) + 1
6*a*b^3*x^6*(1520*A + 1323*B*x)) + (945*a^(9/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[
a]])/Sqrt[1 + (b*x^2)/a]))/(80640*b^(5/2))
```

fricas [A] time = 1.03, size = 302, normalized size = 1.75

$$\left[\frac{945\,Ba^5\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(8064\,Bb^5x^9+8960\,Ab^5x^8+21168\,Bab^4x^7+24320\,Aab^4x^6\cdots\right.}{161280\,b^{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/161280*(945*B*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 2*(8064*B*b^5*x^9 + 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^4*
x^6 + 15624*B*a^2*b^3*x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 1280*
A*a^3*b^2*x^2 - 945*B*a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3, -1/8064
0*(945*B*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8064*B*b^5*x^9
+ 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^4*x^6 + 15624*B*a^2*b^3*
x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 1280*A*a^3*b^2*x^2 - 945*B*
a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.53, size = 140, normalized size = 0.81

$$-\frac{3\,Ba^5\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}-\frac{1}{80640}\left(\frac{2560\,Aa^4}{b^2}+\left(\frac{945\,Ba^4}{b^2}-2\left(\frac{640\,Aa^3}{b}+\left(\frac{315\,Ba^3}{b}+4\left(2400\,Aa^2+\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -3/256*B*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/80640*(2560
*A*a^4/b^2 + (945*B*a^4/b^2 - 2*(640*A*a^3/b + (315*B*a^3/b + 4*(2400*A*a^2
 + (1953*B*a^2 + 2*(1520*A*a*b + 7*(189*B*a*b + 8*(9*B*b^2*x + 10*A*b^2)*x)
*x)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

maple [A] time = 0.01, size = 153, normalized size = 0.88

$$\frac{3B\,a^5\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{256b^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^2+a}\,B\,a^4x}{256b^2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B\,a^3x}{128b^2}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}B\,x^3}{10b}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}A\,x^2}{9b}+\frac{\left(b\,x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/10*B*x^3*(b*x^2+a)^(7/2)/b-3/80*B*a/b^2*x*(b*x^2+a)^(7/2)+1/160*a^2*B*x*(
b*x^2+a)^(5/2)/b^2+1/128*a^3*B*x*(b*x^2+a)^(3/2)/b^2+3/256*a^4*B*x*(b*x^2+a
)^(1/2)/b^2+3/256*B*a^5/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/9*A*x^2*(b*
x^2+a)^(7/2)/b-2/63*A*a/b^2*(b*x^2+a)^(7/2)
```

maxima [A] time = 1.36, size = 145, normalized size = 0.84

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bx^3}{10\,b}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ax^2}{9\,b}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}}Bax}{80\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ba^2x}{160\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ba^3x}{128\,b^2}+\frac{3\sqrt{bx^2+a}\,Ba^4x}{256\,b^2}+\frac{3\,B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(7/2)*B*x^3/b + 1/9*(b*x^2 + a)^(7/2)*A*x^2/b - 3/80*(b*x^
2 + a)^(7/2)*B*a*x/b^2 + 1/160*(b*x^2 + a)^(5/2)*B*a^2*x/b^2 + 1/128*(b*x^2
 + a)^(3/2)*B*a^3*x/b^2 + 3/256*sqrt(b*x^2 + a)*B*a^4*x/b^2 + 3/256*B*a^5*a
rcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/63*(b*x^2 + a)^(7/2)*A*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] int(x^3*(a + b*x^2)^(5/2)*(A + B*x), x)
```

sympy [A] time = 37.44, size = 469, normalized size = 2.71

$$Aa^2\left(\begin{cases} -\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0 \\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+2Aab\left(\begin{cases} \frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} \\ \frac{\sqrt{a}\,x^6}{6} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x*
*2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) +
2*A*a*b*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a
+ b*x**2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**
2)/7, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + A*b**2*Piecewise((-16*a**4*sqrt(
a + b*x**2)/(315*b**4) + 8*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*a**2*x
**4*sqrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt(a + b*x**2)/(63*b) + x**8*sqr
t(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*x**8/8, True)) - 3*B*a**(9/2)*x/(256*b
**2*sqrt(1 + b*x**2/a)) - B*a**(7/2)*x**3/(256*b*sqrt(1 + b*x**2/a)) + 129*
B*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*B*a**(3/2)*b*x**7/(160*sqrt(1
 + b*x**2/a)) + 29*B*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a)) + 3*B*a**5*a
sinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + B*b**3*x**11/(10*sqrt(a)*sqrt(1 +
b*x**2/a))
```

3.16 $\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=150

$$-\frac{5a^4A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{\left(a+bx^2\right)^{7/2}(16aB-63Abx)}{504b^2}-\frac{aAx\left(a+bx^2\right)}{48b}$$

```
[Out] -5/192*a^2*A*x*(b*x^2+a)^(3/2)/b-1/48*a*A*x*(b*x^2+a)^(5/2)/b+1/9*B*x^2*(b*
x^2+a)^(7/2)/b-1/504*(-63*A*b*x+16*B*a)*(b*x^2+a)^(7/2)/b^2-5/128*a^4*A*arc
tanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-5/128*a^3*A*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {833, 780, 195, 217, 206}

$$-\frac{5a^4A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{\left(a+bx^2\right)^{7/2}(16aB-63Abx)}{504b^2}-\frac{aAx\left(a+bx^2\right)}{48b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^3*A*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*A*x*(a + b*x^2)^(3/2))/(192*b
) - (a*A*x*(a + b*x^2)^(5/2))/(48*b) + (B*x^2*(a + b*x^2)^(7/2))/(9*b) - ((
16*a*B - 63*A*b*x)*(a + b*x^2)^(7/2))/(504*b^2) - (5*a^4*A*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
```

```
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{\int x(-2aB+9Abx)\left(a+bx^2\right)^{5/2}\,dx}{9b}$$
$$= \frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{(16aB-63Abx)\left(a+bx^2\right)^{7/2}}{504b^2}-\frac{(aA)\int\left(a+bx^2\right)^{5/2}\,dx}{8b}$$
$$= -\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{(16aB-63Abx)\left(a+bx^2\right)^{7/2}}{504b^2}-\frac{\left(5a^2A\right)}{}$$
$$= -\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{(16aB-63Abx)(}{504b^2}$$
$$= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}$$
$$= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}$$
$$= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}$$

Mathematica [A] time = 0.41, size = 131, normalized size = 0.87

$$\frac{\sqrt{a+bx^2}\left(-\frac{315a^{7/2}A\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}-256a^4B+a^3bx(315A+128Bx)+6a^2b^2x^3(413A+320Bx)+8ab^3x^5(357A+\right.}{8064b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-256*a^4*B + 112*b^4*x^7*(9*A + 8*B*x) + a^3*b*x*(315*A +
 128*B*x) + 8*a*b^3*x^5*(357*A + 304*B*x) + 6*a^2*b^2*x^3*(413*A + 320*B*x)
 - (315*a^(7/2)*A*Sqrt[b]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]
))/(8064*b^2)
```

fricas [A] time = 0.94, size = 271, normalized size = 1.81

$$\left[\frac{315\,Aa^4\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(896\,Bb^4x^8+1008\,Ab^4x^7+2432\,Bab^3x^6+2856\,Aab^3x^5+1\right.}{16128\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16128*(315*A*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6 + 2856*A*a*b^3*x^5
+ 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b*x^2 + 315*A*a^3*b*x
 - 256*B*a^4)*sqrt(b*x^2 + a))/b^2, 1/8064*(315*A*a^4*sqrt(-b)*arctan(sqrt(
```

```
-b)*x/sqrt(b*x^2 + a)) + (896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6
 + 2856*A*a*b^3*x^5 + 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b
*x^2 + 315*A*a^3*b*x - 256*B*a^4)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.46, size = 128, normalized size = 0.85

$$\frac{5\,Aa^4\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}-\frac{1}{8064}\left(\frac{256\,Ba^4}{b^2}-\left(\frac{315\,Aa^3}{b}+2\left(\frac{64\,Ba^3}{b}+\left(1239\,Aa^2+4\left(240\,Ba^2+\left(357\,\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*A*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/8064*(256*B*
a^4/b^2 - (315*A*a^3/b + 2*(64*B*a^3/b + (1239*A*a^2 + 4*(240*B*a^2 + (357*
A*a*b + 2*(152*B*a*b + 7*(8*B*b^2*x + 9*A*b^2)*x)*x)*x)*x)*x)*x)*x)*sqrt(b*
x^2 + a)
```

maple [A] time = 0.01, size = 132, normalized size = 0.88

$$-\frac{5A\,a^4\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{128b^{\frac{3}{2}}}-\frac{5\sqrt{b\,x^2+a}\;A\,a^3x}{128b}-\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}A\,a^2x}{192b}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}Aax}{48b}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}B\,x^2}{9b}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/9*B*x^2*(b*x^2+a)^(7/2)/b-2/63*B*a/b^2*(b*x^2+a)^(7/2)+1/8*A*x*(b*x^2+a)^
(7/2)/b-1/48*a*A*x*(b*x^2+a)^(5/2)/b-5/192*a^2*A*x*(b*x^2+a)^(3/2)/b-5/128*
a^3*A*x*(b*x^2+a)^(1/2)/b-5/128*A*a^4/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.37, size = 124, normalized size = 0.83

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bx^2}{9\,b}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ax}{8\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Aax}{48\,b}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}Aa^2x}{192\,b}-\frac{5\sqrt{bx^2+a}\,Aa^3x}{128\,b}-\frac{5\,Aa^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(7/2)*B*x^2/b + 1/8*(b*x^2 + a)^(7/2)*A*x/b - 1/48*(b*x^2 +
 a)^(5/2)*A*a*x/b - 5/192*(b*x^2 + a)^(3/2)*A*a^2*x/b - 5/128*sqrt(b*x^2 +
a)*A*a^3*x/b - 5/128*A*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/63*(b*x^2 + a
)^(7/2)*B*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(5/2)*(A + B*x), x)
```

sympy [A] time = 61.22, size = 442, normalized size = 2.95

$$\frac{5Aa^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133Aa^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Aa^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23A\sqrt{a}\,b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Aa^4\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Ab^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba^2\left(\begin{cases}-\frac{2a^2\cdots}{\cdots} \\ \frac{\sqrt{a}\,x^4}{4}\end{cases}\right.\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] 5*A*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*A*a**(5/2)*x**3/(384*sqrt(1
 + b*x**2/a)) + 127*A*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*A*sqrt(
a)*b**2*x**7/(48*sqrt(1 + b*x**2/a)) - 5*A*a**4*asinh(sqrt(b)*x/sqrt(a))/(1
28*b**(3/2)) + A*b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**2*Piecewis
e((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x*
*4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + 2*B*a*b*Piecewi
se((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*
b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne(b, 0))
, (sqrt(a)*x**6/6, True)) + B*b**2*Piecewise((-16*a**4*sqrt(a + b*x**2)/(31
5*b**4) + 8*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*a**2*x**4*sqrt(a + b*
x**2)/(105*b**2) + a*x**6*sqrt(a + b*x**2)/(63*b) + x**8*sqrt(a + b*x**2)/9
, Ne(b, 0)), (sqrt(a)*x**8/8, True))
```

3.17 $\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=126

$$-\frac{5a^4B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}+\frac{\left(a+bx^2\right)^{7/2}(8A+7Bx)}{56b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}$$

```
[Out] -5/192*a^2*B*x*(b*x^2+a)^(3/2)/b-1/48*a*B*x*(b*x^2+a)^(5/2)/b+1/56*(7*B*x+8
*A)*(b*x^2+a)^(7/2)/b-5/128*a^4*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2
)-5/128*a^3*B*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {780, 195, 217, 206}

$$-\frac{5a^4B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}+\frac{\left(a+bx^2\right)^{7/2}(8A+7Bx)}{56b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^3*B*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*B*x*(a + b*x^2)^(3/2))/(192*b
) - (a*B*x*(a + b*x^2)^(5/2))/(48*b) + ((8*A + 7*B*x)*(a + b*x^2)^(7/2))/(5
6*b) - (5*a^4*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}dx=\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{(aB)\int\left(a+bx^2\right)^{5/2}dx}{8b}$$
$$=-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{\left(5a^2B\right)\int\left(a+bx^2\right)^{3/2}dx}{48b}$$
$$=-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{\left(5a^3B\right)\int\sqrt{}}{64}$$
$$=-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx\right.}{56b}$$
$$=-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx\right.}{56b}$$
$$=-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx\right.}{56b}$$

Mathematica [A] time = 0.55, size = 112, normalized size = 0.89

$$\frac{\left(a+bx^2\right)^{7/2}\left(-\frac{7aBx\left(\frac{15a^{7/2}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}x}+\left(a+bx^2\right)\left(33a^2+26abx^2+8b^2x^4\right)\right)}{\left(a+bx^2\right)^4}+384A+336Bx\right)}{2688b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(384*A + 336*B*x - (7*a*B*x*((a + b*x^2)*(33*a^2 + 26*a*
b*x^2 + 8*b^2*x^4) + (15*a^(7/2)*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sq
rt[a]])/(Sqrt[b]*x)))/(a + b*x^2)^4))/(2688*b)
```

fricas [A] time = 0.95, size = 253, normalized size = 2.01

$$\left[\frac{105\,Ba^4\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(336\,Bb^4x^7+384\,Ab^4x^6+952\,Bab^3x^5+1152\,Aab^3x^4+826\right.}{5376\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/5376*(105*B*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x^4 + 8
26*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)*sqrt(b
*x^2 + a))/b^2, 1/2688*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) + (336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x^4 +
826*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)*sqrt(
b*x^2 + a))/b^2]
```

giac [A] time = 0.50, size = 114, normalized size = 0.90

$$\frac{5\,Ba^4\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}+\frac{1}{2688}\left(\frac{384\,Aa^3}{b}+\left(\frac{105\,Ba^3}{b}+2\left(576\,Aa^2+\left(413\,Ba^2+4\left(144\,Aab+\left(119\,Bab\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/2688*(384*A*
a^3/b + (105*B*a^3/b + 2*(576*A*a^2 + (413*B*a^2 + 4*(144*A*a*b + (119*B*a*
b + 6*(7*B*b^2*x + 8*A*b^2)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

maple [A] time = 0.01, size = 113, normalized size = 0.90

$$-\frac{5B\,a^4 \ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{128b^{\frac{3}{2}}}-\frac{5\sqrt{b\,x^2+a}\,B\,a^3x}{128b}-\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}B\,a^2x}{192b}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}Bax}{48b}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}Bx}{8b}+\frac{\left(b\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/8*B*x*(b*x^2+a)^(7/2)/b-1/48*a*B*x*(b*x^2+a)^(5/2)/b-5/192*a^2*B*x*(b*x^2
+a)^(3/2)/b-5/128*a^3*B*x*(b*x^2+a)^(1/2)/b-5/128*B*a^4/b^(3/2)*ln(b^(1/2)*
x+(b*x^2+a)^(1/2))+1/7*A/b*(b*x^2+a)^(7/2)
```

maxima [A] time = 1.44, size = 105, normalized size = 0.83

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bx}{8\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bax}{48\,b}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}Ba^2x}{192\,b}-\frac{5\,\sqrt{bx^2+a}\,Ba^3x}{128\,b}-\frac{5\,Ba^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(7/2)*B*x/b - 1/48*(b*x^2 + a)^(5/2)*B*a*x/b - 5/192*(b*x^2
 + a)^(3/2)*B*a^2*x/b - 5/128*sqrt(b*x^2 + a)*B*a^3*x/b - 5/128*B*a^4*arcsi
nh(b*x/sqrt(a*b))/b^(3/2) + 1/7*(b*x^2 + a)^(7/2)*A/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] int(x*(a + b*x^2)^(5/2)*(A + B*x), x)
```

sympy [A] time = 26.54, size = 354, normalized size = 2.81

$$Aa^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\; b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+2Aab\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\; b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+Ab^2\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}- \\ \frac{\sqrt{a}\,x^6}{6}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), Tr
ue)) + 2*A*a*b*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(
a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, T
rue)) + A*b**2*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*
```

```
sqrt(a + b*x**2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a
+ b*x**2)/7, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + 5*B*a**(7/2)*x/(128*b*sqr
t(1 + b*x**2/a)) + 133*B*a**(5/2)*x**3/(384*sqrt(1 + b*x**2/a)) + 127*B*a**
(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*B*sqrt(a)*b**2*x**7/(48*sqrt(1 +
 b*x**2/a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + B*b**3*x**
9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.18 $\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=107

$$\frac{5a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

```
[Out] 5/24*a*A*x*(b*x^2+a)^(3/2)+1/6*A*x*(b*x^2+a)^(5/2)+1/7*B*(b*x^2+a)^(7/2)/b+
5/16*a^3*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+5/16*a^2*A*x*(b*x^2+a
)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*A*x*Sqrt[a + b*x^2])/16 + (5*a*A*x*(a + b*x^2)^(3/2))/24 + (A*x*(a +
 b*x^2)^(5/2))/6 + (B*(a + b*x^2)^(7/2))/(7*b) + (5*a^3*A*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{B\left(a+bx^2\right)^{7/2}}{7b} + A\int \left(a+bx^2\right)^{5/2}\,dx$$
$$= \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{6}(5aA)\int \left(a+bx^2\right)^{3/2}\,dx$$
$$= \frac{5}{24}aAx\left(a+bx^2\right)^{3/2} + \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{8}\left(5a^2A\right)\int \sqrt{a+bx^2}\ d$$
$$= \frac{5}{16}a^2Ax\sqrt{a+bx^2} + \frac{5}{24}aAx\left(a+bx^2\right)^{3/2} + \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{16}$$
$$= \frac{5}{16}a^2Ax\sqrt{a+bx^2} + \frac{5}{24}aAx\left(a+bx^2\right)^{3/2} + \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{1}{16}$$
$$= \frac{5}{16}a^2Ax\sqrt{a+bx^2} + \frac{5}{24}aAx\left(a+bx^2\right)^{3/2} + \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{5a}{}$$

Mathematica [A] time = 0.09, size = 108, normalized size = 1.01

$$\frac{105a^3A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{a+bx^2}\left(48a^3B+3a^2bx(77A+48Bx)+2ab^2x^3(91A+72Bx)+8b^3x^5(7\right.}{336b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*a^3*B + 8*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(77*A + 48
*B*x) + 2*a*b^2*x^3*(91*A + 72*B*x)) + 105*a^3*A*Sqrt[b]*Log[b*x + Sqrt[b]*
Sqrt[a + b*x^2]])/(336*b)
```

fricas [A] time = 0.86, size = 224, normalized size = 2.09

$$\left[\frac{105\,Aa^3\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(48\,Bb^3x^6+56\,Ab^3x^5+144\,Bab^2x^4+182\,Aab^2x^3+144\,Ba^2\right.}{672\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*A*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(48*B*b^3*x^6 + 56*A*b^3*x^5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B
*a^2*b*x^2 + 231*A*a^2*b*x + 48*B*a^3)*sqrt(b*x^2 + a))/b, -1/336*(105*A*a^
3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*B*b^3*x^6 + 56*A*b^3*x^
5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B*a^2*b*x^2 + 231*A*a^2*b*x + 4
8*B*a^3)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.61, size = 101, normalized size = 0.94

$$-\frac{5\,Aa^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{336}\left(\frac{48\,Ba^3}{b}+\left(231\,Aa^2+2\left(72\,Ba^2+\left(91\,Aab+4\left(18\,Bab+\left(6\,Bb^2x+7\,Ab\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/336*(48*B*a^
3/b + (231*A*a^2 + 2*(72*B*a^2 + (91*A*a*b + 4*(18*B*a*b + (6*B*b^2*x + 7*A
*b^2)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

maple [A] time = 0.00, size = 85, normalized size = 0.79

$$\frac{5Aa^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}+\frac{5\sqrt{b\,x^2+a}\,A\,a^2x}{16}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}Aax}{24}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}Ax}{6}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}B}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/7*B*(b*x^2+a)^(7/2)/b+1/6*A*x*(b*x^2+a)^(5/2)+5/24*a*A*x*(b*x^2+a)^(3/2)+
5/16*a^2*A*x*(b*x^2+a)^(1/2)+5/16*A*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2
))
```

maxima [A] time = 1.40, size = 77, normalized size = 0.72

$$\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Ax+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Aax+\frac{5}{16}\sqrt{bx^2+a}\,Aa^2x+\frac{5\,Aa^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*A*x + 5/24*(b*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(b*x^2
+ a)*A*a^2*x + 5/16*A*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/7*(b*x^2 + a)^
(7/2)*B/b
```

mupad [B] time = 1.16, size = 54, normalized size = 0.50

$$\frac{B\left(b\,x^2+a\right)^{7/2}}{7\,b}+\frac{A\,x\left(b\,x^2+a\right)^{5/2}{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\frac{3}{2};-\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] (B*(a + b*x^2)^(7/2))/(7*b) + (A*x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2],
 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

sympy [A] time = 26.05, size = 348, normalized size = 3.25

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Aa^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17A\sqrt{a}\,b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Aa^3\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Ab^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} \\ \frac{\left(a+bx^2\right)}{3b}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + b*x**2/a))
+ 35*A*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a)) + 17*A*sqrt(a)*b**2*x**5/(24
*sqrt(1 + b*x**2/a)) + 5*A*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + A*b
**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**2*Piecewise((sqrt(a)*x**2/2,
 Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + 2*B*a*b*Piecewise((-2*a**2
```

```
*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a
+ b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*b**2*Piecewise((8*a**3*
sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + a*x
**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (sqrt(a)*
x**6/6, True))
```

3.19 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x} \, dx$

Optimal. Leaf size=132

$$-a^{5/2} A \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5a^3 B \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{16}a^2\sqrt{a+bx^2}\,(16A+5Bx)+\frac{1}{24}a\left(a+bx^2\right)^{3/2}(8A+5Bx)+$$

```
[Out] 1/24*a*(5*B*x+8*A)*(b*x^2+a)^(3/2)+1/30*(5*B*x+6*A)*(b*x^2+a)^(5/2)-a^(5/2)
*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))+5/16*a^3*B*arctanh(x*b^(1/2)/(b*x^2+a)^
(1/2))/b^(1/2)+1/16*a^2*(5*B*x+16*A)*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {815, 844, 217, 206, 266, 63, 208}

$$\frac{1}{16}a^2\sqrt{a+bx^2}\,(16A+5Bx)-a^{5/2} A \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5a^3 B \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{24}a\left(a+bx^2\right)^{3/2}(8A+5Bx)+\frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] (a^2*(16*A + 5*B*x)*Sqrt[a + b*x^2])/16 + (a*(8*A + 5*B*x)*(a + b*x^2)^(3/2
))/24 + ((6*A + 5*B*x)*(a + b*x^2)^(5/2))/30 + (5*a^3*B*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(16*Sqrt[b]) - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{\int \frac{(6aAb+5abBx)\left(a+bx^2\right)^{3/2}}{x}\,dx}{6b}$$

$$= \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{\int \frac{\left(24a^2Ab^2+15a^2b^2Bx\right)\sqrt{a\cdot}}{x}}{24b^2}$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5Bx)\left(a+b\right.$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5Bx)\left(a+b\right.$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5Bx)\left(a+b\right.$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5Bx)\left(a+b\right.$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5Bx)\left(a+b\right.$$

Mathematica [A] time = 0.36, size = 139, normalized size = 1.05

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5a^{7/2}B\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}\,\sqrt{a+bx^2}}+\frac{1}{240}\sqrt{a+bx^2}\,\left(a^2(368A+165Bx)+2abx^2(88A+6\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*b^2*x^4*(6*A + 5*B*x) + 2*a*b*x^2*(88*A + 65*B*x) + a^2
*(368*A + 165*B*x)))/240 + (5*a^(7/2)*B*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b
]*x)/Sqrt[a]])/(16*Sqrt[b]*Sqrt[a + b*x^2]) - a^(5/2)*A*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]]
```

fricas [A] time = 0.64, size = 539, normalized size = 4.08

$$\left[\frac{75\,Ba^3\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+240\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(40\,Bb^3x^5+48\,Ab^3x\right.}{480\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/480*(75*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
240*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(40
*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b
*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)) - 120*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A
*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, 1/480*(480*A*
sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 75*B*a^3*sqrt(b)*log(-2*b
*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*B*b^3*x^5 + 48*A*b^3*x^4 +
130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*sqrt(b*x^2
 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 24
0*A*sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (40*B*b^3*x^5 + 48*A*
b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*
sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.58, size = 125, normalized size = 0.95

$$\frac{2\,Aa^3\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\,Ba^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{240}\left(368\,Aa^2+\left(165\,Ba^2+2\left(88\,Aab+\left(65\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 5/16*B*a
^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/240*(368*A*a^2 + (165
*B*a^2 + 2*(88*A*a*b + (65*B*a*b + 4*(5*B*b^2*x + 6*A*b^2)*x)*x)*x)*x)*sqrt
(b*x^2 + a)
```

maple [A] time = 0.01, size = 138, normalized size = 1.05

$$-A\,a^{\frac{5}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\frac{5B\,a^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}+\frac{5\sqrt{b\,x^2+a}\,B\,a^2x}{16}+\sqrt{b\,x^2+a}\,A\,a^2+\frac{5\left(b\,x^2+\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x,x)
```

```
[Out] 1/6*B*x*(b*x^2+a)^(5/2)+5/24*B*a*x*(b*x^2+a)^(3/2)+5/16*B*a^2*x*(b*x^2+a)^(
1/2)+5/16*B*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/5*A*(b*x^2+a)^(5/2)
+1/3*A*a*(b*x^2+a)^(3/2)-A*a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+A*
(b*x^2+a)^(1/2)*a^2
```

maxima [A] time = 1.38, size = 119, normalized size = 0.90

$$\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Bx+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Bax+\frac{5}{16}\sqrt{bx^2+a}\,Ba^2x+\frac{5\,Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}-Aa^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{5}\left(bx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*B*x + 5/24*(b*x^2 + a)^(3/2)*B*a*x + 5/16*sqrt(b*x^2
+ a)*B*a^2*x + 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*a^(5/2)*arcsin
h(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2)*A + 1/3*(b*x^2 + a)^(3/2)*A
*a + sqrt(b*x^2 + a)*A*a^2
```

mupad [B] time = 1.25, size = 101, normalized size = 0.77

$$\frac{A\left(b\,x^2+a\right)^{5/2}}{5}+A\,a^2\,\sqrt{b\,x^2+a}+\frac{A\,a\,\left(b\,x^2+a\right)^{3/2}}{3}+\frac{B\,x\,\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}+A\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+ \ldots}}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x,x)
```

```
[Out] (A*(a + b*x^2)^(5/2))/5 + A*a^2*(a + b*x^2)^(1/2) + A*a^(5/2)*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*1i + (A*a*(a + b*x^2)^(3/2))/3 + (B*x*(a + b*x^2)^(
5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

sympy [A] time = 40.75, size = 323, normalized size = 2.45

$$-Aa^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Aa^3}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{Aa^2\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}+2Aab\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Ab^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b} \ldots\\ \frac{\sqrt{a}\,x^4}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x,x)
```

```
[Out] -A*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**3/(sqrt(b)*x*sqrt(a/(b*x**2)
+ 1)) + A*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*A*a*b*Piecewise((sqrt(a)*
x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + A*b**2*Piecewise((-
2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*s
qrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*a**(5/2)*x*sqrt(1
 + b*x**2/a)/2 + 3*B*a**(5/2)*x/(16*sqrt(1 + b*x**2/a)) + 35*B*a**(3/2)*b*x
**3/(48*sqrt(1 + b*x**2/a)) + 17*B*sqrt(a)*b**2*x**5/(24*sqrt(1 + b*x**2/a)
) + 5*B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + B*b**3*x**7/(6*sqrt(a)
*sqrt(1 + b*x**2/a))
```

3.20 $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x^2} dx$

Optimal. Leaf size=136

$$a^{5/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\frac{1}{8}a\sqrt{a+bx^2}\,(8aB+15Abx)-\frac{(a+bx^2)^{5/2}(5A-$$

```
[Out] 1/12*(15*A*b*x+4*B*a)*(b*x^2+a)^(3/2)-1/5*(-B*x+5*A)*(b*x^2+a)^(5/2)/x-a^(5
/2)*B*arctanh((b*x^2+a)^(1/2)/a^(1/2))+15/8*a^2*A*arctanh(x*b^(1/2)/(b*x^2+
a)^(1/2))*b^(1/2)+1/8*a*(15*A*b*x+8*B*a)*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {813, 815, 844, 217, 206, 266, 63, 208}

$$\frac{15}{8}a^2A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+a^{5/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{8}a\sqrt{a+bx^2}\,(8aB+15Abx)-\frac{(a+bx^2)^{5/2}(5A-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (a*(8*a*B + 15*A*b*x)*Sqrt[a + b*x^2])/8 + ((4*a*B + 15*A*b*x)*(a + b*x^2)^
(3/2))/12 - ((5*A - B*x)*(a + b*x^2)^(5/2))/(5*x) + (15*a^2*A*Sqrt[b]*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - a^(5/2)*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx = -\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x} - \frac{1}{2}\int \frac{(-2aB-10Abx)\left(a+bx^2\right)^{3/2}}{x}\,dx$$

$$= \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x} - \frac{\int \frac{\left(-8a^2bB-30aAb^2x\right)\sqrt{a+bx^2}}{x}}{8b}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx^2\right.}{5x}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx^2\right.}{5x}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx^2\right.}{5x}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx^2\right.}{5x}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx^2\right.}{5x}$$

Mathematica [C] time = 0.23, size = 117, normalized size = 0.86

$$-a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{a^3A\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{a+bx^2}}+\frac{1}{15}B\sqrt{a+bx^2}\left(23a^2+11abx^2+3b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (B*Sqrt[a + b*x^2]*(23*a^2 + 11*a*b*x^2 + 3*b^2*x^4))/15 - a^(5/2)*B*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]] - (a^3*A*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-
5/2, -1/2, 1/2, -((b*x^2)/a)])/(x*Sqrt[a + b*x^2])
```

fricas [A] time = 1.02, size = 519, normalized size = 3.82

$$\left[\frac{225\,Aa^2\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+120\,Ba^{\frac{5}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(24\,Bb^2x^5+30\,A\right.}{240\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/240*(225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 120*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x -
 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/120*(225*A*a^2*sqrt(-b)*x*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - 60*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) - (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^
2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, 1/240*(240*B*sqrt(-a)*a^2*
x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*s
qrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x
^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/120*(2
25*A*a^2*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 120*B*sqrt(-a)*a^2
*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a
*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x]
```

giac [A] time = 0.50, size = 150, normalized size = 1.10

$$\frac{2\,Ba^3\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{15}{8}\,Aa^2\sqrt{b}\,\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)+\frac{2\,Aa^3\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{120}\left(184\,Ba^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*A*a
^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^3*sqrt(b)/((sqrt(
b)*x - sqrt(b*x^2 + a))^2 - a) + 1/120*(184*B*a^2 + (135*A*a*b + 2*(44*B*a*
b + 3*(4*B*b^2*x + 5*A*b^2)*x)*x)*x)*sqrt(b*x^2 + a)
```

maple [A] time = 0.01, size = 158, normalized size = 1.16

$$\frac{15A\,a^2\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8}-B\,a^{\frac{5}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\frac{15\sqrt{b\,x^2+a}\,Aabx}{8}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}Abx}{4}+\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x^2,x)
```

```
[Out] -A/a/x*(b*x^2+a)^(7/2)+A/a*b*x*(b*x^2+a)^(5/2)+5/4*A*b*x*(b*x^2+a)^(3/2)+15
/8*A*a*b*x*(b*x^2+a)^(1/2)+15/8*A*a^2*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
+1/5*B*(b*x^2+a)^(5/2)+1/3*B*a*(b*x^2+a)^(3/2)-B*a^(5/2)*ln((2*a+2*(b*x^2+a
)^(1/2)*a^(1/2))/x)+B*(b*x^2+a)^(1/2)*a^2
```

maxima [A] time = 1.40, size = 120, normalized size = 0.88

$$\frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Abx+\frac{15}{8}\sqrt{bx^2+a}\,Aabx+\frac{15}{8}Aa^2\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-Ba^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}B+\frac{1}{3}\left(bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*A*b*x + 15/8*sqrt(b*x^2 + a)*A*a*b*x + 15/8*A*a^2*sqr
t(b)*arcsinh(b*x/sqrt(a*b)) - B*a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/5
*(b*x^2 + a)^(5/2)*B + 1/3*(b*x^2 + a)^(3/2)*B*a + sqrt(b*x^2 + a)*B*a^2 -
(b*x^2 + a)^(5/2)*A/x
```

mupad [B] time = 2.16, size = 104, normalized size = 0.76

$$\frac{B\left(b\,x^2+a\right)^{5/2}}{5}+B\,a^2\,\sqrt{b\,x^2+a}+\frac{B\,a\left(b\,x^2+a\right)^{3/2}}{3}-\frac{A\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{5/2}}+B\,a^{5/2}\operatorname{atan}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x^2,x)
```

```
[Out] (B*(a + b*x^2)^(5/2))/5 + B*a^2*(a + b*x^2)^(1/2) + B*a^(5/2)*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*1i + (B*a*(a + b*x^2)^(3/2))/3 - (A*(a + b*x^2)^(5/
2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a + 1)^(5/2))
```

sympy [A] time = 18.91, size = 318, normalized size = 2.34

$$-\frac{Aa^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+Aa^{\frac{3}{2}}bx\sqrt{1+\frac{bx^2}{a}}-\frac{7Aa^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}\,b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{Ab^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-Ba^{\frac{5}{2}}\operatorname{asinh}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x**2,x)
```

```
[Out] -A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + A*a**(3/2)*b*x*sqrt(1 + b*x**2/a) - 7*
A*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*b**2*x**3/(8*sqrt(1 + b
*x**2/a)) + 15*A*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/8 + A*b**3*x**5/(4*s
qrt(a)*sqrt(1 + b*x**2/a)) - B*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**3
/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) +
 2*B*a*b*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b),
True)) + B*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt
(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4,
True))
```

3.21 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3} \, dx$

Optimal. Leaf size=141

$$-\frac{5}{2}a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}(2A-Bx)}{4x^2}-\frac{5\left(a+bx^2\right)^{3/2}(3aB-\cdots}{12x}$$

```
[Out] -5/12*(-2*A*b*x+3*B*a)*(b*x^2+a)^(3/2)/x-1/4*(-B*x+2*A)*(b*x^2+a)^(5/2)/x^2
-5/2*a^(3/2)*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))+15/8*a^2*B*arctanh(x*b^(1
/2)/(b*x^2+a)^(1/2))*b^(1/2)+5/8*a*b*(3*B*x+4*A)*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {813, 815, 844, 217, 206, 266, 63, 208}

$$-\frac{5}{2}a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}(2A-Bx)}{4x^2}-\frac{5\left(a+bx^2\right)^{3/2}(3aB-\cdots}{12x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] (5*a*b*(4*A + 3*B*x)*Sqrt[a + b*x^2])/8 - (5*(3*a*B - 2*A*b*x)*(a + b*x^2)^
(3/2))/(12*x) - ((2*A - B*x)*(a + b*x^2)^(5/2))/(4*x^2) + (15*a^2*Sqrt[b]*B
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - (5*a^(3/2)*A*b*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx &= -\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}-\frac{5}{16}\int \frac{(-4aB-8Abx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx\\
&= -\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}+\frac{5}{32}\int \frac{(16aAb+24abBx}{x}\\
&= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}\\
&= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}\\
&= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}\\
&= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}\\
&= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 92, normalized size = 0.65

$$
\frac{Ab\left(a+bx^2\right)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^2}-\frac{a^2B\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] -((a^2*B*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -1/2, 1/2, -((b*x^2)/a)])/
(x*Sqrt[1 + (b*x^2)/a])) + (A*b*(a + b*x^2)^(7/2)*Hypergeometric2F1[2, 7/2,
 9/2, 1 + (b*x^2)/a])/(7*a^2)
```

fricas [A] time = 0.89, size = 535, normalized size = 3.79

$$\left[\frac{45\,Ba^2\sqrt{b}\,x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+60\,Aa^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(6\,Bb^2x^5+8\,Ab^2\right.}{48\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/48*(45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 60*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x -
12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/24*(45*B*a^2*sqrt(-b)*x^2*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - 30*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x
^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, 1/48*(120*A*sqrt(-a)*a*b*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 -
2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*
x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/24*(45
*B*a^2*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 60*A*sqrt(-a)*a*b*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*
b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2]
```

giac [A] time = 0.57, size = 219, normalized size = 1.55

$$\frac{5\,Aa^2b\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{15}{8}\,Ba^2\sqrt{b}\,\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)+\frac{1}{24}\left(56\,Aab+\left(27\,Bab+2\left(3\,Bb^2x+4\,A\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 5*A*a^2*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*B
*a^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/24*(56*A*a*b + (27*
B*a*b + 2*(3*B*b^2*x + 4*A*b^2)*x)*x)*sqrt(b*x^2 + a) + ((sqrt(b)*x - sqrt(
b*x^2 + a))^3*A*a^2*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b) + (
sqrt(b)*x - sqrt(b*x^2 + a))*A*a^3*b - 2*B*a^4*sqrt(b))/((sqrt(b)*x - sqrt(
b*x^2 + a))^2 - a)^2
```

maple [A] time = 0.01, size = 181, normalized size = 1.28

$$-\frac{5A\,a^{\frac{3}{2}}b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2}+\frac{15B\,a^2\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8}+\frac{15\sqrt{b\,x^2+a}\,Babx}{8}+\frac{5\sqrt{b\,x^2+a}\,Aab}{2}+\frac{5\left(b\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x^3,x)
```

```
[Out] -1/2*A/a/x^2*(b*x^2+a)^(7/2)+1/2*A/a*b*(b*x^2+a)^(5/2)+5/6*A*b*(b*x^2+a)^(3
/2)-5/2*A*a^(3/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+5/2*A*a*b*(b*x^2+
a)^(1/2)-B/a/x*(b*x^2+a)^(7/2)+B/a*b*x*(b*x^2+a)^(5/2)+5/4*B*b*x*(b*x^2+a)^
(3/2)+15/8*B*a*b*x*(b*x^2+a)^(1/2)+15/8*B*a^2*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2))
```

maxima [A] time = 1.38, size = 143, normalized size = 1.01

$$\frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bbx+\frac{15}{8}\sqrt{bx^2+a}\,Babx+\frac{15}{8}Ba^2\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{5}{2}Aa^{\frac{3}{2}}b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{5}{6}\left(bx^2+a\right)^{\frac{3}{2}}Ab+\frac{(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*B*b*x + 15/8*sqrt(b*x^2 + a)*B*a*b*x + 15/8*B*a^2*sqr
t(b)*arcsinh(b*x/sqrt(a*b)) - 5/2*A*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x)))
 + 5/6*(b*x^2 + a)^(3/2)*A*b + 1/2*(b*x^2 + a)^(5/2)*A*b/a + 5/2*sqrt(b*x^2
 + a)*A*a*b - (b*x^2 + a)^(5/2)*B/x - 1/2*(b*x^2 + a)^(7/2)*A/(a*x^2)
```

mupad [B] time = 2.59, size = 111, normalized size = 0.79

$$\frac{A\,b\left(b\,x^2+a\right)^{3/2}}{3}+2\,A\,a\,b\,\sqrt{b\,x^2+a}-\frac{A\,a^2\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{B\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{5/2}}+\frac{A\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2\cdots}}{\sqrt{\cdots}}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x^3,x)
```

```
[Out] (A*b*(a + b*x^2)^(3/2))/3 + 2*A*a*b*(a + b*x^2)^(1/2) - (A*a^2*(a + b*x^2)^
(1/2))/(2*x^2) + (A*a^(3/2)*b*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2 -
(B*(a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/
a + 1)^(5/2))
```

sympy [A] time = 12.98, size = 279, normalized size = 1.98

$$-\frac{5Aa^{\frac{3}{2}}b\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{2Aa^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{2Aab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}+Ab^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)-\frac{Ba^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x**3,x)
```

```
[Out] -5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqrt(a/(b*x**
2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + 2*A*a*b**(3/2)*
x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a +
 b*x**2)**(3/2)/(3*b), True)) - B*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + B*a**(3
/2)*b*x*sqrt(1 + b*x**2/a) - 7*B*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*B*
sqrt(a)*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*B*a**2*sqrt(b)*asinh(sqrt(b)*
x/sqrt(a))/8 + B*b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.22 $\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=104

$$\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{a\sqrt{a+bx^2}\,(16A+9Bx)}{24b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

```
[Out] 3/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/3*A*x^2*(b*x^2+a)^(1
/2)/b+1/4*B*x^3*(b*x^2+a)^(1/2)/b-1/24*a*(9*B*x+16*A)*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 780, 217, 206}

$$\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{a\sqrt{a+bx^2}\,(16A+9Bx)}{24b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (A*x^2*Sqrt[a + b*x^2])/(3*b) + (B*x^3*Sqrt[a + b*x^2])/(4*b) - (a*(16*A +
9*B*x)*Sqrt[a + b*x^2])/(24*b^2) + (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{Bx^3\sqrt{a+bx^2}}{4b}+\frac{\int \frac{x^2(-3aB+4Abx)}{\sqrt{a+bx^2}}\,dx}{4b}$$
$$= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}+\frac{\int \frac{x(-8aAb-9abBx)}{\sqrt{a+bx^2}}\,dx}{12b^2}$$
$$= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{(3a^2B)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2}$$
$$= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{(3a^2B)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a\cdot}}\right.}{8b^2}$$
$$= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

Mathematica [**A**] time = 0.05, size = 76, normalized size = 0.73

$$\frac{9a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\sqrt{b}\sqrt{a+bx^2}\left(-16aA-9aBx+8Abx^2+6bBx^3\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(-16*a*A - 9*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) + 9*a^
2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(24*b^(5/2))
```

fricas [**A**] time = 0.67, size = 158, normalized size = 1.52

$$\left[\frac{9\,Ba^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(6\,Bb^2x^3+8\,Ab^2x^2-9\,Babx-16\,Aab\right)\sqrt{bx^2+a}}{48\,b^3},-\frac{9\,Ba^2\sqrt{-b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(6*B*b^2*x^3 + 8*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3, -1
/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b^2*x^3 + 8
*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3]
```

giac [**A**] time = 0.50, size = 74, normalized size = 0.71

$$\frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(\frac{3\,Bx}{b}+\frac{4\,A}{b}\right)x-\frac{9\,Ba}{b^2}\right)x-\frac{16\,Aa}{b^2}\right)-\frac{3\,Ba^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x^2 + a)*((2*(3*B*x/b + 4*A/b)*x - 9*B*a/b^2)*x - 16*A*a/b^2) -
 3/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [**A**] time = 0.01, size = 96, normalized size = 0.92

$$\frac{\sqrt{b\,x^2+a}\;B\,x^3}{4b}+\frac{\sqrt{b\,x^2+a}\;A\,x^2}{3b}+\frac{3B\,a^2\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}-\frac{3\sqrt{b\,x^2+a}\;Bax}{8b^2}-\frac{2\sqrt{b\,x^2+a}\;Aa}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x^3*(b*x^2+a)^(1/2)/b-3/8*B*a/b^2*x*(b*x^2+a)^(1/2)+3/8*B*a^2/b^(5/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/3*A*x^2*(b*x^2+a)^(1/2)/b-2/3*A*a/b^2*(b*x
^2+a)^(1/2)
```

maxima [A] time = 1.36, size = 88, normalized size = 0.85

$$\frac{\sqrt{bx^2+a}\,Bx^3}{4\,b}+\frac{\sqrt{bx^2+a}\,Ax^2}{3\,b}-\frac{3\,\sqrt{bx^2+a}\,Bax}{8\,b^2}+\frac{3\,Ba^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{2\,\sqrt{bx^2+a}\,Aa}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*B*x^3/b + 1/3*sqrt(b*x^2 + a)*A*x^2/b - 3/8*sqrt(b*x^2
+ a)*B*a*x/b^2 + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/3*sqrt(b*x^2
+ a)*A*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(1/2), x)
```

sympy [A] time = 7.91, size = 150, normalized size = 1.44

$$A\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)-\frac{3Ba^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Bx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b),
Ne(b, 0)), (x**4/(4*sqrt(a)), True)) - 3*B*a**(3/2)*x/(8*b**2*sqrt(1 + b*x*
*2/a)) - B*sqrt(a)*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x
/sqrt(a))/(8*b**(5/2)) + B*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.23 $\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}} dx$

Optimal. Leaf size=81

$$-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{\sqrt{a+bx^2}\,(4aB-3Abx)}{6b^2}+\frac{Bx^2\sqrt{a+bx^2}}{3b}$$

```
[Out] -1/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/3*B*x^2*(b*x^2+a)^(1/
2)/b-1/6*(-3*A*b*x+4*B*a)*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 780, 217, 206}

$$-\frac{\sqrt{a+bx^2}\,(4aB-3Abx)}{6b^2}-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx^2\sqrt{a+bx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x^2*Sqrt[a + b*x^2])/(3*b) - ((4*a*B - 3*A*b*x)*Sqrt[a + b*x^2])/(6*b^2)
 - (a*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{Bx^2\sqrt{a+bx^2}}{3b} + \frac{\int \frac{x(-2aB+3Abx)}{\sqrt{a+bx^2}}\,dx}{3b}$$
$$= \frac{Bx^2\sqrt{a+bx^2}}{3b} - \frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2} - \frac{(aA)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$
$$= \frac{Bx^2\sqrt{a+bx^2}}{3b} - \frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2} - \frac{(aA)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}$$
$$= \frac{Bx^2\sqrt{a+bx^2}}{3b} - \frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2} - \frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Mathematica [**A**] time = 0.06, size = 64, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\,(bx(3A+2Bx)-4aB) - 3aA\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-4*a*B + b*x*(3*A + 2*B*x)) - 3*a*A*Sqrt[b]*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(6*b^2)
```

fricas [**A**] time = 0.91, size = 127, normalized size = 1.57

$$\left[\frac{3\,Aa\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,Bbx^2+3\,Abx-4\,Ba\right)\sqrt{bx^2+a}}{12\,b^2}, \frac{3\,Aa\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 + a))/b^2, 1/6*(3*A*a*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 +
a))/b^2]
```

giac [**A**] time = 0.52, size = 61, normalized size = 0.75

$$\frac{1}{6}\,\sqrt{bx^2+a}\left(\left(\frac{2\,Bx}{b}+\frac{3\,A}{b}\right)x-\frac{4\,Ba}{b^2}\right)+\frac{Aa\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*((2*B*x/b + 3*A/b)*x - 4*B*a/b^2) + 1/2*A*a*log(abs(-sq
rt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

maple [**A**] time = 0.01, size = 75, normalized size = 0.93

$$\frac{\sqrt{b\,x^2+a}\;B\,x^2}{3b}-\frac{Aa\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{b\,x^2+a}\;Ax}{2b}-\frac{2\sqrt{b\,x^2+a}\;Ba}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*B*x^2*(b*x^2+a)^(1/2)/b-2/3*B*a/b^2*(b*x^2+a)^(1/2)+1/2*A*x/b*(b*x^2+a)
^(1/2)-1/2*A*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.29, size = 67, normalized size = 0.83

$$\frac{\sqrt{bx^2+a}\,Bx^2}{3\,b}+\frac{\sqrt{bx^2+a}\,Ax}{2\,b}-\frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}-\frac{2\,\sqrt{bx^2+a}\,Ba}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*B*x^2/b + 1/2*sqrt(b*x^2 + a)*A*x/b - 1/2*A*a*arcsinh(b
*x/sqrt(a*b))/b^(3/2) - 2/3*sqrt(b*x^2 + a)*B*a/b^2
```

mupad [B] time = 1.47, size = 93, normalized size = 1.15

$$\begin{cases} \frac{3\,B\,x^4+4\,A\,x^3}{12\,\sqrt{a}} & \text{if } b=0\\ \frac{A\,x\,\sqrt{b\,x^2+a}}{2\,b}-\frac{A\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (4*A*x^3 + 3*B*x^4)/(12*a^(1/2)), b ~= 0, - (A*a*log(2*b^
(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (A*x*(a + b*x^2)^(1/2))/(2*b)
 - (B*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2))
```

sympy [A] time = 6.23, size = 94, normalized size = 1.16

$$\frac{A\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Aa\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+B\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - A*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) + B*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)
/(3*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))
```

3.24 $\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{a+bx^2}\,(2A+Bx)}{2b}-\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*(B*x+2*A)*(b*x^2+a)
^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {780, 217, 206}

$$\frac{\sqrt{a+bx^2}\,(2A+Bx)}{2b}-\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) - (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx &= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b}-\frac{(aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\
&= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b}-\frac{(aB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\
&= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b}-\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 1.02

$$\frac{\sqrt{b}\sqrt{a+bx^2}(2A+Bx)-aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*(2*A + B*x)*Sqrt[a + b*x^2] - a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*b^(3/2))
```

fricas [A] time = 1.19, size = 109, normalized size = 1.95

$$\left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(Bbx+2\,Ab)\sqrt{bx^2+a}}{4\,b^2},\ \frac{Ba\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(Bbx+2\,Ab)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(B*b*
x + 2*A*b)*sqrt(b*x^2 + a))/b^2, 1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) + (B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.48, size = 50, normalized size = 0.89

$$\frac{1}{2}\sqrt{bx^2+a}\left(\frac{Bx}{b}+\frac{2\,A}{b}\right)+\frac{Ba\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*(B*x/b + 2*A/b) + 1/2*B*a*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

maple [A] time = 0.00, size = 55, normalized size = 0.98

$$-\frac{Ba\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{b\,x^2+a}\,Bx}{2b}+\frac{\sqrt{b\,x^2+a}\,A}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*B*x/b*(b*x^2+a)^(1/2)-1/2*B*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+A/b
*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 47, normalized size = 0.84

$$\frac{\sqrt{bx^2+a}\,Bx}{2\,b}-\frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^2+a}\,A}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x/b - 1/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + sqrt(b
*x^2 + a)*A/b
```

mupad [B] time = 1.24, size = 82, normalized size = 1.46

$$\begin{cases} \frac{2\,B\,x^3+3\,A\,x^2}{6\,\sqrt{a}} & \text{if } b=0 \\ \frac{A\,\sqrt{b\,x^2+a}}{b}-\frac{B\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{B\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x))/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (3*A*x^2 + 2*B*x^3)/(6*a^(1/2)), b ~= 0, (A*(a + b*x^2)^(
1/2))/b - (B*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (B*x*(
a + b*x^2)^(1/2))/(2*b))
```

sympy [A] time = 6.26, size = 70, normalized size = 1.25

$$A\left(\begin{cases} \frac{x^2}{2\sqrt{a}} & \text{for } b=0 \\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise} \end{cases}\right)+\frac{B\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, True)) + B*s
qrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - B*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2
))
```

3.25 $\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

```
[Out] A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+B*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {641, 217, 206}

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx &= \frac{B\sqrt{a+bx^2}}{b}+A\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{B\sqrt{a+bx^2}}{b}+A\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{B\sqrt{a+bx^2}}{b}+\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 46, normalized size = 1.07

$$\frac{A\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/Sqrt[b]
```

fricas [A] time = 0.93, size = 92, normalized size = 2.14

$$\left[\frac{A\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\sqrt{bx^2+a}\,B}{2\,b},\,-\frac{A\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,B}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b*
x^2 + a)*B)/b, -(A*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(b*x^2
 + a)*B)/b]
```

giac [A] time = 0.55, size = 39, normalized size = 0.91

$$-\frac{A\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}+\frac{\sqrt{bx^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -A*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

maple [A] time = 0.01, size = 37, normalized size = 0.86

$$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] B*(b*x^2+a)^(1/2)/b+A*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.36, size = 29, normalized size = 0.67

$$\frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}+\frac{\sqrt{bx^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] A*arcsinh(b*x/sqrt(a*b))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

mupad [B] time = 1.14, size = 36, normalized size = 0.84

$$\frac{B\,\sqrt{b\,x^2+a}}{b}+\frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(a + b*x^2)^(1/2),x)
```

```
[Out] (B*(a + b*x^2)^(1/2))/b + (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2)
```

sympy [B] time = 2.64, size = 102, normalized size = 2.37

$$A\left(\begin{cases} \frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0 \wedge b<0 \\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0 \wedge b>0 \\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0 \wedge a<0 \end{cases}\right) + B\left(\begin{cases} \frac{x^2}{2\sqrt{a}} & \text{for } b=0 \\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sq
rt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x
*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + B*Piecewise((x**2/(2*sqrt(a)),
 Eq(b, 0)), (sqrt(a + b*x**2)/b, True))
```

3.26 $\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(
1/2))/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {844, 217, 206, 266, 63, 208}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx &= A\int \frac{1}{x\sqrt{a+bx^2}}\,dx + B\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}A\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) + B\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) \\
&= \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} + \frac{A\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.00

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 0.67, size = 273, normalized size = 5.15

$$\left[\frac{Ba\sqrt{b}\,\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right) + A\sqrt{a}\,b\log\left(-\frac{bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{a} + 2\,a}{x^2}\right)}{2\,ab}, -\frac{2\,Ba\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right) - A\sqrt{}}{2\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + A*sqrt(
a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), -1/2*(2*B*
a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*log(-(b*x^2 - 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), 1/2*(2*A*sqrt(-a)*b*arctan(sqr
t(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a))/(a*b), -(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqr
t(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a*b)]
```

giac [A] time = 0.50, size = 58, normalized size = 1.09

$$\frac{2\,A\arctan\left(-\frac{\sqrt{b}\,x - \sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{B\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*log(abs(-s
qrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

maple [A] time = 0.01, size = 52, normalized size = 0.98

$$-\frac{A\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{B\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(1/2),x)
```

```
[Out] B*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-A/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)
*a^(1/2))/x)
```

maxima [A] time = 1.38, size = 33, normalized size = 0.62

$$\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] B*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

mupad [B] time = 1.30, size = 42, normalized size = 0.79

$$\frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (A*atanh((a + b*x^2)^(1/2)
/a^(1/2)))/a^(1/2)
```

sympy [A] time = 5.14, size = 99, normalized size = 1.87

$$-\frac{A\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}+B\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\,\mathrm{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\,\mathrm{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\,\mathrm{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + B*Piecewise((sqrt(-a/b)*asin(x*sqrt
(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a),
 (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a
 < 0)))
```

3.27 $\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)-A*(b*x^2+a)^(1/2)/a/x
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {807, 266, 63, 208}

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{ax} + B\int \frac{1}{x\sqrt{a+bx^2}}\,dx$$
$$= -\frac{A\sqrt{a+bx^2}}{ax} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= -\frac{A\sqrt{a+bx^2}}{ax} + \frac{B\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}$$
$$= -\frac{A\sqrt{a+bx^2}}{ax} - \frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.00

$$-\frac{A\sqrt{a+bx^2}}{ax} - \frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 0.95, size = 101, normalized size = 2.15

$$\left[\frac{B\sqrt{a}\,x\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}\,A}{2\,ax}, \frac{B\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,A}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*s
qrt(b*x^2 + a)*A)/(a*x), (B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - s
qrt(b*x^2 + a)*A)/(a*x)]
```

giac [A] time = 0.51, size = 65, normalized size = 1.38

$$\frac{2\,B\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\,A\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 2*A*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.01, size = 49, normalized size = 1.04

$$-\frac{B\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}} - \frac{\sqrt{b\,x^2+a}\;A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] -A*(b*x^2+a)^(1/2)/a/x-B/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.31, size = 37, normalized size = 0.79

$$-\frac{B \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}} - \frac{\sqrt{bx^2 + a}\,A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -B*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - sqrt(b*x^2 + a)*A/(a*x)
```

mupad [B] time = 1.20, size = 39, normalized size = 0.83

$$-\frac{B \operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{A\,\sqrt{b\,x^2+a}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (A*(a + b*x^2)^(1/2))/(a*x
)
```

sympy [A] time = 2.80, size = 41, normalized size = 0.87

$$-\frac{A\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{B \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

3.28 $\int \frac{A+Bx}{x^3 \sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=72

$$\frac{Ab \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{A\sqrt{a+bx^2}}{2ax^2} - \frac{B\sqrt{a+bx^2}}{ax}$$

```
[Out] 1/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*A*(b*x^2+a)^(1/2)/a/x^
2-B*(b*x^2+a)^(1/2)/a/x
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {835, 807, 266, 63, 208}

$$\frac{Ab \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{A\sqrt{a+bx^2}}{2ax^2} - \frac{B\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) - (B*Sqrt[a + b*x^2])/(a*x) + (A*b*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{\int \frac{-2aB+Abx}{x^2\sqrt{a+bx^2}}\,dx}{2a}\\ &= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{(Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a}\\ &= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{(Ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a}\\ &= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{A\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a}\\ &= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}+\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 63, normalized size = 0.88

$$\frac{\sqrt{a+bx^2}\left(\frac{Ab\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}-\frac{a(A+2Bx)}{x^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-((a*(A + 2*B*x))/x^2) + (A*b*ArcTanh[Sqrt[1 + (b*x^2)/a]
])/Sqrt[1 + (b*x^2)/a]))/(2*a^2)
```

fricas [A] time = 0.83, size = 123, normalized size = 1.71

$$\left[\frac{A\sqrt{a}\,bx^2\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2(2Bax+Aa)\sqrt{bx^2+a}}{4a^2x^2},-\frac{A\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2Bax+Aa)\sqrt{bx^2+a}}{2a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(A*sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) -
 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2), -1/2*(A*sqrt(-a)*b*x^2*arcta
n(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2)]
```

giac [B] time = 0.43, size = 146, normalized size = 2.03

$$-\frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + ((sqrt(
b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(
b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^2*a)
```

maple [A] time = 0.01, size = 68, normalized size = 0.94

$$\frac{Ab\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}\,B}{ax}-\frac{\sqrt{b\,x^2+a}\,A}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*A*(b*x^2+a)^(1/2)/a/x^2+1/2*A*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)-B*(b*x^2+a)^(1/2)/a/x
```

maxima [A] time = 1.31, size = 56, normalized size = 0.78

$$\frac{Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}\,B}{ax}-\frac{\sqrt{bx^2+a}\,A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - sqrt(b*x^2 + a)*B/(a*x) - 1
/2*sqrt(b*x^2 + a)*A/(a*x^2)
```

mupad [B] time = 1.35, size = 58, normalized size = 0.81

$$\frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}}{a\,x}-\frac{A\,\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (B*(a + b*x^2)^(1/2))/
(a*x) - (A*(a + b*x^2)^(1/2))/(2*a*x^2)
```

sympy [A] time = 7.06, size = 66, normalized size = 0.92

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2
*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

3.29 $\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{a+bx^2}\,(4A+3Bx)}{2b^2}-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

```
[Out] -3/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-x^2*(B*x+A)/b/(b*x^2+a)
^(1/2)+1/2*(3*B*x+4*A)*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {819, 780, 217, 206}

$$\frac{\sqrt{a+bx^2}\,(4A+3Bx)}{2b^2}-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x^2*(A + B*x))/(b*Sqrt[a + b*x^2])) + ((4*A + 3*B*x)*Sqrt[a + b*x^2])/(2
*b^2) - (3*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{\int \frac{x(2aA+3aBx)}{\sqrt{a+bx^2}}\,dx}{ab}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{(3aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{(3aB)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 72, normalized size = 0.89

$$
\frac{a(4A+3Bx)+bx^2(2A+Bx)}{2b^2\sqrt{a+bx^2}}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (b*x^2*(2*A + B*x) + a*(4*A + 3*B*x))/(2*b^2*Sqrt[a + b*x^2]) - (3*a*B*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

fricas [A] time = 1.00, size = 197, normalized size = 2.43

$$
\left[\frac{3\left(Babx^2+Ba^2\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(Bb^2x^3+2\,Ab^2x^2+3\,Babx+4\,Aab\right)\sqrt{bx^2+a}}{4\left(b^4x^2+ab^3\right)},\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) + 2*(B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqrt(b*x^2 + a
))/(b^4*x^2 + a*b^3), 1/2*(3*(B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) + (B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqrt(b*
x^2 + a))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 0.51, size = 70, normalized size = 0.86

$$
\frac{\left(\left(\frac{Bx}{b}+\frac{2\,A}{b}\right)x+\frac{3\,Ba}{b^2}\right)x+\frac{4\,Aa}{b^2}}{2\,\sqrt{bx^2+a}}+\frac{3\,Ba\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((B*x/b + 2*A/b)*x + 3*B*a/b^2)*x + 4*A*a/b^2)/sqrt(b*x^2 + a) + 3/2*B
*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 93, normalized size = 1.15

$$
\frac{B\,x^3}{2\sqrt{b\,x^2+a}\;b}+\frac{A\,x^2}{\sqrt{b\,x^2+a}\;b}+\frac{3Bax}{2\sqrt{b\,x^2+a}\;b^2}-\frac{3Ba\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{5}{2}}}+\frac{2Aa}{\sqrt{b\,x^2+a}\;b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*B*x^3/b/(b*x^2+a)^(1/2)+3/2*B*a/b^2*x/(b*x^2+a)^(1/2)-3/2*B*a/b^(5/2)*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))+A*x^2/b/(b*x^2+a)^(1/2)+2*A*a/b^2/(b*x^2+a)^(1
/2)
```

maxima [A] time = 1.35, size = 85, normalized size = 1.05

$$\frac{Bx^3}{2\sqrt{bx^2+a}\,b}+\frac{Ax^2}{\sqrt{bx^2+a}\,b}+\frac{3\,Bax}{2\sqrt{bx^2+a}\,b^2}-\frac{3\,Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}+\frac{2\,Aa}{\sqrt{bx^2+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^3/(sqrt(b*x^2 + a)*b) + A*x^2/(sqrt(b*x^2 + a)*b) + 3/2*B*a*x/(sqrt
(b*x^2 + a)*b^2) - 3/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 2*A*a/(sqrt(b*x
^2 + a)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x\right)}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(3/2),x)
```

```
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(3/2), x)
```

sympy [A] time = 10.34, size = 117, normalized size = 1.44

$$A\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{3\sqrt{a}\,x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b,
 0)), (x**4/(4*a**(3/2)), True)) + B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a
)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 +
 b*x**2/a)))
```

3.30 $\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}$$

```
[Out] A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-x*(B*x+A)/b/(b*x^2+a)^(1/2)+2*
B*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {819, 641, 217, 206}

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x*(A + B*x))/(b*Sqrt[a + b*x^2])) + (2*B*Sqrt[a + b*x^2])/b^2 + (A*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{x(A+Bx)}{b\sqrt{a+bx^2}} + \frac{\int \frac{aA+2aBx}{\sqrt{a+bx^2}}\,dx}{ab}$$
$$= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}} + \frac{2B\sqrt{a+bx^2}}{b^2} + \frac{A\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}$$
$$= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}} + \frac{2B\sqrt{a+bx^2}}{b^2} + \frac{A\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b}$$
$$= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}} + \frac{2B\sqrt{a+bx^2}}{b^2} + \frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 1.02

$$\frac{A\sqrt{b}\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+2aB+bx(Bx-A)}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*a*B + b*x*(-A + B*x) + A*Sqrt[b]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(b^2*Sqrt[a + b*x^2])
```

fricas [A] time = 0.98, size = 164, normalized size = 2.48

$$\left[\frac{\left(Abx^2+Aa\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(Bbx^2-Abx+2\,Ba\right)\sqrt{bx^2+a}}{2\left(b^3x^2+ab^2\right)}, -\frac{\left(Abx^2+Aa\right)\sqrt{-b}\,\text{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b*x^2 + A*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) + 2*(B*b*x^2 - A*b*x + 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((A*b
*x^2 + A*a)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*x^2 - A*b*x
+ 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2)]
```

giac [A] time = 0.55, size = 58, normalized size = 0.88

$$\frac{\left(\frac{Bx}{b}-\frac{A}{b}\right)x+\frac{2\,Ba}{b^2}}{\sqrt{bx^2+a}} - \frac{A\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((B*x/b - A/b)*x + 2*B*a/b^2)/sqrt(b*x^2 + a) - A*log(abs(-sqrt(b)*x + sqrt
(b*x^2 + a)))/b^(3/2)
```

maple [A] time = 0.01, size = 72, normalized size = 1.09

$$\frac{B\,x^2}{\sqrt{b\,x^2+a}\;b} - \frac{Ax}{\sqrt{b\,x^2+a}\;b} + \frac{A\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}} + \frac{2Ba}{\sqrt{b\,x^2+a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] B*x^2/b/(b*x^2+a)^(1/2)+2*B*a/b^2/(b*x^2+a)^(1/2)-A*x/b/(b*x^2+a)^(1/2)+A/b
^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.30, size = 64, normalized size = 0.97

$$\frac{Bx^2}{\sqrt{bx^2+a}\,b}-\frac{Ax}{\sqrt{bx^2+a}\,b}+\frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{2\,Ba}{\sqrt{bx^2+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] B*x^2/(sqrt(b*x^2 + a)*b) - A*x/(sqrt(b*x^2 + a)*b) + A*arcsinh(b*x/sqrt(a*
b))/b^(3/2) + 2*B*a/(sqrt(b*x^2 + a)*b^2)
```

mupad [B] time = 1.34, size = 61, normalized size = 0.92

$$\frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}-\frac{A\,x}{b\,\sqrt{b\,x^2+a}}+\frac{B\,\left(b\,x^2+2\,a\right)}{b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(3/2),x)
```

```
[Out] (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - (A*x)/(b*(a + b*x^2)^(1/2)
) + (B*(2*a + b*x^2))/(b^2*(a + b*x^2)^(1/2))
```

sympy [A] time = 16.61, size = 83, normalized size = 1.26

$$A\left(\frac{\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
B*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b,
 0)), (x**4/(4*a**(3/2)), True))
```

3.31 $\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

[Out] B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+(-B*x-A)/b/(b*x^2+a)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {778, 217, 206}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x))/(a + b*x^2)^(3/2),x]

[Out] -((A + B*x)/(b*Sqrt[a + b*x^2])) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{A+Bx}{b\sqrt{a+bx^2}}+\frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\ &= -\frac{A+Bx}{b\sqrt{a+bx^2}}+\frac{B\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\ &= -\frac{A+Bx}{b\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 64, normalized size = 1.33

$$\frac{\sqrt{a}B\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{b}(A+Bx)}{b^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[b]*(A + B*x)) + Sqrt[a]*B*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/S
qrt[a]])/(b^(3/2)*Sqrt[a + b*x^2])
```

fricas [A] time = 1.42, size = 147, normalized size = 3.06

$$\left[\frac{\left(Bbx^2+Ba\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(Bbx+Ab)\sqrt{bx^2+a}}{2\left(b^3x^2+ab^2\right)}, -\frac{\left(Bbx^2+Ba\right)\sqrt{-b}\arctan\left(\frac{\sqrt{}}{\sqrt{b}}\right.}{b^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*x^2 + B*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) - 2*(B*b*x + A*b)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((B*b*x^2 + B*a)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b*x + A*b)*sqrt(b*x^2 + a)
)/(b^3*x^2 + a*b^2)]
```

giac [A] time = 0.50, size = 48, normalized size = 1.00

$$-\frac{\frac{Bx}{b}+\frac{A}{b}}{\sqrt{bx^2+a}}-\frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*x/b + A/b)/sqrt(b*x^2 + a) - B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(3/2)
```

maple [A] time = 0.01, size = 54, normalized size = 1.12

$$-\frac{Bx}{\sqrt{b\,x^2+a}\;b}+\frac{B\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}-\frac{A}{\sqrt{b\,x^2+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -B*x/b/(b*x^2+a)^(1/2)+B/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-A/b/(b*x^2+a
)^(1/2)
```

maxima [A] time = 1.31, size = 46, normalized size = 0.96

$$-\frac{Bx}{\sqrt{bx^2+a}\,b}+\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}-\frac{A}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -B*x/(sqrt(b*x^2 + a)*b) + B*arcsinh(b*x/sqrt(a*b))/b^(3/2) - A/(sqrt(b*x^2
 + a)*b)
```

mupad [B] time = 1.06, size = 53, normalized size = 1.10

$$\frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}-\frac{A}{b\,\sqrt{b\,x^2+a}}-\frac{B\,x}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x))/(a + b*x^2)^(3/2),x)
```

```
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - A/(b*(a + b*x^2)^(1/2)) -
(B*x)/(b*(a + b*x^2)^(1/2))
```

sympy [A] time = 15.18, size = 66, normalized size = 1.38

$$A\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
 + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

3.32 $\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

```
[Out] (A*b*x-B*a)/a/b/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {637}

$$-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((a*B - A*b*x)/(a*b*Sqrt[a + b*x^2]))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 0.96

$$\frac{Abx-aB}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(a + b*x^2)^(3/2),x]
```

```
[Out] (-(a*B) + A*b*x)/(a*b*Sqrt[a + b*x^2])
```

fricas [A] time = 0.78, size = 35, normalized size = 1.25

$$\frac{(Abx-Ba)\sqrt{bx^2+a}}{ab^2x^2+a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (A*b*x - B*a)*sqrt(b*x^2 + a)/(a*b^2*x^2 + a^2*b)
```

giac [A] time = 0.44, size = 23, normalized size = 0.82

$$\frac{\frac{Ax}{a}-\frac{B}{b}}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (A*x/a - B/b)/sqrt(b*x^2 + a)
```

maple [A] time = 0.00, size = 26, normalized size = 0.93

$$\frac{Abx - Ba}{\sqrt{b\,x^2 + a}\;ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] (A*b*x-B*a)/a/b/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.37, size = 31, normalized size = 1.11

$$\frac{Ax}{\sqrt{bx^2 + a}\,a} - \frac{B}{\sqrt{bx^2 + a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] A*x/(sqrt(b*x^2 + a)*a) - B/(sqrt(b*x^2 + a)*b)
```

mupad [B] time = 0.91, size = 24, normalized size = 0.86

$$-\frac{\frac{B}{b} - \frac{A\,x}{a}}{\sqrt{b\,x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(a + b*x^2)^(3/2),x)
```

```
[Out] -(B/b - (A*x)/a)/(a + b*x^2)^(1/2)
```

sympy [A] time = 10.22, size = 46, normalized size = 1.64

$$\frac{Ax}{a^{\frac{3}{2}}\sqrt{1 + \frac{bx^2}{a}}} + B\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)), N
e(b, 0)), (x**2/(2*a**(3/2)), True))
```

3.33 $\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+(B*x+A)/a/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {823, 12, 266, 63, 208}

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx &= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{\int \frac{aAb}{x\sqrt{a+bx^2}}\,dx}{a^2b}\\ &= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a}\\ &= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\ &= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\ &= \frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 47, normalized size = 1.00

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2
)
```

fricas [A] time = 1.16, size = 146, normalized size = 3.11

$$\left[\frac{\left(Abx^2+Aa\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right)+2\left(Bax+Aa\right)\sqrt{bx^2+a}}{2\left(a^2bx^2+a^3\right)},\ \frac{\left(Abx^2+Aa\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(Ba\right.}{a^2bx^2+a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b*x^2 + A*a)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^2 + a^3), ((A*b*x^2 + A*
a)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (B*a*x + A*a)*sqrt(b*x^2 + a
))/(a^2*b*x^2 + a^3)]
```

giac [A] time = 0.44, size = 59, normalized size = 1.26

$$\frac{\frac{Bx}{a}+\frac{A}{a}}{\sqrt{bx^2+a}}+\frac{2A\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (B*x/a + A/a)/sqrt(b*x^2 + a) + 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/s
qrt(-a))/(sqrt(-a)*a)
```

maple [A] time = 0.01, size = 60, normalized size = 1.28

$$\frac{Bx}{\sqrt{b\,x^{2}+a}\;a}-\frac{A\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{a^{\frac{3}{2}}}+\frac{A}{\sqrt{b\,x^{2}+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(3/2),x)
```

```
[Out] B*x/a/(b*x^2+a)^(1/2)+A/a/(b*x^2+a)^(1/2)-A/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/
2)*a^(1/2))/x)
```

maxima [A] time = 1.30, size = 48, normalized size = 1.02

$$\frac{Bx}{\sqrt{bx^{2}+a}\,a}-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{3}{2}}}+\frac{A}{\sqrt{bx^{2}+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] B*x/(sqrt(b*x^2 + a)*a) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + A/(sqrt
(b*x^2 + a)*a)
```

mupad [B] time = 1.29, size = 50, normalized size = 1.06

$$\frac{A}{a\,\sqrt{b\,x^2+a}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{B\,x}{a\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x*(a + b*x^2)^(3/2)),x)
```

```
[Out] A/(a*(a + b*x^2)^(1/2)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) + (B
*x)/(a*(a + b*x^2)^(1/2))
```

sympy [B] time = 11.29, size = 206, normalized size = 4.38

$$A\left(\frac{2a^{3}\sqrt{1+\frac{bx^{2}}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}+\frac{a^{3}\log\left(\frac{bx^{2}}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}-\frac{2a^{3}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}+\frac{a^{2}bx^{2}\log\left(\frac{bx^{2}}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}-\frac{2a^{2}bx^{2}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*
x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) +
1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2)
 + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/
2) + 2*a**(7/2)*b*x**2)) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

3.34 $\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=70

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax\sqrt{a+bx^2}}$$

```
[Out] -B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+(B*x+A)/a/x/(b*x^2+a)^(1/2)-2*A
*(b*x^2+a)^(1/2)/a^2/x
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {823, 807, 266, 63, 208}

$$-\frac{2A\sqrt{a+bx^2}}{a^2x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{A+Bx}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x*Sqrt[a + b*x^2]) - (2*A*Sqrt[a + b*x^2])/(a^2*x) - (B*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx &= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{\int \frac{-2aAb-abBx}{x^2\sqrt{a+bx^2}}\,dx}{a^2b}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 1.03

$$-\frac{a(A-Bx)+\sqrt{a}\,Bx\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+2Abx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -((2*A*b*x^2 + a*(A - B*x) + Sqrt[a]*B*x*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(a^2*x*Sqrt[a + b*x^2]))
```

fricas [A] time = 1.15, size = 169, normalized size = 2.41

$$\left[\frac{\left(Bbx^3+Bax\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(2\,Abx^2-Bax+Aa\right)\sqrt{bx^2+a}}{2\left(a^2bx^3+a^3x\right)},\ \frac{\left(Bbx^3+Bax\right)\sqrt{-a}\,\arctan\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*x^3 + B*a*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) - 2*(2*A*b*x^2 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*x)
, ((B*b*x^3 + B*a*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*A*b*x^2
 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*x)]
```

giac [A] time = 0.50, size = 96, normalized size = 1.37

$$-\frac{\frac{Abx}{a^2}-\frac{B}{a}}{\sqrt{bx^2+a}}+\frac{2\,B\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(A*b*x/a^2 - B/a)/sqrt(b*x^2 + a) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 +
a))/sqrt(-a))/(sqrt(-a)*a) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)*a)
```

maple [A] time = 0.01, size = 80, normalized size = 1.14

$$-\frac{2Abx}{\sqrt{b\,x^2+a}\;a^2}-\frac{B\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)}{a^{\frac{3}{2}}}+\frac{B}{\sqrt{b\,x^2+a}\;a}-\frac{A}{\sqrt{b\,x^2+a}\;ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] -A/a/x/(b*x^2+a)^(1/2)-2*A*b/a^2*x/(b*x^2+a)^(1/2)+B/a/(b*x^2+a)^(1/2)-B/a^
(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.32, size = 68, normalized size = 0.97

$$-\frac{2\,Abx}{\sqrt{bx^2+a}\,a^2}-\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}}+\frac{B}{\sqrt{bx^2+a}\,a}-\frac{A}{\sqrt{bx^2+a}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*A*b*x/(sqrt(b*x^2 + a)*a^2) - B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) +
B/(sqrt(b*x^2 + a)*a) - A/(sqrt(b*x^2 + a)*a*x)
```

mupad [B] time = 1.45, size = 70, normalized size = 1.00

$$\frac{B}{a\,\sqrt{b\,x^2+a}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{A}{a\,x\,\sqrt{b\,x^2+a}}-\frac{2\,A\,b\,x}{a^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(3/2)),x)
```

```
[Out] B/(a*(a + b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) - A/
(a*x*(a + b*x^2)^(1/2)) - (2*A*b*x)/(a^2*(a + b*x^2)^(1/2))
```

sympy [B] time = 15.83, size = 235, normalized size = 3.36

$$A\left(-\frac{1}{a\sqrt{b}\,x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log}{2a^{\frac{9}{2}}+2a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**
2) + 1))) + B*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) +
 a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 +
b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)
/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) +
1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2))
```

3.35 $\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax^2\sqrt{a+bx^2}}$$

```
[Out] 3/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+(B*x+A)/a/x^2/(b*x^2+a)^(1
/2)-3/2*A*(b*x^2+a)^(1/2)/a^2/x^2-2*B*(b*x^2+a)^(1/2)/a^2/x
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {823, 835, 807, 266, 63, 208}

$$-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}+\frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x^2*Sqrt[a + b*x^2]) - (3*A*Sqrt[a + b*x^2])/(2*a^2*x^2) - (2*
B*Sqrt[a + b*x^2])/(a^2*x) + (3*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^
(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
```

```
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx &= \frac{A+Bx}{ax^2\sqrt{a+bx^2}}-\frac{\int \frac{-3aAb-2abBx}{x^3\sqrt{a+bx^2}}\,dx}{a^2b}\\
&= \frac{A+Bx}{ax^2\sqrt{a+bx^2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}+\frac{\int \frac{4a^2bB-3aAb^2x}{x^2\sqrt{a+bx^2}}\,dx}{2a^3b}\\
&= \frac{A+Bx}{ax^2\sqrt{a+bx^2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}-\frac{(3Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a^2}\\
&= \frac{A+Bx}{ax^2\sqrt{a+bx^2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}-\frac{(3Ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a^2}\\
&= \frac{A+Bx}{ax^2\sqrt{a+bx^2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}-\frac{(3A)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a^2}\\
&= \frac{A+Bx}{ax^2\sqrt{a+bx^2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 75, normalized size = 0.79

$$\frac{3Ab\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)-\frac{a(A+2Bx)}{x^2}-b(3A+4Bx)}{2a^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-((a*(A + 2*B*x))/x^2) - b*(3*A + 4*B*x) + 3*A*b*Sqrt[1 + (b*x^2)/a]*ArcTa
nh[Sqrt[1 + (b*x^2)/a]])/(2*a^2*Sqrt[a + b*x^2])
```

fricas [A] time = 1.23, size = 211, normalized size = 2.22

$$\left[\frac{3\left(Ab^2x^4+Aabx^2\right)\sqrt{a}\,\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(4\,Babx^3+3\,Aabx^2+2\,Ba^2x+Aa^2\right)\sqrt{bx^2+a}}{4\left(a^3bx^4+a^4x^2\right)},-\frac{3\left(Ab^2x^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) - 2*(4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2*x + A*a^2)*sqrt(b
*x^2 + a))/(a^3*b*x^4 + a^4*x^2), -1/2*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2*x +
 A*a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

giac [B] time = 0.50, size = 171, normalized size = 1.80

$$-\frac{\frac{Bbx}{a^2}+\frac{Ab}{a^2}}{\sqrt{bx^2+a}}-\frac{3\,Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3 Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{\cdots}\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*b*x/a^2 + A*b/a^2)/sqrt(b*x^2 + a) - 3*A*b*arctan(-(sqrt(b)*x - sqrt(b*
x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b +
 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 +
a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^2)
```

maple [A] time = 0.01, size = 101, normalized size = 1.06

$$-\frac{2Bbx}{\sqrt{b\,x^2+a}\;a^2}+\frac{3Ab\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)}{2a^{\frac{5}{2}}}-\frac{3Ab}{2\sqrt{b\,x^2+a}\;a^2}-\frac{B}{\sqrt{b\,x^2+a}\;ax}-\frac{A}{2\sqrt{b\,x^2+a}\;a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(1/2)-3/2*A/a^2*b/(b*x^2+a)^(1/2)+3/2*A/a^(5/2)*b*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-B/a/x/(b*x^2+a)^(1/2)-2*B*b/a^2*x/(b*x^
2+a)^(1/2)
```

maxima [A] time = 1.35, size = 89, normalized size = 0.94

$$-\frac{2\,Bbx}{\sqrt{bx^2+a}\,a^2}+\frac{3\,Ab\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{5}{2}}}-\frac{3\,Ab}{2\,\sqrt{bx^2+a}\,a^2}-\frac{B}{\sqrt{bx^2+a}\,ax}-\frac{A}{2\,\sqrt{bx^2+a}\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*B*b*x/(sqrt(b*x^2 + a)*a^2) + 3/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5
/2) - 3/2*A*b/(sqrt(b*x^2 + a)*a^2) - B/(sqrt(b*x^2 + a)*a*x) - 1/2*A/(sqrt
(b*x^2 + a)*a*x^2)
```

mupad [B] time = 1.59, size = 94, normalized size = 0.99

$$\frac{3\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}-\frac{3\,A\,b}{2\,a^2\,\sqrt{b\,x^2+a}}-\frac{A}{2\,a\,x^2\,\sqrt{b\,x^2+a}}-\frac{\sqrt{b\,x^2+a}\,\left(\frac{B}{a}+\frac{2\,B\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(3/2)),x)
```

```
[Out] (3*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2)) - (3*A*b)/(2*a^2*(a +
b*x^2)^(1/2)) - A/(2*a*x^2*(a + b*x^2)^(1/2)) - ((a + b*x^2)^(1/2)*(B/a + (
2*B*b*x^2)/a^2))/(a*x + b*x^3)
```

sympy [**A**] time = 10.79, size = 124, normalized size = 1.31

$$A\left(-\frac{1}{2a\sqrt{b}\,x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{5}{2}}}\right)+B\left(-\frac{1}{a\sqrt{b}\,x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/
(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(-1/(a*sq
rt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1)))
```

3.36 $\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}-\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

```
[Out] -1/3*x^2*(B*x+A)/b/(b*x^2+a)^(3/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
5/2)+1/3*(-3*B*x-2*A)/b^2/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {819, 778, 217, 206}

$$-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}-\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(x^2*(A + B*x))/(3*b*(a + b*x^2)^(3/2)) - (2*A + 3*B*x)/(3*b^2*Sqrt[a + b*
x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{x(2aA+3aBx)}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}$$
$$= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}} - \frac{2A+3Bx}{3b^2\sqrt{a+bx^2}} + \frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}$$
$$= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}} - \frac{2A+3Bx}{3b^2\sqrt{a+bx^2}} + \frac{B\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b^2}$$
$$= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}} - \frac{2A+3Bx}{3b^2\sqrt{a+bx^2}} + \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.12, size = 69, normalized size = 0.87

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}} - \frac{a(2A+3Bx)+bx^2(3A+4Bx)}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(a*(2*A + 3*B*x) + b*x^2*(3*A + 4*B*x))/(b^2*(a + b*x^2)^(3/2)) + (B*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

fricas [A] time = 1.06, size = 239, normalized size = 3.03

$$\left[\frac{3\left(Bb^2x^4 + 2\,Babx^2 + Ba^2\right)\sqrt{b}\,\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right) - 2\left(4\,Bb^2x^3 + 3\,Ab^2x^2 + 3\,Babx + 2\,Aab\right)\sqrt{b}}{6\left(b^5x^4 + 2\,ab^4x^2 + a^2b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*b^2*x^4 + 2*B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) - 2*(4*B*b^2*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a*b
)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3), -1/3*(3*(B*b^2*x^4 +
2*B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (4*B*b^2
*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a*b)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*b
^4*x^2 + a^2*b^3)]
```

giac [A] time = 0.48, size = 70, normalized size = 0.89

$$-\frac{\left(\left(\frac{4\,Bx}{b} + \frac{3\,A}{b}\right)x + \frac{3\,Ba}{b^2}\right)x + \frac{2\,Aa}{b^2}}{3\left(bx^2+a\right)^{\frac{3}{2}}} - \frac{B\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((4*B*x/b + 3*A/b)*x + 3*B*a/b^2)*x + 2*A*a/b^2)/(b*x^2 + a)^(3/2) -
B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 91, normalized size = 1.15

$$-\frac{B\,x^{3}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}} b}-\frac{A\,x^{2}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}} b}-\frac{2Aa}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}} b^{2}}-\frac{Bx}{\sqrt{b\,x^{2}+a}\;b^{2}}+\frac{B\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*B*x^3/b/(b*x^2+a)^(3/2)-B/b^2*x/(b*x^2+a)^(1/2)+B/b^(5/2)*ln(b^(1/2)*x
+(b*x^2+a)^(1/2))-A*x^2/b/(b*x^2+a)^(3/2)-2/3*A*a/b^2/(b*x^2+a)^(3/2)
```

maxima [A] time = 1.41, size = 102, normalized size = 1.29

$$-\frac{1}{3}\,Bx\left(\frac{3\,x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}\right)-\frac{Ax^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}-\frac{Bx}{3\,\sqrt{bx^{2}+a}\,b^{2}}+\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{2\,Aa}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*B*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - A*x^
2/((b*x^2 + a)^(3/2)*b) - 1/3*B*x/(sqrt(b*x^2 + a)*b^2) + B*arcsinh(b*x/sqr
t(a*b))/b^(5/2) - 2/3*A*a/((b*x^2 + a)^(3/2)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3}\,\left(A+B\,x\right)}{{\left(b\,x^{2}+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(5/2),x)
```

```
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(5/2), x)
```

sympy [A] time = 18.32, size = 400, normalized size = 5.06

$$A\left(\begin{cases} -\frac{2a}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}}-\frac{3bx^{2}}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}} & \text{for}\ b\neq 0\\ \frac{x^{4}}{4a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^{2}}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)
) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne
(b, 0)), (x**4/(4*a**(5/2)), True)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/
a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a
**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1
 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**
2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x
/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqr
t(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b
*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)))
```

3.37 $\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^2}}$$

```
[Out] -1/3*x^2*(-A*b*x+B*a)/a/b/(b*x^2+a)^(3/2)-2/3*B/b^2/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {805, 261}

$$-\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(x^2*(a*B - A*b*x))/(3*a*b*(a + b*x^2)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*x^
2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{(2B)\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx}{3b}\\ &= -\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 0.83

$$\frac{-2a^2B-3abBx^2+Ab^2x^3}{3ab^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*a^2*B - 3*a*b*B*x^2 + A*b^2*x^3)/(3*a*b^2*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.92, size = 63, normalized size = 1.19

$$\frac{\left(Ab^2x^3 - 3\,Babx^2 - 2\,Ba^2\right)\sqrt{bx^2 + a}}{3\left(ab^4x^4 + 2\,a^2b^3x^2 + a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(A*b^2*x^3 - 3*B*a*b*x^2 - 2*B*a^2)*sqrt(b*x^2 + a)/(a*b^4*x^4 + 2*a^2*
b^3*x^2 + a^3*b^2)
```

giac [A] time = 0.48, size = 36, normalized size = 0.68

$$\frac{\left(\frac{Ax}{a} - \frac{3\,B}{b}\right)x^2 - \frac{2\,Ba}{b^2}}{3\left(bx^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((A*x/a - 3*B/b)*x^2 - 2*B*a/b^2)/(b*x^2 + a)^(3/2)
```

maple [A] time = 0.01, size = 41, normalized size = 0.77

$$\frac{A\,x^3b^2 - 3Bab\,x^2 - 2B\,a^2}{3\left(b\,x^2 + a\right)^{\frac{3}{2}}\,a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(A*b^2*x^3-3*B*a*b*x^2-2*B*a^2)/(b*x^2+a)^(3/2)/a/b^2
```

maxima [A] time = 1.36, size = 70, normalized size = 1.32

$$-\frac{Bx^2}{\left(bx^2 + a\right)^{\frac{3}{2}}b} - \frac{Ax}{3\left(bx^2 + a\right)^{\frac{3}{2}}b} + \frac{Ax}{3\sqrt{bx^2 + a}\,ab} - \frac{2\,Ba}{3\left(bx^2 + a\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -B*x^2/((b*x^2 + a)^(3/2)*b) - 1/3*A*x/((b*x^2 + a)^(3/2)*b) + 1/3*A*x/(sqr
t(b*x^2 + a)*a*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2)
```

mupad [B] time = 0.97, size = 51, normalized size = 0.96

$$\frac{B\,a^2 - 3\,B\,a\,\left(b\,x^2 + a\right) + A\,b\,x\,\left(b\,x^2 + a\right) - A\,a\,b\,x}{3\,a\,b^2\left(b\,x^2 + a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(5/2),x)
```

```
[Out] (B*a^2 - 3*B*a*(a + b*x^2) + A*b*x*(a + b*x^2) - A*a*b*x)/(3*a*b^2*(a + b*x
^2)^(3/2))
```

sympy [B] time = 17.32, size = 141, normalized size = 2.66

$$\frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+B\left(\begin{cases}-\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}}-\frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a
)) + B*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*
x**2)) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)
), Ne(b, 0)), (x**4/(4*a**(5/2)), True))
```

3.38 $\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}} \, dx$

Optimal. Leaf size=50

$$\frac{-A-Bx}{3b(a+bx^2)^{3/2}}+\frac{Bx}{3ab\sqrt{a+bx^2}}$$

```
[Out] 1/3*(-B*x-A)/b/(b*x^2+a)^(3/2)+1/3*B*x/a/b/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {778, 191}

$$\frac{Bx}{3ab\sqrt{a+bx^2}}-\frac{A+Bx}{3b(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(A + B*x)/(3*b*(a + b*x^2)^(3/2)) + (B*x)/(3*a*b*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}} \, dx &= -\frac{A+Bx}{3b(a+bx^2)^{3/2}}+\frac{B\int \frac{1}{(a+bx^2)^{3/2}} \, dx}{3b}\\ &= -\frac{A+Bx}{3b(a+bx^2)^{3/2}}+\frac{Bx}{3ab\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.64

$$\frac{bBx^3-aA}{3ab(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(a*A) + b*B*x^3)/(3*a*b*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.66, size = 49, normalized size = 0.98

$$\frac{(Bbx^3-Aa)\sqrt{bx^2+a}}{3(ab^3x^4+2a^2b^2x^2+a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^3 - A*a)*sqrt(b*x^2 + a)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

giac [A] time = 0.52, size = 26, normalized size = 0.52

$$\frac{\frac{Bx^3}{a} - \frac{A}{b}}{3\left(bx^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(B*x^3/a - A/b)/(b*x^2 + a)^(3/2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.58

$$-\frac{-Bb\,x^3 + Aa}{3\left(b\,x^2 + a\right)^{\frac{3}{2}}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-B*b*x^3+A*a)/(b*x^2+a)^(3/2)/a/b
```

maxima [A] time = 1.31, size = 51, normalized size = 1.02

$$-\frac{Bx}{3\left(bx^2 + a\right)^{\frac{3}{2}}b} + \frac{Bx}{3\sqrt{bx^2 + a}\,ab} - \frac{A}{3\left(bx^2 + a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*B*x/((b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b) - 1/3*A/((b*
x^2 + a)^(3/2)*b)
```

mupad [B] time = 0.92, size = 34, normalized size = 0.68

$$\frac{B\,x^3}{3\,a\,{\left(b\,x^2+a\right)}^{3/2}} - \frac{A}{3\,b\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x))/(a + b*x^2)^(5/2),x)
```

```
[Out] (B*x^3)/(3*a*(a + b*x^2)^(3/2)) - A/(3*b*(a + b*x^2)^(3/2))
```

sympy [A] time = 13.98, size = 95, normalized size = 1.90

$$A\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2} + 3b^2x^2\sqrt{a+bx^2}} & \text{for}\ b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + \frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{3}{2}}bx^2\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne
(b, 0)), (x**2/(2*a**(5/2)), True)) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a)
 + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

3.39 $\int \frac{A+Bx}{(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=51

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}} + \frac{Abx - aB}{3ab\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/3*(A*b*x-B*a)/a/b/(b*x^2+a)^(3/2)+2/3*A*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {639, 191}

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}} - \frac{aB - Abx}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(a + b*x^2)^(5/2),x]
```

```
[Out] -(a*B - A*b*x)/(3*a*b*(a + b*x^2)^(3/2)) + (2*A*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}} dx &= -\frac{aB - Abx}{3ab\left(a+bx^2\right)^{3/2}} + \frac{(2A)\int \frac{1}{\left(a+bx^2\right)^{3/2}} dx}{3a} \\ &= -\frac{aB - Abx}{3ab\left(a+bx^2\right)^{3/2}} + \frac{2Ax}{3a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 0.84

$$\frac{-a^2B + 3aAbx + 2Ab^2x^3}{3a^2b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(a^2*B) + 3*a*A*b*x + 2*A*b^2*x^3)/(3*a^2*b*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.61, size = 62, normalized size = 1.22

$$\frac{\left(2\,Ab^2x^3 + 3\,Aabx - Ba^2\right)\sqrt{bx^2+a}}{3\left(a^2b^3x^4 + 2\,a^3b^2x^2 + a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*A*b^2*x^3 + 3*A*a*b*x - B*a^2)*sqrt(b*x^2 + a)/(a^2*b^3*x^4 + 2*a^3*
b^2*x^2 + a^4*b)
```

giac [A] time = 0.48, size = 37, normalized size = 0.73

$$\frac{\left(\frac{2\,Abx^2}{a^2}+\frac{3\,A}{a}\right)x-\frac{B}{b}}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*A*b*x^2/a^2 + 3*A/a)*x - B/b)/(b*x^2 + a)^(3/2)
```

maple [A] time = 0.00, size = 40, normalized size = 0.78

$$\frac{2A\,x^3b^2+3Axab-B\,a^2}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(2*A*b^2*x^3+3*A*a*b*x-B*a^2)/(b*x^2+a)^(3/2)/a^2/b
```

maxima [A] time = 1.37, size = 48, normalized size = 0.94

$$\frac{2\,Ax}{3\,\sqrt{bx^2+a}\,a^2}+\frac{Ax}{3\left(bx^2+a\right)^{\frac{3}{2}}a}-\frac{B}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B/((b*x
^2 + a)^(3/2)*b)
```

mupad [B] time = 0.93, size = 41, normalized size = 0.80

$$\frac{2\,A\,b\,x\,\left(b\,x^2+a\right)-B\,a^2+A\,a\,b\,x}{3\,a^2\,b\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(a + b*x^2)^(5/2),x)
```

```
[Out] (2*A*b*x*(a + b*x^2) - B*a^2 + A*a*b*x)/(3*a^2*b*(a + b*x^2)^(3/2))
```

sympy [B] time = 13.20, size = 146, normalized size = 2.86

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } l \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{othe}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a
 + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

3.40 $\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=76

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/3*(B*x+A)/a/(b*x^2+a)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1/
3*(2*B*x+3*A)/a^2/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {823, 12, 266, 63, 208}

$$\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*(a + b*x^2)^(3/2)) + (3*A + 2*B*x)/(3*a^2*Sqrt[a + b*x^2]) -
 (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx &= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-3aAb-2abBx}{x\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b} \\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} + \frac{3A+2Bx}{3a^2\sqrt{a+bx^2}} + \frac{\int \frac{3a^2Ab^2}{x\sqrt{a+bx^2}}\,dx}{3a^4b^2} \\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} + \frac{3A+2Bx}{3a^2\sqrt{a+bx^2}} + \frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^2} \\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} + \frac{3A+2Bx}{3a^2\sqrt{a+bx^2}} + \frac{A\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a^2} \\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} + \frac{3A+2Bx}{3a^2\sqrt{a+bx^2}} + \frac{A\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{a^2b} \\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}} + \frac{3A+2Bx}{3a^2\sqrt{a+bx^2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 69, normalized size = 0.91

$$\frac{a(4A+3Bx)+bx^2(3A+2Bx)}{3a^2\left(a+bx^2\right)^{3/2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (b*x^2*(3*A + 2*B*x) + a*(4*A + 3*B*x))/(3*a^2*(a + b*x^2)^(3/2)) - (A*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 0.67, size = 239, normalized size = 3.14

$$\left[\frac{3\left(Ab^2x^4+2\,Aabx^2+Aa^2\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,Babx^3+3\,Aabx^2+3\,Ba^2x+4\,Aa^2\right)\sqrt{bx^2+a}}{6\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(A*b^2*x^4 + 2*A*a*b*x^2 + A*a^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*a*b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4*A
*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5), 1/3*(3*(A*b^2*x^4
 + 2*A*a*b*x^2 + A*a^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*
```

```
b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 +
2*a^4*b*x^2 + a^5)]
```

giac [A] time = 0.54, size = 82, normalized size = 1.08

$$\frac{\left(\left(\frac{2\,Bbx}{a^2} + \frac{3\,Ab}{a^2}\right)x + \frac{3\,B}{a}\right)x + \frac{4\,A}{a}}{3\left(bx^2 + a\right)^{\frac{3}{2}}} + \frac{2\,A\arctan\left(-\frac{\sqrt{b}x - \sqrt{bx^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((((2*B*b*x/a^2 + 3*A*b/a^2)*x + 3*B/a)*x + 4*A/a)/(b*x^2 + a)^(3/2) + 2
*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2)
```

maple [A] time = 0.01, size = 92, normalized size = 1.21

$$\frac{Bx}{3\left(b\,x^2 + a\right)^{\frac{3}{2}} a} + \frac{A}{3\left(b\,x^2 + a\right)^{\frac{3}{2}} a} + \frac{2Bx}{3\sqrt{b\,x^2 + a}\;a^2} - \frac{A\ln\left(\frac{2a + 2\sqrt{b\,x^2 + a}\;\sqrt{a}}{x}\right)}{a^{\frac{5}{2}}} + \frac{A}{\sqrt{b\,x^2 + a}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*B*x/a/(b*x^2+a)^(3/2)+2/3*B/a^2*x/(b*x^2+a)^(1/2)+1/3*A/a/(b*x^2+a)^(3/
2)+A/a^2/(b*x^2+a)^(1/2)-A/a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.37, size = 80, normalized size = 1.05

$$\frac{2\,Bx}{3\,\sqrt{bx^2 + a}\,a^2} + \frac{Bx}{3\left(bx^2 + a\right)^{\frac{3}{2}} a} - \frac{A\,\text{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}} + \frac{A}{\sqrt{bx^2 + a}\,a^2} + \frac{A}{3\left(bx^2 + a\right)^{\frac{3}{2}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*B*x/(sqrt(b*x^2 + a)*a^2) + 1/3*B*x/((b*x^2 + a)^(3/2)*a) - A*arcsinh(a
/(sqrt(a*b)*abs(x)))/a^(5/2) + A/(sqrt(b*x^2 + a)*a^2) + 1/3*A/((b*x^2 + a)
^(3/2)*a)
```

mupad [B] time = 1.38, size = 80, normalized size = 1.05

$$\frac{\frac{A}{3\,a} + \frac{A\left(b\,x^2 + a\right)}{a^2}}{\left(b\,x^2 + a\right)^{3/2}} + \frac{2\,B\,x\,\left(b\,x^2 + a\right) + B\,a\,x}{3\,a^2\left(b\,x^2 + a\right)^{3/2}} - \frac{A\,\text{atanh}\left(\frac{\sqrt{b\,x^2 + a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x*(a + b*x^2)^(5/2)),x)
```

```
[Out] (A/(3*a) + (A*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) + (2*B*x*(a + b*x^2) + B*
a*x)/(3*a^2*(a + b*x^2)^(3/2)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(5/
2)
```

sympy [B] time = 25.92, size = 840, normalized size = 11.05

$$A\left(\frac{8a^7\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{3a^7\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{6a^7\log\left(\sqrt{1}\right.}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15
/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2)
+ 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6
*a**7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**
2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13
/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*
x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log
(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*
b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6
*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**
3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2
 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(1
9/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6
) - 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2
)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)) + B*(3*a*x/(3*a
**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)) + 2*b*x*
*3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)))
```

3.41 $\int \frac{A+Bx}{x^2(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}+\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/3*(B*x+A)/a/x/(b*x^2+a)^(3/2)-B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+
1/3*(3*B*x+4*A)/a^2/x/(b*x^2+a)^(1/2)-8/3*A*(b*x^2+a)^(1/2)/a^3/x
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {823, 807, 266, 63, 208}

$$\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*x*(a + b*x^2)^(3/2)) + (4*A + 3*B*x)/(3*a^2*x*Sqrt[a + b*x^2
]) - (8*A*Sqrt[a + b*x^2])/(3*a^3*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/a^(5/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx &= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{-4aAb-3abBx}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}+\frac{\int \frac{8a^2Ab^2+3a^2b^2Bx}{x^2\sqrt{a+bx^2}}\,dx}{3a^4b^2}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{B\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^2}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{B\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{B\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{a^2b}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 95, normalized size = 0.91

$$
\frac{a^2(4Bx-3A)+3abx^2(Bx-4A)-3\sqrt{a}\,Bx\left(a+bx^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-8Ab^2x^4}{3a^3x\left(a+bx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-8*A*b^2*x^4 + 3*a*b*x^2*(-4*A + B*x) + a^2*(-3*A + 4*B*x) - 3*Sqrt[a]*B*x
*(a + b*x^2)^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(3*a^3*x*(a + b*x^2)^(
3/2))
```

fricas [A] time = 0.74, size = 264, normalized size = 2.54

$$
\left[\frac{3\left(Bb^2x^5+2\,Babx^3+Ba^2x\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(8\,Ab^2x^4-3\,Babx^3+12\,Aabx^2-4\,Ba^2x+3\,Aa^2\right)}{6\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2
- 4*B*a^2*x + 3*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
, 1/3*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(
b*x^2 + a)) - (8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2 - 4*B*a^2*x + 3*A*a
^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)]
```

giac [A] time = 0.55, size = 119, normalized size = 1.14

$$-\frac{\left(\left(\frac{5\,Ab^2x}{a^3}-\frac{3\,Bb}{a^2}\right)x+\frac{6\,Ab}{a^2}\right)x-\frac{4\,B}{a}}{3\,\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\,B\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((5*A*b^2*x/a^3 - 3*B*b/a^2)*x + 6*A*b/a^2)*x - 4*B/a)/(b*x^2 + a)^(3
/2) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) +
2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^2)
```

maple [A] time = 0.01, size = 112, normalized size = 1.08

$$-\frac{4Abx}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^2}-\frac{8Abx}{3\sqrt{b\,x^2+a}\;a^3}+\frac{B}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a}-\frac{B\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)}{a^{\frac{5}{2}}}-\frac{A}{\left(b\,x^2+a\right)^{\frac{3}{2}}ax}+\frac{B}{\sqrt{b\,x^2+a}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(5/2),x)
```

```
[Out] -A/a/x/(b*x^2+a)^(3/2)-4/3*A/a^2*b*x/(b*x^2+a)^(3/2)-8/3*A/a^3*b*x/(b*x^2+a
)^(1/2)+1/3*B/a/(b*x^2+a)^(3/2)+B/a^2/(b*x^2+a)^(1/2)-B/a^(5/2)*ln((2*a+2*(
b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.34, size = 100, normalized size = 0.96

$$-\frac{8\,Abx}{3\,\sqrt{bx^2+a}\,a^3}-\frac{4\,Abx}{3\,\left(bx^2+a\right)^{\frac{3}{2}}a^2}-\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}}+\frac{B}{\sqrt{bx^2+a}\,a^2}+\frac{B}{3\,\left(bx^2+a\right)^{\frac{3}{2}}a}-\frac{A}{\left(bx^2+a\right)^{\frac{3}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*A*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*A*b*x/((b*x^2 + a)^(3/2)*a^2) - B*ar
csinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + B/(sqrt(b*x^2 + a)*a^2) + 1/3*B/((b*x
^2 + a)^(3/2)*a) - A/((b*x^2 + a)^(3/2)*a*x)
```

mupad [B] time = 1.58, size = 96, normalized size = 0.92

$$\frac{\frac{B}{3\,a}+\frac{B\,\left(b\,x^2+a\right)}{a^2}}{{\left(b\,x^2+a\right)}^{3/2}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A\,a^2-8\,A\,{\left(b\,x^2+a\right)}^2+4\,A\,a\,\left(b\,x^2+a\right)}{3\,a^3\,x\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(5/2)),x)
```

```
[Out] (B/(3*a) + (B*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) - (B*atanh((a + b*x^2)^(1
/2)/a^(1/2)))/a^(5/2) + (A*a^2 - 8*A*(a + b*x^2)^2 + 4*A*a*(a + b*x^2))/(3*
a^3*x*(a + b*x^2)^(3/2))
```

sympy [B] time = 24.18, size = 910, normalized size = 8.75

$$A\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)+B\left(\frac{8}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 +
 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) +
1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)) + B*(8*a**7*sqrt(1
+ b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6
*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*
x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**7*log(sqrt(1
+ b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**
4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a)/(6*a**(19/2)
 + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) +
9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/
2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sqrt(1 + b*x**2/
a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a
**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 9*a**5*
b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(sqrt(1 + b*x**2/a
) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(
13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/
2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**4*b**3*x
**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**
(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6))
```

3.42 $\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=129

$$\frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}+\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/3*(B*x+A)/a/x^2/(b*x^2+a)^(3/2)+5/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/
a^(7/2)+1/3*(4*B*x+5*A)/a^2/x^2/(b*x^2+a)^(1/2)-5/2*A*(b*x^2+a)^(1/2)/a^3/x
^2-8/3*B*(b*x^2+a)^(1/2)/a^3/x
```

Rubi [A] time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {823, 835, 807, 266, 63, 208}

$$\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}+\frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*x^2*(a + b*x^2)^(3/2)) + (5*A + 4*B*x)/(3*a^2*x^2*Sqrt[a + b
*x^2]) - (5*A*Sqrt[a + b*x^2])/(2*a^3*x^2) - (8*B*Sqrt[a + b*x^2])/(3*a^3*x
) + (5*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-5aAb-4abBx}{x^3\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} + \frac{\int \frac{15a^2Ab^2+8a^2b^2Bx}{x^3\sqrt{a+bx^2}}\,dx}{3a^4b^2}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{\int \frac{-16a^3b^2B+15a^2Ab^3x}{x^2\sqrt{a+bx^2}}\,dx}{6a^5b^2}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a^3}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5Ab)\,\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{4a^3}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5A)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+}\right.}{2}$$

$$= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} + \frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a}}\right.}{2a^{7/2}}$$

Mathematica [A] time = 0.22, size = 106, normalized size = 0.82

$$\frac{-\frac{3a^3(A+2Bx)}{x^2} - 4a^2b(5A+6Bx) - ab^2x^2(15A+16Bx) + \frac{15Ab\left(a+bx^2\right)^2\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}}{6a^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] ((-3*a^3*(A + 2*B*x))/x^2 - 4*a^2*b*(5*A + 6*B*x) - a*b^2*x^2*(15*A + 16*B*
x) + (15*A*b*(a + b*x^2)^2*ArcTanh[Sqrt[1 + (b*x^2)/a]])/Sqrt[1 + (b*x^2)/a
])/(6*a^4*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.73, size = 307, normalized size = 2.38

$$\left[\frac{15\left(Ab^3x^6+2\,Aab^2x^4+Aa^2bx^2\right)\sqrt{a}\,\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(16\,Bab^2x^5+15\,Aab^2x^4+24\,Ba^2bx^3+20\right.}{12\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(A*b^3*x^6 + 2*A*a*b^2*x^4 + A*a^2*b*x^2)*sqrt(a)*log(-(b*x^2 + 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(16*B*a*b^2*x^5 + 15*A*a*b^2*x^4 +
 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^3)*sqrt(b*x^2 + a))/(a
^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), -1/6*(15*(A*b^3*x^6 + 2*A*a*b^2*x^4 +
A*a^2*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (16*B*a*b^2*x^5 +
15*A*a*b^2*x^4 + 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^3)*sqr
t(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)]
```

giac [A] time = 0.53, size = 197, normalized size = 1.53

$$-\frac{\left(\left(\frac{5\,Bb^2x}{a^3}+\frac{6\,Ab^2}{a^3}\right)x+\frac{6\,Bb}{a^2}\right)x+\frac{7\,Ab}{a^2}}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{5\,Ab\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}\,x-\sqrt{bx^2+\right.}{\left(\left(\sqrt{b}\,x-\sqrt{b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((5*B*b^2*x/a^3 + 6*A*b^2/a^3)*x + 6*B*b/a^2)*x + 7*A*b/a^2)/(b*x^2 +
 a)^(3/2) - 5*A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)
*a^3) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/
(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^3)
```

maple [A] time = 0.01, size = 134, normalized size = 1.04

$$-\frac{4Bbx}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^2}-\frac{5Ab}{6\left(b\,x^2+a\right)^{\frac{3}{2}}a^2}-\frac{8Bbx}{3\sqrt{b\,x^2+a}\;a^3}+\frac{5Ab\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)}{2a^{\frac{7}{2}}}-\frac{5Ab}{2\sqrt{b\,x^2+a}\;a^3}-\frac{B}{\left(b\,x^2+a\right)^{\frac{3}{2}}ax}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(3/2)-5/6*A/a^2*b/(b*x^2+a)^(3/2)-5/2*A/a^3*b/(b*x^2
+a)^(1/2)+5/2*A/a^(7/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-B/a/x/(b*x^
2+a)^(3/2)-4/3*B/a^2*b*x/(b*x^2+a)^(3/2)-8/3*B/a^3*b*x/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.34, size = 122, normalized size = 0.95

$$-\frac{8\,Bbx}{3\,\sqrt{bx^2+a}\,a^3}-\frac{4\,Bbx}{3\left(bx^2+a\right)^{\frac{3}{2}}a^2}+\frac{5\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{7}{2}}}-\frac{5\,Ab}{2\,\sqrt{bx^2+a}\,a^3}-\frac{5\,Ab}{6\left(bx^2+a\right)^{\frac{3}{2}}a^2}-\frac{B}{\left(bx^2+a\right)^{\frac{3}{2}}ax}-\frac{}{2\left(bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*B*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*B*b*x/((b*x^2 + a)^(3/2)*a^2) + 5/2*
A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 5/2*A*b/(sqrt(b*x^2 + a)*a^3) -
 5/6*A*b/((b*x^2 + a)^(3/2)*a^2) - B/((b*x^2 + a)^(3/2)*a*x) - 1/2*A/((b*x^
2 + a)^(3/2)*a*x^2)
```

mupad [B] time = 1.62, size = 123, normalized size = 0.95

$$\frac{B\,a^2-8\,B\,{\left(b\,x^2+a\right)}^2+4\,B\,a\,\left(b\,x^2+a\right)}{3\,a^3\,x\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{10\,A\,b}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{A}{2\,a\,x^2\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{5\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}-\frac{5\,A\,b}{2\,a^3\,\left(b\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(5/2)),x)
```

```
[Out] (B*a^2 - 8*B*(a + b*x^2)^2 + 4*B*a*(a + b*x^2))/(3*a^3*x*(a + b*x^2)^(3/2))
 - (10*A*b)/(3*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^2)^(3/2)) + (5*
A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) - (5*A*b^2*x^2)/(2*a^3*(a
 + b*x^2)^(3/2))
```

sympy [B] time = 24.76, size = 1034, normalized size = 8.02

$$A\left(-\frac{6a^{17}\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{46a^{16}bx^2\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{}{12a^{\frac{39}{2}}x^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 +
b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x*
*2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8)
 + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(
37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*
b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*
a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x*
*2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 1
2*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12
*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/
2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 +
36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45
*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sq
rt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/
2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(
12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(3
3/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39
/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3
*x**8)) + B*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b*
*5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a
**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(
b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4))
```

3.43 $\int \frac{(1-x)x}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {780, 216}

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x)/Sqrt[1 - x^2],x]
```

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 24, normalized size = 0.89

$$\frac{1}{2}\left((x-2)\sqrt{1-x^2}-\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x)/Sqrt[1 - x^2],x]
```

```
[Out] ((-2 + x)*Sqrt[1 - x^2] - ArcSin[x])/2
```

fricas [A] time = 0.59, size = 31, normalized size = 1.15

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.47, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

maple [A] time = 0.01, size = 29, normalized size = 1.07

$$\frac{\sqrt{-x^2+1}\,x}{2}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x/(-x^2+1)^(1/2),x)
[Out] 1/2*x*(-x^2+1)^(1/2)-1/2*arcsin(x)-(-x^2+1)^(1/2)
```

maxima [A] time = 2.97, size = 28, normalized size = 1.04

$$\frac{1}{2}\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.74

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x - 1))/(1 - x^2)^(1/2),x)
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

sympy [A] time = 0.24, size = 24, normalized size = 0.89

$$\frac{x\sqrt{1-x^2}}{2}-\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x**2+1)**(1/2),x)
[Out] x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

3.44 $\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1593, 780, 216}

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - x^2)/Sqrt[1 - x^2],x]
```

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx &= \int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.89

$$\frac{1}{2}\left((x-2)\sqrt{1-x^2}-\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^2)/Sqrt[1 - x^2],x]
```

```
[Out] ((-2 + x)*Sqrt[1 - x^2] - ArcSin[x])/2
```

fricas [A] time = 0.62, size = 31, normalized size = 1.15

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.42, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.07

$$\frac{\sqrt{-x^2+1}\,x}{2}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x)/(-x^2+1)^(1/2),x)
[Out] 1/2*(-x^2+1)^(1/2)*x-1/2*arcsin(x)-(-x^2+1)^(1/2)
```

maxima [A] time = 2.89, size = 28, normalized size = 1.04

$$\frac{1}{2}\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.74

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^2)/(1 - x^2)^(1/2),x)
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

sympy [A] time = 0.29, size = 24, normalized size = 0.89

$$\frac{x\sqrt{1-x^2}}{2}-\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x)/(-x**2+1)**(1/2),x)
[Out] x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

3.45 $\int \frac{3+x^2}{-3+x^2}\,dx$

Optimal. Leaf size=17

$$x-2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] x-2*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {388, 207}

$$x-2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2)/(-3 + x^2),x]
```

```
[Out] x - 2*Sqrt[3]*ArcTanh[x/Sqrt[3]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3+x^2}{-3+x^2}\,dx &= x+6\int \frac{1}{-3+x^2}\,dx\\ &= x-2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.94

$$x+\sqrt{3}\log\left(\sqrt{3}-x\right)-\sqrt{3}\log\left(x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2)/(-3 + x^2),x]
```

```
[Out] x + Sqrt[3]*Log[Sqrt[3] - x] - Sqrt[3]*Log[Sqrt[3] + x]
```

fricas [A] time = 0.62, size = 26, normalized size = 1.53

$$\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="fricas")
[Out] sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3)/(x^2 - 3)) + x
```

giac [B] time = 0.41, size = 30, normalized size = 1.76

$$\sqrt{3} \log\left(\frac{\left|2 x - 2 \sqrt{3}\right|}{\left|2 x + 2 \sqrt{3}\right|}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="giac")
[Out] sqrt(3)*log(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + x
```

maple [A] time = 0.00, size = 15, normalized size = 0.88

$$x - 2\sqrt{3}\ \operatorname{arctanh}\left(\frac{\sqrt{3}\, x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^2-3),x)
[Out] x-2*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

maxima [A] time = 2.88, size = 22, normalized size = 1.29

$$\sqrt{3} \log\left(\frac{x - \sqrt{3}}{x + \sqrt{3}}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="maxima")
[Out] sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + x
```

mupad [B] time = 0.92, size = 14, normalized size = 0.82

$$x - 2 \sqrt{3}\, \operatorname{atanh}\left(\frac{\sqrt{3}\, x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3)/(x^2 - 3),x)
[Out] x - 2*3^(1/2)*atanh((3^(1/2)*x)/3)
```

sympy [A] time = 0.19, size = 27, normalized size = 1.59

$$x + \sqrt{3} \log\left(x - \sqrt{3}\right) - \sqrt{3} \log\left(x + \sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)/(x**2-3),x)
[Out] x + sqrt(3)*log(x - sqrt(3)) - sqrt(3)*log(x + sqrt(3))
```

3.46 $\int \frac{-1+x^2}{1+x^2}\,dx$

Optimal. Leaf size=6

$$x - 2\tan^{-1}(x)$$

```
[Out] x-2*arctan(x)
```

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {388, 203}

$$x - 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(1 + x^2),x]
```

```
[Out] x - 2*ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{-1+x^2}{1+x^2}\,dx &= x - 2\int \frac{1}{1+x^2}\,dx \\ &= x - 2\tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 1.00

$$x - 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^2),x]
```

```
[Out] x - 2*ArcTan[x]
```

fricas [A] time = 0.69, size = 6, normalized size = 1.00

$$x - 2\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="fricas")
```

```
[Out] x - 2*arctan(x)
```

giac [**A**] time = 0.46, size = 6, normalized size = 1.00

$$x - 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="giac")
[Out] x - 2*arctan(x)
```

maple [**A**] time = 0.00, size = 7, normalized size = 1.17

$$x - 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1),x)
[Out] x-2*arctan(x)
```

maxima [**A**] time = 2.97, size = 6, normalized size = 1.00

$$x - 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="maxima")
[Out] x - 2*arctan(x)
```

mupad [**B**] time = 0.04, size = 6, normalized size = 1.00

$$x - 2\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^2 + 1),x)
[Out] x - 2*atan(x)
```

sympy [**A**] time = 0.15, size = 5, normalized size = 0.83

$$x - 2\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1),x)
[Out] x - 2*atan(x)
```

3.47 $\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{16\sqrt{a+bx^2}\,(Ab-8aC)}{35ab^5}-\frac{x(35aB-8x(Ab-8aC))}{35ab^4\sqrt{a+bx^2}}-\frac{x^3(35aB-6x(Ab-8aC))}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^5(7aB-x(Ab-8aC))}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^7(\cdots}{7\cdots}$$

```
[Out] -1/7*x^7*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^5*(7*a*B-(A*b-8*C*a)*
x)/a/b^2/(b*x^2+a)^(5/2)-1/105*x^3*(35*a*B-6*(A*b-8*C*a)*x)/a/b^3/(b*x^2+a)
^(3/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)-1/35*x*(35*a*B-8*(A*b-8
*C*a)*x)/a/b^4/(b*x^2+a)^(1/2)-16/35*(A*b-8*C*a)*(b*x^2+a)^(1/2)/a/b^5
```

Rubi [A] time = 0.32, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1804, 819, 641, 217, 206}

$$-\frac{x^5(7aB-x(Ab-8aC))}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^3(35aB-6x(Ab-8aC))}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x(35aB-8x(Ab-8aC))}{35ab^4\sqrt{a+bx^2}}-\frac{16\sqrt{a+bx^2}\,(Ab-8aC)}{35ab^5}-\frac{x^7(\cdots}{7\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^7*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^5*(7*a*B - (A*b
 - 8*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (x^3*(35*a*B - 6*(A*b - 8*a*C)
*x))/(105*a*b^3*(a + b*x^2)^(3/2)) - (x*(35*a*B - 8*(A*b - 8*a*C)*x))/(35*a
*b^4*Sqrt[a + b*x^2]) - (16*(A*b - 8*a*C)*Sqrt[a + b*x^2])/(35*a*b^5) + (B*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
```

```
!ILtQ[m + 2*p + 3, 0])
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^6(-7aB+(Ab-8aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{x^4\left(-35a^2B+6a(Ab-8aC)x\right)}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{x^2}{}}{}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35}{}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35}{}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35}{}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35}{}$$

Mathematica [A] time = 0.65, size = 165, normalized size = 0.77

$$\frac{384a^4C-3a^3b(16A+7x(5B-64Cx))+14a^2b^2x^2(5x(24Cx-5B)-12A)+14ab^3x^4(x(60Cx-29B)-15A)+10}{105b^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (384*a^4*C - 3*a^3*b*(16*A + 7*x*(5*B - 64*C*x)) + 14*a^2*b^2*x^2*(-12*A +
5*x*(-5*B + 24*C*x)) + 14*a*b^3*x^4*(-15*A + x*(-29*B + 60*C*x)) + b^4*x^6*
(-105*A + x*(-176*B + 105*C*x)) + 105*Sqrt[a]*Sqrt[b]*B*(a + b*x^2)^3*Sqrt[
1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(105*b^5*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.11, size = 522, normalized size = 2.45

$$\left[\frac{105\left(Bb^4x^8 + 4\,Bab^3x^6 + 6\,Ba^2b^2x^4 + 4\,Ba^3bx^2 + Ba^4\right)\sqrt{b}\,\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right) + 2\,(105\,Cb^4x^8 \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(B*b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 +
B*a^4)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(105*C*b
^4*x^8 - 176*B*b^4*x^7 - 406*B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b
^3 - A*b^4)*x^6 - 105*B*a^3*b*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2
 - A*a*b^3)*x^4 + 168*(8*C*a^3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^
8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(105*(B*
b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 + B*a^4)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (105*C*b^4*x^8 - 176*B*b^4*x^7 - 406*
B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b^3 - A*b^4)*x^6 - 105*B*a^3*b
*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2 - A*a*b^3)*x^4 + 168*(8*C*a^
3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x
^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

giac [A] time = 0.54, size = 204, normalized size = 0.96

$$\frac{(((((((\frac{105\,Cx}{b} - \frac{176\,B}{b})x + \frac{105\,(8\,Ca^4b^7 - Aa^3b^8)}{a^3b^9})x - \frac{406\,Ba}{b^2})x + \frac{210\,(8\,Ca^5b^6 - Aa^4b^7)}{a^3b^9})x - \frac{350\,Ba^2}{b^3})x + \frac{168\,(8\,Ca^6b^5 - Aa^5b^6)}{a^3b^9})x \cdots}{105\,(bx^2+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((((((((105*C*x/b - 176*B/b)*x + 105*(8*C*a^4*b^7 - A*a^3*b^8)/(a^3*b
^9))*x - 406*B*a/b^2)*x + 210*(8*C*a^5*b^6 - A*a^4*b^7)/(a^3*b^9))*x - 350*
B*a^2/b^3)*x + 168*(8*C*a^6*b^5 - A*a^5*b^6)/(a^3*b^9))*x - 105*B*a^3/b^4)*
x + 48*(8*C*a^7*b^4 - A*a^6*b^5)/(a^3*b^9))/(b*x^2 + a)^(7/2) - B*log(abs(-
sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

maple [A] time = 0.05, size = 265, normalized size = 1.24

$$\frac{C\,x^8}{(b\,x^2+a)^{\frac{7}{2}}\,b} - \frac{B\,x^7}{7\,(b\,x^2+a)^{\frac{7}{2}}\,b} - \frac{A\,x^6}{(b\,x^2+a)^{\frac{7}{2}}\,b} + \frac{8Ca\,x^6}{(b\,x^2+a)^{\frac{7}{2}}\,b^2} - \frac{2Aa\,x^4}{(b\,x^2+a)^{\frac{7}{2}}\,b^2} - \frac{B\,x^5}{5\,(b\,x^2+a)^{\frac{5}{2}}\,b^2} + \frac{16C\,a^2x^4}{(b\,x^2+a)^{\frac{7}{2}}\,b^3} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] C*x^8/b/(b*x^2+a)^(7/2)+8*C*a/b^2*x^6/(b*x^2+a)^(7/2)+16*C*a^2/b^3*x^4/(b*x
^2+a)^(7/2)+64/5*C*a^3/b^4*x^2/(b*x^2+a)^(7/2)+128/35*C*a^4/b^5/(b*x^2+a)^(
7/2)-1/7*B*x^7/b/(b*x^2+a)^(7/2)-1/5*B/b^2*x^5/(b*x^2+a)^(5/2)-1/3*B/b^3*x^
3/(b*x^2+a)^(3/2)-B/b^4*x/(b*x^2+a)^(1/2)+B/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^
(1/2))-A*x^6/b/(b*x^2+a)^(7/2)-2*A*a/b^2*x^4/(b*x^2+a)^(7/2)-8/5*A*a^2/b^3*
x^2/(b*x^2+a)^(7/2)-16/35*A*a^3/b^4/(b*x^2+a)^(7/2)
```

maxima [B] time = 1.68, size = 435, normalized size = 2.04

$$\frac{Cx^8}{(bx^2+a)^{\frac{7}{2}}b} - \frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b} + \frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2} + \frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3} + \frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Bx + \frac{8\,Cax^6}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^6}{(bx^2+a)^{\cdots}} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] C*x^8/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4
/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*
x^2 + a)^(7/2)*b^4))*B*x + 8*C*a*x^6/((b*x^2 + a)^(7/2)*b^2) - A*x^6/((b*x^
2 + a)^(7/2)*b) - 1/15*B*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2
 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/3*B*x*(3*x^2/((b*x^
2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 16*C*a^2*x^4/((b*x^2 +
 a)^(7/2)*b^3) - 2*A*a*x^4/((b*x^2 + a)^(7/2)*b^2) - B*a*x^3/((b*x^2 + a)^(
5/2)*b^3) + 64/5*C*a^3*x^2/((b*x^2 + a)^(7/2)*b^4) - 8/5*A*a^2*x^2/((b*x^2
+ a)^(7/2)*b^3) + 139/105*B*x/(sqrt(b*x^2 + a)*b^4) + 17/105*B*a*x/((b*x^2
+ a)^(3/2)*b^4) - 29/35*B*a^2*x/((b*x^2 + a)^(5/2)*b^4) + B*arcsinh(b*x/sqr
t(a*b))/b^(9/2) + 128/35*C*a^4/((b*x^2 + a)^(7/2)*b^5) - 16/35*A*a^3/((b*x^
2 + a)^(7/2)*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\left(C\,x^2+B\,x+A\right)}{{\left(b\,x^2+a\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.48 $\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=150

$$-\frac{x^6(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}+\frac{C\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}-\frac{x^2(24B+35Cx)}{105b^3(a+bx^2)^{3/2}}-\frac{x^4(6B+7Cx)}{35b^2(a+bx^2)^{5/2}}$$

```
[Out] -1/7*x^6*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^4*(7*C*x+6*B)/b^2/(b*
x^2+a)^(5/2)-1/105*x^2*(35*C*x+24*B)/b^3/(b*x^2+a)^(3/2)+C*arctanh(x*b^(1/2
)/(b*x^2+a)^(1/2))/b^(9/2)+1/35*(-35*C*x-16*B)/b^4/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1804, 819, 778, 217, 206}

$$-\frac{x^6(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}-\frac{x^4(6B+7Cx)}{35b^2(a+bx^2)^{5/2}}-\frac{x^2(24B+35Cx)}{105b^3(a+bx^2)^{3/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}+\frac{C\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^6*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(6*B + 7*C*x)
)/(35*b^2*(a + b*x^2)^(5/2)) - (x^2*(24*B + 35*C*x))/(105*b^3*(a + b*x^2)^(
3/2)) - (16*B + 35*C*x)/(35*b^4*Sqrt[a + b*x^2]) + (C*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^5(-6aB-7aCx)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{\int \frac{x^3\left(-24a^2B-35a^2Cx\right)}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{x\left(-48a^3B-105a^3Cx\right)}{\left(a+bx^2\right)^{3/2}}}{105a^3b^3}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}+$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}+$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}+$$

Mathematica [A] time = 0.41, size = 147, normalized size = 0.98

$$\frac{\sqrt{a}\,C\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}\sqrt{a+bx^2}}-\frac{3a^4(16B+35Cx)+14a^3bx^2(12B+25Cx)+14a^2b^2x^4(15B+29Cx)+ab^3x^6(105B}{105ab^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/105*(-15*A*b^4*x^7 + 14*a^3*b*x^2*(12*B + 25*C*x) + 14*a^2*b^2*x^4*(15*B
 + 29*C*x) + 3*a^4*(16*B + 35*C*x) + a*b^3*x^6*(105*B + 176*C*x))/(a*b^4*(a
 + b*x^2)^(7/2)) + (Sqrt[a]*C*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[
a]])/(b^(9/2)*Sqrt[a + b*x^2])
```

fricas [A] time = 0.89, size = 467, normalized size = 3.11

$$\left[\frac{105\left(Cab^4x^8+4\,Ca^2b^3x^6+6\,Ca^3b^2x^4+4\,Ca^4bx^2+Ca^5\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(105\,Bab^4x\right.}{210\left(ab^9x^8+4\,a^2b^8x^6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x^4 + 4*C*a^4*b*x^
2 + C*a^5)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(105
*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3 +
168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B*a^4
*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^4*b^6
*x^2 + a^5*b^5), -1/105*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x
^4 + 4*C*a^4*b*x^2 + C*a^5)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (
105*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3
 + 168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B*
a^4*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^4*
b^6*x^2 + a^5*b^5)]
```

giac [A] time = 0.53, size = 138, normalized size = 0.92

$$-\frac{\left(\left(\left(\left(\left(x\left(\frac{105\,B}{b}+\frac{\left(176\,Ca^3b^7-15\,Aa^2b^8\right)x}{a^3b^8}\right)+\frac{406\,Ca}{b^2}\right)x+\frac{210\,Ba}{b^2}\right)x+\frac{350\,Ca^2}{b^3}\right)x+\frac{168\,Ba^2}{b^3}\right)x+\frac{105\,Ca^3}{b^4}\right)x+\frac{48\,Ba^3}{b^4}}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{C\log\Big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((((((x*(105*B/b + (176*C*a^3*b^7 - 15*A*a^2*b^8)*x/(a^3*b^8)) + 406
*C*a/b^2)*x + 210*B*a/b^2)*x + 350*C*a^2/b^3)*x + 168*B*a^2/b^3)*x + 105*C*
a^3/b^4)*x + 48*B*a^3/b^4)/(b*x^2 + a)^(7/2) - C*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(9/2)
```

maple [B] time = 0.01, size = 277, normalized size = 1.85

$$-\frac{C\,x^7}{7\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{B\,x^6}{\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{A\,x^5}{2\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{2Ba\,x^4}{\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}-\frac{C\,x^5}{5\left(b\,x^2+a\right)^{\frac{5}{2}}b^2}-\frac{5Aa\,x^3}{8\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}-\frac{8B\,a^2x}{5\left(b\,x^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*C*x^7/b/(b*x^2+a)^(7/2)-1/5*C/b^2*x^5/(b*x^2+a)^(5/2)-1/3*C/b^3*x^3/(b
*x^2+a)^(3/2)-C/b^4*x/(b*x^2+a)^(1/2)+C/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2
))-B*x^6/b/(b*x^2+a)^(7/2)-2*B*a/b^2*x^4/(b*x^2+a)^(7/2)-8/5*B*a^2/b^3*x^2/
(b*x^2+a)^(7/2)-16/35*B*a^3/b^4/(b*x^2+a)^(7/2)-1/2*A*x^5/b/(b*x^2+a)^(7/2)
-5/8*A*a/b^2*x^3/(b*x^2+a)^(7/2)-15/56*A*a^2/b^3*x/(b*x^2+a)^(7/2)+3/56*A*a
/b^3*x/(b*x^2+a)^(5/2)+1/14*A/b^3*x/(b*x^2+a)^(3/2)+1/7*A/a/b^3*x/(b*x^2+a)
^(1/2)
```

maxima [B] time = 1.63, size = 447, normalized size = 2.98

$$-\frac{1}{35}\left(\frac{35\,x^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{70\,ax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{\left(bx^2+a\right)^{\frac{7}{2}}b^4}\right)Cx-\frac{Bx^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Cx\left(\frac{15\,x^4}{\left(bx^2+a\right)^{\frac{5}{2}}b}+\frac{20\,ax^2}{\left(bx^2+a\right)^{\frac{5}{2}}}\right.}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*C*x - B*
```

```
x^6/((b*x^2 + a)^(7/2)*b) - 1/15*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x
^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*A*x^5/(
(b*x^2 + a)^(7/2)*b) - 1/3*C*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 +
 a)^(3/2)*b^2))/b^2 - 2*B*a*x^4/((b*x^2 + a)^(7/2)*b^2) - C*a*x^3/((b*x^2 +
 a)^(5/2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 8/5*B*a^2*x^2/((b*x^
2 + a)^(7/2)*b^3) + 139/105*C*x/(sqrt(b*x^2 + a)*b^4) + 17/105*C*a*x/((b*x^
2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*A*x/((b*x^
2 + a)^(3/2)*b^3) + 1/7*A*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*A*a*x/((b*x^2 +
a)^(5/2)*b^3) - 15/56*A*a^2*x/((b*x^2 + a)^(7/2)*b^3) + C*arcsinh(b*x/sqrt(
a*b))/b^(9/2) - 16/35*B*a^3/((b*x^2 + a)^(7/2)*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6\,\left(C\,x^2+B\,x+A\right)}{{\left(b\,x^2+a\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
[Out] int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

3.49 $\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=132

$$-\frac{4(6aC+Ab)}{35ab^4\sqrt{a+bx^2}}+\frac{4(6aC+Ab)}{105b^4\left(a+bx^2\right)^{3/2}}-\frac{x^4(6aC+Ab-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/7*x^5*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^4*(-5*B*b*x+A*b+6*C*a
)/a/b^2/(b*x^2+a)^(5/2)+4/105*(A*b+6*C*a)/b^4/(b*x^2+a)^(3/2)-4/35*(A*b+6*C
*a)/a/b^4/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1804, 805, 266, 43}

$$-\frac{x^4(6aC+Ab-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{4(6aC+Ab)}{35ab^4\sqrt{a+bx^2}}+\frac{4(6aC+Ab)}{105b^4\left(a+bx^2\right)^{3/2}}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^5*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(A*b + 6*a*C
- 5*b*B*x))/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*(A*b + 6*a*C))/(105*b^4*(a +
b*x^2)^(3/2)) - (4*(A*b + 6*a*C))/(35*a*b^4*Sqrt[a + b*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^5\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx=-\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^4(-5aB-(Ab+6aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$=-\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(4(Ab+6aC))\int \frac{x^3}{\left(a+bx^2\right)^{5/2}}\,dx}{35ab^2}$$

$$=-\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(2(Ab+6aC))\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{5/2}}\,d\right.}{35ab^2}$$

$$=-\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(2(Ab+6aC))\operatorname{Subst}\left(\int\left(-\frac{a}{b(a+bx)^5}\right.\right.}{35ab^2}$$

$$=-\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{4(Ab+6aC)}{105b^4\left(a+bx^2\right)^{3/2}}-\frac{4(Ab+6aC}{35ab^4\sqrt{a+b}}$$

Mathematica [A] time = 0.11, size = 89, normalized size = 0.67

$$\frac{-48a^4C-8a^3b\left(A+21Cx^2\right)-14a^2b^2x^2\left(2A+15Cx^2\right)-35ab^3x^4\left(A+3Cx^2\right)+15b^4Bx^7}{105ab^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-48*a^4*C + 15*b^4*B*x^7 - 35*a*b^3*x^4*(A + 3*C*x^2) - 14*a^2*b^2*x^2*(2*
A + 15*C*x^2) - 8*a^3*b*(A + 21*C*x^2))/(105*a*b^4*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.03, size = 137, normalized size = 1.04

$$\frac{\left(15\,Bb^4x^7-105\,Cab^3x^6-48\,Ca^4-8\,Aa^3b-35\left(6\,Ca^2b^2+Aab^3\right)x^4-28\left(6\,Ca^3b+Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{105\left(ab^8x^8+4\,a^2b^7x^6+6\,a^3b^6x^4+4\,a^4b^5x^2+a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^4*x^7 - 105*C*a*b^3*x^6 - 48*C*a^4 - 8*A*a^3*b - 35*(6*C*a^2*
b^2 + A*a*b^3)*x^4 - 28*(6*C*a^3*b + A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a*b^8
*x^8 + 4*a^2*b^7*x^6 + 6*a^3*b^6*x^4 + 4*a^4*b^5*x^2 + a^5*b^4)
```

giac [A] time = 0.62, size = 112, normalized size = 0.85

$$\frac{\left(5\left(3\left(\frac{Bx}{a}-\frac{7\,C}{b}\right)x^2-\frac{7\left(6\,Ca^4b^2+Aa^3b^3\right)}{a^3b^4}\right)x^2-\frac{28\left(6\,Ca^5b+Aa^4b^2\right)}{a^3b^4}\right)x^2-\frac{8\left(6\,Ca^6+Aa^5b\right)}{a^3b^4}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((5*(3*(B*x/a - 7*C/b)*x^2 - 7*(6*C*a^4*b^2 + A*a^3*b^3)/(a^3*b^4))*x
^2 - 28*(6*C*a^5*b + A*a^4*b^2)/(a^3*b^4))*x^2 - 8*(6*C*a^6 + A*a^5*b)/(a^3
*b^4))/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.00, size = 95, normalized size = 0.72

$$-\frac{-15B\,x^7b^4+105C\,x^6a\,b^3+35Aa\,b^3x^4+210C\,a^2b^2x^4+28A\,a^2b^2x^2+168C\,a^3b\,x^2+8A\,a^3b+48C\,a^4}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-15*B*b^4*x^7+105*C*a*b^3*x^6+35*A*a*b^3*x^4+210*C*a^2*b^2*x^4+28*A
*a^2*b^2*x^2+168*C*a^3*b*x^2+8*A*a^3*b+48*C*a^4)/(b*x^2+a)^(7/2)/a/b^4
```

maxima [B] time = 1.44, size = 240, normalized size = 1.82

$$-\frac{Cx^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{2\,Cax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Ax^4}{3\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5\,Bax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{8\,Ca^2x^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{4\,Aax^2}{15\left(bx^2+a\right)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -C*x^6/((b*x^2 + a)^(7/2)*b) - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) - 2*C*a*x^4/
((b*x^2 + a)^(7/2)*b^2) - 1/3*A*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*B*a*x^3/((b
*x^2 + a)^(7/2)*b^2) - 8/5*C*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) - 4/15*A*a*x^2
/((b*x^2 + a)^(7/2)*b^2) + 1/14*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt
(b*x^2 + a)*a*b^3) + 3/56*B*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b
*x^2 + a)^(7/2)*b^3) - 16/35*C*a^3/((b*x^2 + a)^(7/2)*b^4) - 8/105*A*a^2/((
b*x^2 + a)^(7/2)*b^3)
```

mupad [B] time = 1.27, size = 196, normalized size = 1.48

$$\frac{\frac{a\left(\frac{C}{3\,b^3}-\frac{7\,A\,b-14\,C\,a}{21\,a\,b^3}\right)}{b}-\frac{3\,B\,x}{7\,b^3}}{\left(b\,x^2+a\right)^{3/2}}-\frac{\frac{a^2\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b^2}+\frac{B\,a^2\,x}{7\,b^3}}{\left(b\,x^2+a\right)^{7/2}}-\frac{\frac{C}{b^4}-\frac{B\,x}{7\,a\,b^3}}{\sqrt{b\,x^2+a}}-\frac{\frac{a\left(\frac{7\,C\,a^2-7\,A\,a\,b}{35\,a\,b^3}+\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b^2-7\,C\,a\,b}{35\,a\,b^3}\right)}{b}\right)}{b}-\frac{3\,B\,a\,x}{7\,b^3}}{\left(b\,x^2+a\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] ((a*(C/(3*b^3) - (7*A*b - 14*C*a)/(21*a*b^3)))/b - (3*B*x)/(7*b^3))/(a + b*
x^2)^(3/2) - ((a^2*(A/(7*b) - (C*a)/(7*b^2)))/b^2 + (B*a^2*x)/(7*b^3))/(a +
 b*x^2)^(7/2) - (C/b^4 - (B*x)/(7*a*b^3))/(a + b*x^2)^(1/2) - ((a*((7*C*a^2
 - 7*A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a*b^3)))/b
))/b - (3*B*a*x)/(7*b^3))/(a + b*x^2)^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.50 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=149

$$\frac{x(5aC+2Ab)}{35a^2b^3\sqrt{a+bx^2}}-\frac{3x(5aC+2Ab)+8aB}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^2(x(5aC+2Ab)+4aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/7*x^4*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^2*(4*a*B+(2*A*b+5*C*a
)*x)/a/b^2/(b*x^2+a)^(5/2)+1/105*(-8*a*B-3*(2*A*b+5*C*a)*x)/a/b^3/(b*x^2+a)
^(3/2)+1/35*(2*A*b+5*C*a)*x/a^2/b^3/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1804, 819, 778, 191}

$$\frac{x(5aC+2Ab)}{35a^2b^3\sqrt{a+bx^2}}-\frac{x^2(x(5aC+2Ab)+4aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{3x(5aC+2Ab)+8aB}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^4*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^2*(4*a*B + (2*A
*b + 5*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (8*a*B + 3*(2*A*b + 5*a*C)*x
)/(105*a*b^3*(a + b*x^2)^(3/2)) + ((2*A*b + 5*a*C)*x)/(35*a^2*b^3*Sqrt[a +
b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
```

```
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}dx = -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^3(-4aB-(2Ab+5aC)x)}{\left(a+bx^2\right)^{7/2}}dx}{7ab}$$

$$= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{x\left(-8a^2B-3a(2Ab+5aC)x\right)}{\left(a+bx^2\right)^{5/2}}dx}{35a^2b^2}$$

$$= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{8aB+3(2Ab+5aC)x}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{(2A}{\ldots}$$

$$= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{8aB+3(2Ab+5aC)x}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{(2}{35\ldots}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 0.52

$$\frac{-8a^4B-28a^3bBx^2-35a^2b^2Bx^4+3ab^3x^5\left(7A+5Cx^2\right)+6Ab^4x^7}{105a^2b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*B - 28*a^3*b*B*x^2 - 35*a^2*b^2*B*x^4 + 6*A*b^4*x^7 + 3*a*b^3*x^5*(
7*A + 5*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.59, size = 122, normalized size = 0.82

$$\frac{\left(21\,Aab^3x^5-35\,Ba^2b^2x^4+3\left(5\,Cab^3+2\,Ab^4\right)x^7-28\,Ba^3bx^2-8\,Ba^4\right)\sqrt{bx^2+a}}{105\left(a^2b^7x^8+4\,a^3b^6x^6+6\,a^4b^5x^4+4\,a^5b^4x^2+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(21*A*a*b^3*x^5 - 35*B*a^2*b^2*x^4 + 3*(5*C*a*b^3 + 2*A*b^4)*x^7 - 28
*B*a^3*b*x^2 - 8*B*a^4)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b^6*x^6 + 6*a^
4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

giac [A] time = 0.58, size = 81, normalized size = 0.54

$$\frac{\left(\left(3\,x\left(\frac{7\,A}{a}+\frac{\left(5\,Ca^2b^3+2\,Aab^4\right)x^2}{a^3b^3}\right)-\frac{35\,B}{b}\right)x^2-\frac{28\,Ba}{b^2}\right)x^2-\frac{8\,Ba^2}{b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*x*(7*A/a + (5*C*a^2*b^3 + 2*A*a*b^4)*x^2/(a^3*b^3)) - 35*B/b)*x^
2 - 28*B*a/b^2)*x^2 - 8*B*a^2/b^3)/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 76, normalized size = 0.51

$$\frac{6A\,b^{4}x^{7}+15Ca\,b^{3}x^{7}+21A\,x^{5}a\,b^{3}-35B\,a^{2}b^{2}x^{4}-28B\,a^{3}b\,x^{2}-8B\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(6*A*b^4*x^7+15*C*a*b^3*x^7+21*A*a*b^3*x^5-35*B*a^2*b^2*x^4-28*B*a^3*
b*x^2-8*B*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

maxima [A] time = 1.42, size = 253, normalized size = 1.70

$$-\frac{Cx^{5}}{2\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{Bx^{4}}{3\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{5\,Cax^{3}}{8\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}-\frac{Ax^{3}}{4\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{4\,Bax^{2}}{15\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}+\frac{Cx}{14\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}}+\frac{Cx}{7\sqrt{bx^{2}+a}\,a\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*B*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*C*
a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*B*a*
x^2/((b*x^2 + a)^(7/2)*b^2) + 1/14*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(s
qrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/
((b*x^2 + a)^(7/2)*b^3) + 3/140*A*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqr
t(b*x^2 + a)*a^2*b^2) + 1/35*A*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b
*x^2 + a)^(7/2)*b^2) - 8/105*B*a^2/((b*x^2 + a)^(7/2)*b^3)
```

mupad [B] time = 1.19, size = 186, normalized size = 1.25

$$\frac{x\left(\frac{C\,a^{2}-A\,a\,b}{35\,a\,b^{3}}+\frac{a\left(\frac{C}{5\,b^{2}}-\frac{7\,A\,b^{2}-7\,C\,a\,b}{35\,a\,b^{3}}\right)}{b}\right)+\frac{2\,B\,a}{5\,b^{3}}}{\left(b\,x^{2}+a\right)^{5/2}}-\frac{\frac{B}{3\,b^{3}}+x\left(\frac{C}{3\,b^{3}}-\frac{3\,A\,b-10\,C\,a}{105\,a\,b^{3}}\right)}{\left(b\,x^{2}+a\right)^{3/2}}-\frac{\frac{B\,a^{2}}{7\,b^{3}}-\frac{a\,x\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^{2}}\right)}{b}}{\left(b\,x^{2}+a\right)^{7/2}}+\frac{x\,\left(2\,A\,b+5\,C\,a\right)}{35\,a^{2}\,b^{3}\,\sqrt{b\,x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*((C*a^2 - A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a*
b^3)))/b) + (2*B*a)/(5*b^3))/(a + b*x^2)^(5/2) - (B/(3*b^3) + x*(C/(3*b^3)
- (3*A*b - 10*C*a)/(105*a*b^3)))/(a + b*x^2)^(3/2) - ((B*a^2)/(7*b^3) - (a*
x*(A/(7*b) - (C*a)/(7*b^2)))/b)/(a + b*x^2)^(7/2) + (x*(2*A*b + 5*C*a))/(35
*a^2*b^3*(a + b*x^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.51 $\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=139

$$\frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}-\frac{2(4aC+3Ab)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x(x(4aC+3Ab)+3aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^3(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/7*x^3*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x*(3*a*B+(3*A*b+4*C*a)*
x)/a/b^2/(b*x^2+a)^(5/2)+1/105*(3*B*b*x-6*A*b-8*C*a)/a/b^3/(b*x^2+a)^(3/2)+
2/35*B*x/a^2/b^2/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1804, 819, 639, 191}

$$\frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}-\frac{x(x(4aC+3Ab)+3aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{2(4aC+3Ab)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^3(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^3*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x*(3*a*B + (3*A*b
 + 4*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (2*(3*A*b + 4*a*C) - 3*b*B*x)/
(105*a*b^3*(a + b*x^2)^(3/2)) + (2*B*x)/(35*a^2*b^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
```

```
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2(-3aB-(3Ab+4aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{-3a^2B-2a(3Ab+4aC)x}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{2(3Ab+4aC)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{(2B)\int}{3}$$

$$= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{2(3Ab+4aC)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{2}{35a^2b^2}$$

Mathematica [A] time = 0.13, size = 84, normalized size = 0.60

$$\frac{-8a^4C-2a^3b\left(3A+14Cx^2\right)-7a^2b^2x^2\left(3A+5Cx^2\right)+21ab^3Bx^5+6b^4Bx^7}{105a^2b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*C + 21*a*b^3*B*x^5 + 6*b^4*B*x^7 - 7*a^2*b^2*x^2*(3*A + 5*C*x^2) -
2*a^3*b*(3*A + 14*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.63, size = 131, normalized size = 0.94

$$\frac{\left(6\,Bb^4x^7+21\,Bab^3x^5-35\,Ca^2b^2x^4-8\,Ca^4-6\,Aa^3b-7\left(4\,Ca^3b+3\,Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{105\left(a^2b^7x^8+4\,a^3b^6x^6+6\,a^4b^5x^4+4\,a^5b^4x^2+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(6*B*b^4*x^7 + 21*B*a*b^3*x^5 - 35*C*a^2*b^2*x^4 - 8*C*a^4 - 6*A*a^3*
b - 7*(4*C*a^3*b + 3*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b
^6*x^6 + 6*a^4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

giac [A] time = 0.59, size = 95, normalized size = 0.68

$$\frac{\left(\left(3\left(\frac{2\,Bbx^2}{a^2}+\frac{7\,B}{a}\right)x-\frac{35\,C}{b}\right)x^2-\frac{7\left(4\,Ca^4b+3\,Aa^3b^2\right)}{a^3b^3}\right)x^2-\frac{2\left(4\,Ca^5+3\,Aa^4b\right)}{a^3b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*(2*B*b*x^2/a^2 + 7*B/a)*x - 35*C/b)*x^2 - 7*(4*C*a^4*b + 3*A*a^3
*b^2)/(a^3*b^3))*x^2 - 2*(4*C*a^5 + 3*A*a^4*b)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 85, normalized size = 0.61

$$-\frac{-6B\,x^7b^4-21B\,x^5a\,b^3+35C\,a^2b^2x^4+21A\,a^2b^2x^2+28C\,a^3b\,x^2+6A\,a^3b+8C\,a^4}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-6*B*b^4*x^7-21*B*a*b^3*x^5+35*C*a^2*b^2*x^4+21*A*a^2*b^2*x^2+28*C*
a^3*b*x^2+6*A*a^3*b+8*C*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

maxima [A] time = 1.38, size = 179, normalized size = 1.29

$$-\frac{Cx^4}{3\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{4\,Cax^2}{15\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Ax^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{3\,Bx}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{2\,Bx}{35\sqrt{bx^2+a}\,a^2b^2}+\frac{}{35\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3*C*x^4/((b*x^2 + a)^(7/2)*b) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*C
*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 1/5*A*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*B*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/(
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 8/105*C*a^2
/((b*x^2 + a)^(7/2)*b^3) - 2/35*A*a/((b*x^2 + a)^(7/2)*b^2)
```

mupad [B] time = 1.14, size = 133, normalized size = 0.96

$$\frac{\frac{a\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b}+\frac{B\,a\,x}{7\,b^2}}{\left(b\,x^2+a\right)^{7/2}}-\frac{\frac{C}{3\,b^3}-\frac{B\,x}{35\,a\,b^2}}{\left(b\,x^2+a\right)^{3/2}}+\frac{\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b-7\,C\,a}{35\,a\,b^2}\right)}{b}-\frac{8\,B\,x}{35\,b^2}}{\left(b\,x^2+a\right)^{5/2}}+\frac{2\,B\,x}{35\,a^2\,b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] ((a*(A/(7*b) - (C*a)/(7*b^2)))/b + (B*a*x)/(7*b^2))/(a + b*x^2)^(7/2) - (C/
(3*b^3) - (B*x)/(35*a*b^2))/(a + b*x^2)^(3/2) + ((a*(C/(5*b^2) - (7*A*b - 7
*C*a)/(35*a*b^2)))/b - (8*B*x)/(35*b^2))/(a + b*x^2)^(5/2) + (2*B*x)/(35*a^
2*b^2*(a + b*x^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.52 $\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=139

$$\frac{2x(3aC+4Ab)}{105a^3b^2\sqrt{a+bx^2}}+\frac{x(3aC+4Ab)}{105a^2b^2\left(a+bx^2\right)^{3/2}}-\frac{x(3aC+4Ab)+2aB}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/7*x^2*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(-2*a*B-(4*A*b+3*C*a)*x
)/a/b^2/(b*x^2+a)^(5/2)+1/105*(4*A*b+3*C*a)*x/a^2/b^2/(b*x^2+a)^(3/2)+2/105
*(4*A*b+3*C*a)*x/a^3/b^2/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1804, 778, 192, 191}

$$\frac{2x(3aC+4Ab)}{105a^3b^2\sqrt{a+bx^2}}+\frac{x(3aC+4Ab)}{105a^2b^2\left(a+bx^2\right)^{3/2}}-\frac{x(3aC+4Ab)+2aB}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^2*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (2*a*B + (4*A*b +
3*a*C)*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + ((4*A*b + 3*a*C)*x)/(105*a^2*b^2*(
a + b*x^2)^(3/2)) + (2*(4*A*b + 3*a*C)*x)/(105*a^3*b^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x(-2aB-(4Ab+3aC)x)}{\left(a+bx^2\right)^{7/2}}dx}{7ab}$$

$$= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4Ab+3aC)\int \frac{1}{\left(a+bx^2\right)^{5/2}}dx}{35ab^2}$$

$$= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4Ab+3aC)x}{105a^2b^2\left(a+bx^2\right)^{3/2}} + \frac{(2(4Ab\ \cdots}{\cdots}$$

$$= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4Ab+3aC)x}{105a^2b^2\left(a+bx^2\right)^{3/2}} + \frac{2(4Ab\ \cdots}{105a^3b^2\cdots}$$

Mathematica [**A**] time = 0.14, size = 87, normalized size = 0.63

$$\frac{-6a^4B-21a^3bBx^2+7a^2b^2x^3\left(5A+3Cx^2\right)+2ab^3x^5\left(14A+3Cx^2\right)+8Ab^4x^7}{105a^3b^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^4*B - 21*a^3*b*B*x^2 + 8*A*b^4*x^7 + 7*a^2*b^2*x^3*(5*A + 3*C*x^2) +
2*a*b^3*x^5*(14*A + 3*C*x^2))/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

fricas [**A**] time = 0.51, size = 134, normalized size = 0.96

$$\frac{\left(35\,Aa^2b^2x^3+2\left(3\,Cab^3+4\,Ab^4\right)x^7-21\,Ba^3bx^2+7\left(3\,Ca^2b^2+4\,Aab^3\right)x^5-6\,Ba^4\right)\sqrt{bx^2+a}}{105\left(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*A*a^2*b^2*x^3 + 2*(3*C*a*b^3 + 4*A*b^4)*x^7 - 21*B*a^3*b*x^2 + 7*
(3*C*a^2*b^2 + 4*A*a*b^3)*x^5 - 6*B*a^4)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a
^4*b^5*x^6 + 6*a^5*b^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

giac [**A**] time = 0.52, size = 94, normalized size = 0.68

$$\frac{\left(\left(x^2\left(\frac{2\left(3\,Cab^4+4\,Ab^5\right)x^2}{a^3b^3}+\frac{7\left(3\,Ca^2b^3+4\,Aab^4\right)}{a^3b^3}\right)+\frac{35\,A}{a}\right)x-\frac{21\,B}{b}\right)x^2-\frac{6\,Ba}{b^2}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((x^2*(2*(3*C*a*b^4 + 4*A*b^5)*x^2/(a^3*b^3) + 7*(3*C*a^2*b^3 + 4*A*
a*b^4)/(a^3*b^3)) + 35*A/a)*x - 21*B/b)*x^2 - 6*B*a/b^2)/(b*x^2 + a)^(7/2)
```

maple [**A**] time = 0.01, size = 88, normalized size = 0.63

$$\frac{8A\,b^4x^7+6Ca\,b^3x^7+28A\,x^5a\,b^3+21C\,a^2b^2x^5+35A\,x^3a^2b^2-21B\,a^3b\,x^2-6B\,a^4}{105\left(b\,x^2+a\right)^{\frac{7}{2}}\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(8*A*b^4*x^7+6*C*a*b^3*x^7+28*A*a*b^3*x^5+21*C*a^2*b^2*x^5+35*A*a^2*b
^2*x^3-21*B*a^3*b*x^2-6*B*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

maxima [A] time = 1.42, size = 197, normalized size = 1.42

$$-\frac{Cx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{3\,Cx}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{2\,Cx}{35\,\sqrt{bx^2+a}\,a^2b^2}+\frac{Cx}{35\left(bx^2+a\right)^{\frac{3}{2}}ab^2}-\frac{3\,Cax}{28\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{}{7\,(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*C*x^3/((b*x^2 + a)^(7/2)*b) - 1/5*B*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*
C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C*x
/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/(
(b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^
2 + a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) - 2/35*B*a/((b*x^2 +
 a)^(7/2)*b^2)
```

mupad [B] time = 1.09, size = 133, normalized size = 0.96

$$\frac{x\,\left(4\,A\,b+3\,C\,a\right)}{105\,a^2\,b^2\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{\frac{B}{5\,b^2}+x\,\left(\frac{C}{5\,b^2}-\frac{A\,b-C\,a}{35\,a\,b^2}\right)}{{\left(b\,x^2+a\right)}^{5/2}}-\frac{x\,\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)-\frac{B\,a}{7\,b^2}}{{\left(b\,x^2+a\right)}^{7/2}}+\frac{x\,\left(8\,A\,b+6\,C\,a\right)}{105\,a^3\,b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*(4*A*b + 3*C*a))/(105*a^2*b^2*(a + b*x^2)^(3/2)) - (B/(5*b^2) + x*(C/(5*
b^2) - (A*b - C*a)/(35*a*b^2)))/(a + b*x^2)^(5/2) - (x*(A/(7*b) - (C*a)/(7*
b^2)) - (B*a)/(7*b^2))/(a + b*x^2)^(7/2) + (x*(8*A*b + 6*C*a))/(105*a^3*b^2
*(a + b*x^2)^(1/2))
```

sympy [B] time = 118.65, size = 904, normalized size = 6.50

$$A\left(\frac{35a^5x^3}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
```

```
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + B*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 1
05*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*
b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105
*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b*
*5*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(9/2)), True)) + C*(7*a*x
**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/
a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt
(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*
b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*
a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)))
```

3.53 $\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=119

$$\frac{8Bx}{105a^3b\sqrt{a+bx^2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}-\frac{2aC+5Ab-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/7*x*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(B*b*x-5*A*b-2*C*a)/a/b^2
/(b*x^2+a)^(5/2)+4/105*B*x/a^2/b/(b*x^2+a)^(3/2)+8/105*B*x/a^3/b/(b*x^2+a)^
(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1804, 639, 192, 191}

$$\frac{8Bx}{105a^3b\sqrt{a+bx^2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}-\frac{2aC+5Ab-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (5*A*b + 2*a*C - b*B
*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2)) +
(8*B*x)/(105*a^3*b*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-aB-(5Ab+2aC)x}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4B)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35ab}$$

$$= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}} + \frac{(8B)\int \frac{1}{\left(a+bx^2\right)^{3}}}{105a^2b}$$

$$= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}} + \frac{8Bx}{105a^3b\sqrt{a+b}}$$

Mathematica [**A**] time = 0.09, size = 75, normalized size = 0.63

$$\frac{-6a^4C-3a^3b\left(5A+7Cx^2\right)+35a^2b^2Bx^3+28ab^3Bx^5+8b^4Bx^7}{105a^3b^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^4*C + 35*a^2*b^2*B*x^3 + 28*a*b^3*B*x^5 + 8*b^4*B*x^7 - 3*a^3*b*(5*A
+ 7*C*x^2))/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

fricas [**A**] time = 0.69, size = 119, normalized size = 1.00

$$\frac{\left(8\,Bb^4x^7+28\,Bab^3x^5+35\,Ba^2b^2x^3-21\,Ca^3bx^2-6\,Ca^4-15\,Aa^3b\right)\sqrt{bx^2+a}}{105\left(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*B*b^4*x^7 + 28*B*a*b^3*x^5 + 35*B*a^2*b^2*x^3 - 21*C*a^3*b*x^2 - 6
*C*a^4 - 15*A*a^3*b)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a^4*b^5*x^6 + 6*a^5*b
^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

giac [**A**] time = 0.60, size = 82, normalized size = 0.69

$$\frac{\left(\left(4\left(\frac{2\,Bb^2x^2}{a^3}+\frac{7\,Bb}{a^2}\right)x^2+\frac{35\,B}{a}\right)x-\frac{21\,C}{b}\right)x^2-\frac{3\left(2\,Ca^4b+5\,Aa^3b^2\right)}{a^3b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*(2*B*b^2*x^2/a^3 + 7*B*b/a^2)*x^2 + 35*B/a)*x - 21*C/b)*x^2 - 3*
(2*C*a^4*b + 5*A*a^3*b^2)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

maple [**A**] time = 0.00, size = 73, normalized size = 0.61

$$-\frac{-8B\,x^7b^4-28B\,x^5a\,b^3-35B\,x^3a^2b^2+21C\,a^3b\,x^2+15A\,a^3b+6C\,a^4}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-8*B*b^4*x^7-28*B*a*b^3*x^5-35*B*a^2*b^2*x^3+21*C*a^3*b*x^2+15*A*a^
3*b+6*C*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

maxima [A] time = 1.34, size = 123, normalized size = 1.03

$$-\frac{Cx^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,Bx}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Bx}{105\left(bx^2+a\right)^{\frac{3}{2}}a^2b}+\frac{Bx}{35\left(bx^2+a\right)^{\frac{5}{2}}ab}-\frac{2\,Ca}{35\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{}{7\left(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/5*C*x^2/((b*x^2 + a)^(7/2)*b) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*
x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/
((b*x^2 + a)^(5/2)*a*b) - 2/35*C*a/((b*x^2 + a)^(7/2)*b^2) - 1/7*A/((b*x^2
+ a)^(7/2)*b)
```

mupad [B] time = 1.05, size = 99, normalized size = 0.83

$$\frac{8\,B\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}}-\frac{\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}+\frac{B\,x}{7\,b}}{\left(b\,x^2+a\right)^{7/2}}-\frac{\frac{C}{5\,b^2}-\frac{B\,x}{35\,a\,b}}{\left(b\,x^2+a\right)^{5/2}}+\frac{4\,B\,x}{105\,a^2\,b\,\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (8*B*x)/(105*a^3*b*(a + b*x^2)^(1/2)) - (A/(7*b) - (C*a)/(7*b^2) + (B*x)/(7
*b))/(a + b*x^2)^(7/2) - (C/(5*b^2) - (B*x)/(35*a*b))/(a + b*x^2)^(5/2) + (
4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2))
```

sympy [A] time = 85.30, size = 796, normalized size = 6.69

$$A\left(\begin{cases}-\frac{1}{7a^3b\sqrt{a+bx^2}+21a^2b^2x^2\sqrt{a+bx^2}+21ab^3x^4\sqrt{a+bx^2}+7b^4x^6\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{9}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x
**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(
b, 0)), (x**2/(2*a**(9/2)), True)) + B*(35*a**5*x**3/(105*a**(19/2)*sqrt(1
+ b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*
x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*
a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqr
t(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b
**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19
/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(
15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**
2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*
a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 63
0*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 +
 b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))) + C*Piecewise((-2
```

```
*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 1
05*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2
/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 105
*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)), Ne(b, 0)),
(x**4/(4*a**(9/2)), True))
```

3.54 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=127

$$\frac{8x(aC+6Ab)}{105a^4b\sqrt{a+bx^2}}+\frac{4x(aC+6Ab)}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{x(aC+6Ab)}{35a^2b\left(a+bx^2\right)^{5/2}}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] 1/7*(-a*B+(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(6*A*b+C*a)*x/a^2/b/(b*x^2+
a)^(5/2)+4/105*(6*A*b+C*a)*x/a^3/b/(b*x^2+a)^(3/2)+8/105*(6*A*b+C*a)*x/a^4/
b/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 192, 191}

$$\frac{8x(aC+6Ab)}{105a^4b\sqrt{a+bx^2}}+\frac{4x(aC+6Ab)}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{x(aC+6Ab)}{35a^2b\left(a+bx^2\right)^{5/2}}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

```
[Out] -(a*B - (A*b - a*C)*x)/(7*a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*C)*x)/(35*a^
2*b*(a + b*x^2)^(5/2)) + (4*(6*A*b + a*C)*x)/(105*a^3*b*(a + b*x^2)^(3/2))
+ (8*(6*A*b + a*C)*x)/(105*a^4*b*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-6A-\frac{aC}{b}}{\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aC)\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}} + \frac{(4(6Ab+aC))\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b}$$

$$= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}} + \frac{4(6Ab+aC)x}{105a^3b\left(a+bx^2\right)^{3/2}} + \frac{(8(6Ab+aC))\int \frac{}{(a+b}}{105a^3b}$$

$$= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}} + \frac{4(6Ab+aC)x}{105a^3b\left(a+bx^2\right)^{3/2}} + \frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.09, size = 92, normalized size = 0.72

$$\frac{-15a^4B+35a^3bx\left(3A+Cx^2\right)+14a^2b^2x^3\left(15A+2Cx^2\right)+8ab^3x^5\left(21A+Cx^2\right)+48Ab^4x^7}{105a^4b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

```
[Out] (-15*a^4*B + 48*A*b^4*x^7 + 35*a^3*b*x*(3*A + C*x^2) + 8*a*b^3*x^5*(21*A +
C*x^2) + 14*a^2*b^2*x^3*(15*A + 2*C*x^2))/(105*a^4*b*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.71, size = 137, normalized size = 1.08

$$\frac{\left(8\left(Cab^3+6\,Ab^4\right)x^7+105\,Aa^3bx+28\left(Ca^2b^2+6\,Aab^3\right)x^5-15\,Ba^4+35\left(Ca^3b+6\,Aa^2b^2\right)x^3\right)\sqrt{bx^2+a}}{105\left(a^4b^5x^8+4\,a^5b^4x^6+6\,a^6b^3x^4+4\,a^7b^2x^2+a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(C*a*b^3 + 6*A*b^4)*x^7 + 105*A*a^3*b*x + 28*(C*a^2*b^2 + 6*A*a*b^
3)*x^5 - 15*B*a^4 + 35*(C*a^3*b + 6*A*a^2*b^2)*x^3)*sqrt(b*x^2 + a)/(a^4*b^
5*x^8 + 4*a^5*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^7*b^2*x^2 + a^8*b)
```

giac [A] time = 0.50, size = 112, normalized size = 0.88

$$\frac{\left(\left(4\,x^2\left(\frac{2\left(Cab^5+6\,Ab^6\right)x^2}{a^4b^3}+\frac{7\left(Ca^2b^4+6\,Aab^5\right)}{a^4b^3}\right)+\frac{35\left(Ca^3b^3+6\,Aa^2b^4\right)}{a^4b^3}\right)x^2+\frac{105\,A}{a}\right)x-\frac{15\,B}{b}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*(C*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(C*a^2*b^4 + 6*A*a*
b^5)/(a^4*b^3)) + 35*(C*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x
- 15*B/b)/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 96, normalized size = 0.76

$$\frac{48A\,b^4x^7+8Ca\,b^3x^7+168A\,x^5a\,b^3+28C\,a^2b^2x^5+210A\,x^3a^2b^2+35C\,a^3b\,x^3+105Ax\,a^3b-15B\,a^4}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(48*A*b^4*x^7+8*C*a*b^3*x^7+168*A*a*b^3*x^5+28*C*a^2*b^2*x^5+210*A*a^
2*b^2*x^3+35*C*a^3*b*x^3+105*A*a^3*b*x-15*B*a^4)/(b*x^2+a)^(7/2)/a^4/b
```

maxima [A] time = 1.33, size = 153, normalized size = 1.20

$$\frac{16\,Ax}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Ax}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Ax}{7\left(bx^2+a\right)^{\frac{7}{2}}a}-\frac{Cx}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,Cx}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,C}{105\,(bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)*a) - 1/7*C*x/((b*x^
2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*C*x/((b*x^2 + a
)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b) - 1/7*B/((b*x^2 + a)^(7/2
)*b)
```

mupad [B] time = 1.03, size = 115, normalized size = 0.91

$$\frac{x\,\left(6\,A\,b+C\,a\right)}{35\,a^2\,b\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{\frac{B}{7\,b}-x\,\left(\frac{A}{7\,a}-\frac{C}{7\,b}\right)}{{\left(b\,x^2+a\right)}^{7/2}}+\frac{x\,\left(24\,A\,b+4\,C\,a\right)}{105\,a^3\,b\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{x\,\left(48\,A\,b+8\,C\,a\right)}{105\,a^4\,b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*(6*A*b + C*a))/(35*a^2*b*(a + b*x^2)^(5/2)) - (B/(7*b) - x*(A/(7*a) - C/
(7*b)))/(a + b*x^2)^(7/2) + (x*(24*A*b + 4*C*a))/(105*a^3*b*(a + b*x^2)^(3/
2)) + (x*(48*A*b + 8*C*a))/(105*a^4*b*(a + b*x^2)^(1/2))
```

sympy [B] time = 94.22, size = 1880, normalized size = 14.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(
1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) +
 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt
(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a
**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**
2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8
*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(
25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
```

```
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b*
*3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*
x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x
**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a
**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b
*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x*
*13/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2
/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*
sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(2
7/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**
2/a))) + B*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqr
t(a + b*x**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x*
*2)), Ne(b, 0)), (x**2/(2*a**(9/2)), True)) + C*(35*a**5*x**3/(105*a**(19/2
)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15
/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/
a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(
19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a*
*(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x
**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(1
05*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) +
 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(
1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x
**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**
2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)))
```

3.55 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[Out] 1/7*(B*b*x+A*b-C*a)/a/b/(b*x^2+a)^(7/2)+1/35*(6*B*x+7*A)/a^2/(b*x^2+a)^(5/2
)+1/105*(24*B*x+35*A)/a^3/(b*x^2+a)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2)
)/a^(9/2)+1/35*(16*B*x+35*A)/a^4/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1805, 823, 12, 266, 63, 208}

$$\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (A*b - a*C + b*B*x)/(7*a*b*(a + b*x^2)^(7/2)) + (7*A + 6*B*x)/(35*a^2*(a +
b*x^2)^(5/2)) + (35*A + 24*B*x)/(105*a^3*(a + b*x^2)^(3/2)) + (35*A + 16*B*
x)/(35*a^4*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
```

```
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-6Bx}{x\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35aAb+24abBx}{x\left(a+bx^2\right)^{5/2}}\,dx}{35a^3b}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105a^2Ab^2-48a^2b^2Bx}{x\left(a+bx^2\right)^{3/2}}\,dx}{105a^5b^2}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{\int \frac{105a^3A}{x\sqrt{a+b}}}{105a^7}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\int \frac{}{x\sqrt{a}}}{a}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\,\mathrm{Subs}}{}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\,\mathrm{Subs}}{}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} - \frac{A\tanh^{}}{a}$$

Mathematica [A] time = 0.23, size = 120, normalized size = 0.87

$$\frac{-15a^4C+a^3b(176A+105Bx)+14a^2b^2x^2(29A+15Bx)+14ab^3x^4(25A+12Bx)+3b^4x^6(35A+16Bx)}{105a^4b\left(a+bx^2\right)^{7/2}} - \frac{A\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-15*a^4*C + 14*a*b^3*x^4*(25*A + 12*B*x) + 14*a^2*b^2*x^2*(29*A + 15*B*x)
+ 3*b^4*x^6*(35*A + 16*B*x) + a^3*b*(176*A + 105*B*x))/(105*a^4*b*(a + b*x^
2)^(7/2)) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

fricas [A] time = 0.77, size = 465, normalized size = 3.37

$$\left[\frac{105\left(Ab^5x^8 + 4\,Aab^4x^6 + 6\,Aa^2b^3x^4 + 4\,Aa^3b^2x^2 + Aa^4b\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right) + 2\left(48\,Bab^4x^7 + 105\,A\right.}{210\left(a^5b^5x^8 + 4\,a^6b^4x^6 + 6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4 + 4*A*a^3*b^2*x^2
+ A*a^4*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(48*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^4 +
 210*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176*A*a
^4*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8
*b^2*x^2 + a^9*b), 1/105*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4
+ 4*A*a^3*b^2*x^2 + A*a^4*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (4
8*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^4 + 2
10*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176*A*a^4
*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8*b
^2*x^2 + a^9*b)]
```

giac [A] time = 0.57, size = 152, normalized size = 1.10

$$\frac{\left(\left(\left(\left(3\left(\left(\frac{16\,Bb^3x}{a^4} + \frac{35\,Ab^3}{a^4}\right)x + \frac{56\,Bb^2}{a^3}\right)x + \frac{350\,Ab^2}{a^3}\right)x + \frac{210\,Bb}{a^2}\right)x + \frac{406\,Ab}{a^2}\right)x + \frac{105\,B}{a}\right)x - \frac{15\,Ca^{14}b^2-176\,Aa^{13}b^3}{a^{14}b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}} + \frac{2\,A\arctan\left(\right.}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((((3*((16*B*b^3*x/a^4 + 35*A*b^3/a^4)*x + 56*B*b^2/a^3)*x + 350*A*b
^2/a^3)*x + 210*B*b/a^2)*x + 406*A*b/a^2)*x + 105*B/a)*x - (15*C*a^14*b^2 -
 176*A*a^13*b^3)/(a^14*b^3))/(b*x^2 + a)^(7/2) + 2*A*arctan(-(sqrt(b)*x - s
qrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4)
```

maple [A] time = 0.01, size = 169, normalized size = 1.22

$$\frac{Bx}{7\left(b\,x^2+a\right)^{\frac{7}{2}}a} + \frac{A}{7\left(b\,x^2+a\right)^{\frac{7}{2}}a} + \frac{6Bx}{35\left(b\,x^2+a\right)^{\frac{5}{2}}a^2} - \frac{C}{7\left(b\,x^2+a\right)^{\frac{7}{2}}b} + \frac{A}{5\left(b\,x^2+a\right)^{\frac{5}{2}}a^2} + \frac{8Bx}{35\left(b\,x^2+a\right)^{\frac{3}{2}}a^3} + \frac{A}{3\left(b\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*C/b/(b*x^2+a)^(7/2)+1/7*B*x/a/(b*x^2+a)^(7/2)+6/35*B/a^2*x/(b*x^2+a)^(
5/2)+8/35*B/a^3*x/(b*x^2+a)^(3/2)+16/35*B/a^4*x/(b*x^2+a)^(1/2)+1/7*A/a/(b*
x^2+a)^(7/2)+1/5*A/a^2/(b*x^2+a)^(5/2)+1/3*A/a^3/(b*x^2+a)^(3/2)+A/a^4/(b*x
^2+a)^(1/2)-A/a^(9/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.45, size = 157, normalized size = 1.14

$$\frac{16\,Bx}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Bx}{35\,\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Bx}{35\,\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Bx}{7\,\left(bx^2+a\right)^{\frac{7}{2}}a}-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{9}{2}}}+\frac{A}{\sqrt{bx^2+a}\,a^4}+\frac{A}{3\,\left(bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b*x^2 + a)^(7/2)*a) - A*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(9/2) + A/(sqrt(b*x^2 + a)*a^4) + 1/3*A/((b*x^2 + a)^(3
/2)*a^3) + 1/5*A/((b*x^2 + a)^(5/2)*a^2) + 1/7*A/((b*x^2 + a)^(7/2)*a) - 1/
7*C/((b*x^2 + a)^(7/2)*b)
```

mupad [B] time = 1.62, size = 159, normalized size = 1.15

$$\frac{\frac{A}{7\,a}+\frac{A\,\left(b\,x^2+a\right)^2}{3\,a^3}+\frac{A\,\left(b\,x^2+a\right)^3}{a^4}+\frac{A\,\left(b\,x^2+a\right)}{5\,a^2}}{\left(b\,x^2+a\right)^{7/2}}-\frac{C}{7\,b\,\left(b\,x^2+a\right)^{7/2}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{16\,B\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,B\,x}{35\,a^3\,\left(b\,x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x)
```

```
[Out] (A/(7*a) + (A*(a + b*x^2)^2)/(3*a^3) + (A*(a + b*x^2)^3)/a^4 + (A*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - C/(7*b*(a + b*x^2)^(7/2)) - (A*atanh((a +
b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*B*x)/(35*a^4*(a + b*x^2)^(1/2)) + (8*B
*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*B*x)/(35*a^2*(a + b*x^2)^(5/2)) + (B*x)
/(7*a*(a + b*x^2)^(7/2))
```

sympy [B] time = 107.41, size = 6613, normalized size = 47.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 94
50*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x
**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(5
9/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 2
10*a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a
**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44
100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6
*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(
55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x*
*2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**
4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)
*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 945
0*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x*
*20) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)
*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(
65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 +
 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b*
*9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(21
0*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(
67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 +
```

```
44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b*
*8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a*
*31*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x*
*2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)
*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2520
0*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x*
*18 + 210*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)
/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*
a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**1
0 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2
)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 472
5*a**30*b**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 94
50*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x
**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(5
9/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 2
10*a**(53/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1
)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200
*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**
10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/
2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23
630*a**29*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x*
*2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)
*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2520
0*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x*
*18 + 210*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210
*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(6
7/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 4
4100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**
8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a*
*29*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b
*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65
/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2
5200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9
*x**18 + 210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2
/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) +
22050*a**28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
+ 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*x**2/a
) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 +
25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**
5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a*
*(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)
 + 31442*a**27*b**5*x**10*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2
)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**
(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
+ 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b
**9*x**18 + 210*a**(53/2)*b**10*x**20) + 26460*a**27*b**5*x**10*log(b*x**2/
a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 2520
0*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x*
*10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57
/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 5
2920*a**27*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**
(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 4410
0*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x
```

```
**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55
/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 19924*a**26*b**6*x**12*sqrt(1
 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x
**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/
2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9
450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*
x**20) + 22050*a**26*b**6*x**12*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/
2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a*
*(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*
b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 44100*a**26*b**6*x**12*log(sqrt(1
 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b
**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a*
*(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**1
4 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b
**10*x**20) + 8162*a**25*b**7*x**14*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 210
0*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 +
 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b
**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a
**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**25*b**7*x**14*l
og(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x
**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/
2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9
450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*
x**20) - 25200*a**25*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2)
+ 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x
**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61
/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2
100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1960*a**24*b**8*x**
16*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/
2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 5292
0*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*
x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/
2)*b**10*x**20) + 4725*a**24*b**8*x**16*log(b*x**2/a)/(210*a**(73/2) + 2100
*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 +
44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b*
*6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a*
*(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 9450*a**24*b**8*x**16*log
(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(
69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 5
2920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b*
*7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(
53/2)*b**10*x**20) + 210*a**23*b**9*x**18*sqrt(1 + b*x**2/a)/(210*a**(73/2)
 + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*
x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(6
1/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**23*b**9*x*
*18*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b
**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a*
*(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**1
4 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b
**10*x**20) - 2100*a**23*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73
/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b*
*3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a*
*(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16
 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 105*a**22*b**10
*x**20*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2
)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920
*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x
```

```
**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2
)*b**10*x**20) - 210*a**22*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(210*a**
(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)
*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100
*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x*
*16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)) + B*(35*a**14
*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/
a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*s
qrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27
/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2
/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*
x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*
a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*
x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*x*
*2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*s
qrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29
/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2
/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(35
*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 5
25*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1
+ b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b*
*5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**
2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**
(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x*
*2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**
12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**2/
a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)
*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a)
 + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a
**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*
x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x
**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))) + C*
Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x**
2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(b,
 0)), (x**2/(2*a**(9/2)), True))
```

3.56 $\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}-\frac{A\sqrt{a+bx^2}}{a^5x}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{35a^4\sqrt{a+bx^2}}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{B-\ldots}{7a\left(\ldots\right.}$$

```
[Out] 1/7*(B-(A*b/a-C)*x)/a/(b*x^2+a)^(7/2)+1/35*(7*B-(13*A*b/a-6*C)*x)/a^2/(b*x^
2+a)^(5/2)+1/105*(35*B-3*(29*A*b/a-8*C)*x)/a^3/(b*x^2+a)^(3/2)-B*arctanh((b
*x^2+a)^(1/2)/a^(1/2))/a^(9/2)+1/35*(35*B-(93*A*b/a-16*C)*x)/a^4/(b*x^2+a)^
(1/2)-A*(b*x^2+a)^(1/2)/a^5/x
```

Rubi [A] time = 0.38, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1805, 807, 266, 63, 208}

$$\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{35a^4\sqrt{a+bx^2}}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{35a^2\left(a+bx^2\right)^{5/2}}-\frac{A\sqrt{a+bx^2}}{a^5x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{B-x\ldots}{7a\left(a\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (B - ((A*b)/a - C)*x)/(7*a*(a + b*x^2)^(7/2)) + (7*B - ((13*A*b)/a - 6*C)*x
)/(35*a^2*(a + b*x^2)^(5/2)) + (35*B - 3*((29*A*b)/a - 8*C)*x)/(105*a^3*(a
+ b*x^2)^(3/2)) + (35*B - ((93*A*b)/a - 16*C)*x)/(35*a^4*Sqrt[a + b*x^2]) -
 (A*Sqrt[a + b*x^2])/(a^5*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-7Bx+6\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35A+35Bx-4\left(\frac{13Ab}{a}-6C\right)x^2}{x^2\left(a+bx^2\right)^{5/2}}\,dx}{35a^2}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105A-105Bx+6\left(\frac{29Ab}{a}-8C\right)\ldots}{x^2\left(a+bx^2\right)^{3/2}}\ldots}{105a^3}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.22, size = 158, normalized size = 0.84

$$\frac{a^4(x(176B+105Cx)-105A)+14a^3bx^2(x(29B+15Cx)-60A)+14a^2b^2x^4(x(25B+12Cx)-120A)+3ab^3x^6(x(\ldots}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 14*a^2*b^2*x^4*(-120*A + x*(25*B + 12*C*x)) + 14*a^3*b*x^
2*(-60*A + x*(29*B + 15*C*x)) + 3*a*b^3*x^6*(-448*A + x*(35*B + 16*C*x)) +
```

```
a^4*(-105*A + x*(176*B + 105*C*x)) - 105*Sqrt[a]*B*x*(a + b*x^2)^(7/2)*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(105*a^5*x*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.76, size = 525, normalized size = 2.79

$$\left[\frac{105\left(Bb^4x^9+4\,Bab^3x^7+6\,Ba^2b^2x^5+4\,Ba^3bx^3+Ba^4x\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(105\,Bab^3x^7+350\,\ldots\right.}{21\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 +
B*a^4*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(1
05*B*a*b^3*x^7 + 350*B*a^2*b^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3
*b*x^3 + 168*(C*a^2*b^2 - 8*A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C
*a^3*b - 8*A*a^2*b^2)*x^4 + 105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(
a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), 1/105*(
105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 + B*a^4*x)
*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (105*B*a*b^3*x^7 + 350*B*a^2*b
^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3*b*x^3 + 168*(C*a^2*b^2 - 8*
A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C*a^3*b - 8*A*a^2*b^2)*x^4 +
105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7
+ 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

giac [A] time = 0.51, size = 239, normalized size = 1.27

$$\frac{\left(\left(\left(\left(3\left(x\left(\frac{35\,Bb^3}{a^4}+\frac{\left(16\,Ca^{20}b^6-93\,Aa^{19}b^7\right)x}{a^{24}b^3}\right)+\frac{28\left(2\,Ca^{21}b^5-11\,Aa^{20}b^6\right)}{a^{24}b^3}\right)x+\frac{350\,Bb^2}{a^3}\right)x+\frac{210\left(Ca^{22}b^4-5\,Aa^{21}b^5\right)}{a^{24}b^3}\right)x+\frac{406\,Bb}{a^2}\right)x+\ldots}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((((3*(x*(35*B*b^3/a^4 + (16*C*a^20*b^6 - 93*A*a^19*b^7)*x/(a^24*b^3
)) + 28*(2*C*a^21*b^5 - 11*A*a^20*b^6)/(a^24*b^3))*x + 350*B*b^2/a^3)*x + 2
10*(C*a^22*b^4 - 5*A*a^21*b^5)/(a^24*b^3))*x + 406*B*b/a^2)*x + 105*(C*a^23
*b^3 - 4*A*a^22*b^4)/(a^24*b^3))*x + 176*B/a)/(b*x^2 + a)^(7/2) + 2*B*arcta
n(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) + 2*A*sqrt(b)/(((
sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

maple [A] time = 0.01, size = 240, normalized size = 1.28

$$-\frac{8Abx}{7\left(b\,x^2+a\right)^{\frac{7}{2}}a^2}+\frac{Cx}{7\left(b\,x^2+a\right)^{\frac{7}{2}}a}-\frac{48Abx}{35\left(b\,x^2+a\right)^{\frac{5}{2}}a^3}+\frac{B}{7\left(b\,x^2+a\right)^{\frac{7}{2}}a}+\frac{6Cx}{35\left(b\,x^2+a\right)^{\frac{5}{2}}a^2}-\frac{A}{\left(b\,x^2+a\right)^{\frac{7}{2}}ax}-\frac{\ldots}{35\left(\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7*C*x/a/(b*x^2+a)^(7/2)+6/35*C/a^2*x/(b*x^2+a)^(5/2)+8/35*C/a^3*x/(b*x^2+
a)^(3/2)+16/35*C/a^4*x/(b*x^2+a)^(1/2)-A/a/x/(b*x^2+a)^(7/2)-8/7*A/a^2*b*x/
(b*x^2+a)^(7/2)-48/35*A/a^3*b*x/(b*x^2+a)^(5/2)-64/35*A/a^4*b*x/(b*x^2+a)^(
3/2)-128/35*A/a^5*b*x/(b*x^2+a)^(1/2)+1/7*B/a/(b*x^2+a)^(7/2)+1/5*B/a^2/(b*
x^2+a)^(5/2)+1/3*B/a^3/(b*x^2+a)^(3/2)+B/a^4/(b*x^2+a)^(1/2)-B/a^(9/2)*ln((
2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.41, size = 228, normalized size = 1.21

$$\frac{16\,Cx}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Cx}{35\,\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Cx}{35\,\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Cx}{7\,\left(bx^2+a\right)^{\frac{7}{2}}a}-\frac{128\,Abx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Abx}{35\,\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{48\,A}{35\,\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*C
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 128/35*A*b*x/(
sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*A*b*x/((
b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)*a^2) - B*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(9/2) + B/(sqrt(b*x^2 + a)*a^4) + 1/3*B/((b*x^2 + a)^(3/
2)*a^3) + 1/5*B/((b*x^2 + a)^(5/2)*a^2) + 1/7*B/((b*x^2 + a)^(7/2)*a) - A/(
(b*x^2 + a)^(7/2)*a*x)
```

mupad [B] time = 2.10, size = 225, normalized size = 1.20

$$\frac{\frac{B}{7\,a}+\frac{B\,{\left(b\,x^2+a\right)}^2}{3\,a^3}+\frac{B\,{\left(b\,x^2+a\right)}^3}{a^4}+\frac{B\,\left(b\,x^2+a\right)}{5\,a^2}}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{\frac{A}{a^4}+\frac{128\,A\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{16\,C\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,C\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x)
```

```
[Out] (B/(7*a) + (B*(a + b*x^2)^2)/(3*a^3) + (B*(a + b*x^2)^3)/a^4 + (B*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - (A/a^4 + (128*A*b*x^2)/(35*a^5))/(x*(a + b
*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*C*x)/(35*
a^4*(a + b*x^2)^(1/2)) + (8*C*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*C*x)/(35*a
^2*(a + b*x^2)^(5/2)) + (C*x)/(7*a*(a + b*x^2)^(7/2)) - (29*A*b*x)/(35*a^4*
(a + b*x^2)^(3/2)) - (13*A*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (A*b*x)/(7*a^2
*(a + b*x^2)^(7/2))
```

sympy [B] time = 165.70, size = 6922, normalized size = 36.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 28
0*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 56
0*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 44
8*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b
**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 2
10*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) + B*(352*a*
*32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69
/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 529
20*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7
*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53
/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*
b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(6
```

```
5/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 +
25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**
9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x**2/a) + 1
)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200
*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**
10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/
2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 29
24*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 +
 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**
4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a*
*(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
+ 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(210*a**(73/
2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**
3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**
(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16
+ 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a**31*b*x**
2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450
*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**
8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/
2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210
*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)/(210*a**
(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)
*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100
*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x*
*16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 4725*a**30*b
**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69
/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 529
20*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7
*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53
/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(210*a*
*(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2
)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 4410
0*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x
**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23630*a**29
*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450
*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**
8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/
2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210
*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210*a**(73/2
) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3
*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(
61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a**29*b**3*
x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9
450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*
x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(
59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 +
210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2/a)/(210*
a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67
/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44
100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8
*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 22050*a**
28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a*
*(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 +
 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*
b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a*
*(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(2
10*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**
(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 +
```

```
 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b
**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 31442*
a**27*b**5*x**10*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
+ 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) + 26460*a**27*b**5*x**10*log(b*x**2/a)/(210*a
**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/
2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 441
00*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*
x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 52920*a**2
7*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*
x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/
2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25
200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*
x**18 + 210*a**(53/2)*b**10*x**20) + 19924*a**26*b**6*x**12*sqrt(1 + b*x**2
/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) +
22050*a**26*b**6*x**12*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b
**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*
a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**1
8 + 210*a**(53/2)*b**10*x**20) - 44100*a**26*b**6*x**12*log(sqrt(1 + b*x**2
/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
+ 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b
**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*
a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**2
0) + 8162*a**25*b**7*x**14*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/
2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a*
*(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*
b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**25*b**7*x**14*log(b*x**2
/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) -
25200*a**25*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a*
*(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 441
00*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*
x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(5
5/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1960*a**24*b**8*x**16*sqrt(1
 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x
**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/
2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9
450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*
x**20) + 4725*a**24*b**8*x**16*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2
)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**
(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
+ 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b
**9*x**18 + 210*a**(53/2)*b**10*x**20) - 9450*a**24*b**8*x**16*log(sqrt(1 +
 b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**
2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(
63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14
+ 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**
10*x**20) + 210*a**23*b**9*x**18*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a
**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44
100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6
```

```
*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(
55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**23*b**9*x**18*log(b
*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
+ 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b
**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*
a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**2
0) - 2100*a**23*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 210
0*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 +
 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b
**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a
**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 105*a**22*b**10*x**20*lo
g(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x*
*4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2
)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 94
50*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x
**20) - 210*a**22*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) +
2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**
6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2
)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 210
0*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)) + C*(35*a**14*x/(35*a*
*(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b
*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*
x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 17
5*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)
*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a)
+ 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqr
t(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 2
10*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b
*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*
x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*
a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(35*a**(37/2
)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33
/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/
a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*
sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 286*a**1
0*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1
+ b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b*
*3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 2
10*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1
 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*
a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x*
*2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**
8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**
(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)))
```

3.57 $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=219

$$\frac{(9Ab-2aC)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}-\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{2a^5x^2}-\frac{B\sqrt{a+bx^2}}{a^5x}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{7(2\ldots}{3\ldots}$$

```
[Out] 1/7*(-a*(A*b/a-C)-b*B*x)/a^2/(b*x^2+a)^(7/2)+1/35*(-13*B*b*x-14*A*b+7*C*a)/
a^3/(b*x^2+a)^(5/2)+1/105*(-87*B*b*x-105*A*b+35*C*a)/a^4/(b*x^2+a)^(3/2)+1/
2*(9*A*b-2*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(11/2)+1/35*(-93*B*b*x-1
40*A*b+35*C*a)/a^5/(b*x^2+a)^(1/2)-1/2*A*(b*x^2+a)^(1/2)/a^5/x^2-B*(b*x^2+a
)^(1/2)/a^5/x
```

Rubi [A] time = 0.48, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1805, 1807, 807, 266, 63, 208}

$$-\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}+\frac{(9Ab-2aC)\tanh\ldots}{2a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(a*((A*b)/a - C) + b*B*x)/(7*a^2*(a + b*x^2)^(7/2)) - (7*(2*A*b - a*C) + 1
3*b*B*x)/(35*a^3*(a + b*x^2)^(5/2)) - (35*(3*A*b - a*C) + 87*b*B*x)/(105*a^
4*(a + b*x^2)^(3/2)) - (35*(4*A*b - a*C) + 93*b*B*x)/(35*a^5*Sqrt[a + b*x^2
]) - (A*Sqrt[a + b*x^2])/(2*a^5*x^2) - (B*Sqrt[a + b*x^2])/(a^5*x) + ((9*A*
b - 2*a*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-7Bx+7\left(\frac{Ab}{a}-C\right)x^2+\frac{6bBx^3}{a}}{x^3\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35A+35Bx-35\left(\frac{2Ab}{a}-C\right)x^2-\frac{52bBx^3}{a}}{x^3\left(a+bx^2\right)^{5/2}}\,dx}{35a^2}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105A-105Bx+1\cdots}{x^3(\cdots}}{\cdots}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

Mathematica [A] time = 0.53, size = 178, normalized size = 0.81

$$\frac{\frac{a^5\left(-105A-210Bx+352Cx^2\right)}{x^2}-4a^4b(396A+7x(60B-29Cx))+14a^3b^2x^2(10x(5Cx-24B)-261A)+42a^2b^3x^4(x(5Cx\cdots}{210a^6\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-3*a*b^4*x^6*(315*A + 256*B*x) + (a^5*(-105*A - 210*B*x + 352*C*x^2))/x^2
- 4*a^4*b*(396*A + 7*x*(60*B - 29*C*x)) + 42*a^2*b^3*x^4*(-75*A + x*(-64*B
+ 5*C*x)) + 14*a^3*b^2*x^2*(-261*A + 10*x*(-24*B + 5*C*x)) + (105*(9*A*b -
2*a*C)*(a + b*x^2)^4*ArcTanh[Sqrt[1 + (b*x^2)/a]])/Sqrt[1 + (b*x^2)/a])/(21
0*a^6*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.93, size = 688, normalized size = 3.14

$$\left[-\frac{105\left(\left(2\,Cab^{4}-9\,Ab^{5}\right)x^{10}+4\left(2\,Ca^{2}b^{3}-9\,Aab^{4}\right)x^{8}+6\left(2\,Ca^{3}b^{2}-9\,Aa^{2}b^{3}\right)x^{6}+4\left(2\,Ca^{4}b-9\,Aa^{3}b^{2}\right)x^{4}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [-1/420*(105*((2*C*a*b^4 - 9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8
+ 6*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*
C*a^5 - 9*A*a^4*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(768*B*a*b^4*x^9 + 2688*B*a^2*b^3*x^7 + 3360*B*a^3*b^2*x^5 + 1
680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 210*B*a^5*x - 350*(2*
C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*(2*C*a^4*b - 9*A*a^3*b^2)*x^
4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b
^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2), 1/210*(105*((2*C*a*b^4 -
9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 6*(2*C*a^3*b^2 - 9*A*a^2*
b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*C*a^5 - 9*A*a^4*b)*x^2)*sqr
t(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (768*B*a*b^4*x^9 + 2688*B*a^2*b^3*
x^7 + 3360*B*a^3*b^2*x^5 + 1680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)
*x^8 + 210*B*a^5*x - 350*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*
(2*C*a^4*b - 9*A*a^3*b^2)*x^4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 +
 a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2
)]
```

giac [A] time = 0.48, size = 325, normalized size = 1.48

$$-\frac{\left(\left(\left(\left(3\left(\left(\frac{93\,Bb^{4}x}{a^{5}}-\frac{35\left(Ca^{24}b^{6}-4\,Aa^{23}b^{7}\right)}{a^{28}b^{3}}\right)x+\frac{308\,Bb^{3}}{a^{4}}\right)x-\frac{35\left(10\,Ca^{25}b^{5}-39\,Aa^{24}b^{6}\right)}{a^{28}b^{3}}\right)x+\frac{1050\,Bb^{2}}{a^{3}}\right)x-\frac{14\left(29\,Ca^{26}b^{4}-108\,Aa^{25}b^{5}\right)}{a^{28}b^{3}}\right.}{105\left(bx^{2}+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(((((3*((93*B*b^4*x/a^5 - 35*(C*a^24*b^6 - 4*A*a^23*b^7)/(a^28*b^3))
*x + 308*B*b^3/a^4)*x - 35*(10*C*a^25*b^5 - 39*A*a^24*b^6)/(a^28*b^3))*x +
1050*B*b^2/a^3)*x - 14*(29*C*a^26*b^4 - 108*A*a^25*b^5)/(a^28*b^3))*x + 420
*B*b/a^2)*x - 2*(88*C*a^27*b^3 - 291*A*a^26*b^4)/(a^28*b^3))/(b*x^2 + a)^(7
/2) + (2*C*a - 9*A*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt
(-a)*a^5) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(
b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^5)
```

maple [A] time = 0.02, size = 288, normalized size = 1.32

$$-\frac{8Bbx}{7\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{2}}-\frac{9Ab}{14\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{2}}-\frac{48Bbx}{35\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{3}}+\frac{C}{7\left(b\,x^{2}+a\right)^{\frac{7}{2}}a}-\frac{9Ab}{10\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{3}}-\frac{B}{\left(b\,x^{2}+a\right)^{\frac{7}{2}}ax}-\frac{}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(7/2)-9/14*A/a^2*b/(b*x^2+a)^(7/2)-9/10*A/a^3*b/(b*x
^2+a)^(5/2)-3/2*A/a^4*b/(b*x^2+a)^(3/2)-9/2*A/a^5*b/(b*x^2+a)^(1/2)+9/2*A/a
```

```
^(11/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-B/a/x/(b*x^2+a)^(7/2)-8/7*B
/a^2*b*x/(b*x^2+a)^(7/2)-48/35*B/a^3*b*x/(b*x^2+a)^(5/2)-64/35*B/a^4*b*x/(b
*x^2+a)^(3/2)-128/35*B/a^5*b*x/(b*x^2+a)^(1/2)+1/7*C/a/(b*x^2+a)^(7/2)+1/5*
C/a^2/(b*x^2+a)^(5/2)+1/3*C/a^3/(b*x^2+a)^(3/2)+C/a^4/(b*x^2+a)^(1/2)-C/a^(
9/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.49, size = 265, normalized size = 1.21

$$-\frac{128\,Bbx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Bbx}{35\,\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{48\,Bbx}{35\,\left(bx^2+a\right)^{\frac{5}{2}}a^3}-\frac{8\,Bbx}{7\,\left(bx^2+a\right)^{\frac{7}{2}}a^2}-\frac{C\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{9}{2}}}+\frac{9\,Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{11}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -128/35*B*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4) -
 48/35*B*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2) -
C*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 9/2*A*b*arcsinh(a/(sqrt(a*b)*abs(
x)))/a^(11/2) + C/(sqrt(b*x^2 + a)*a^4) + 1/3*C/((b*x^2 + a)^(3/2)*a^3) + 1
/5*C/((b*x^2 + a)^(5/2)*a^2) + 1/7*C/((b*x^2 + a)^(7/2)*a) - 9/2*A*b/(sqrt(
b*x^2 + a)*a^5) - 3/2*A*b/((b*x^2 + a)^(3/2)*a^4) - 9/10*A*b/((b*x^2 + a)^(
5/2)*a^3) - 9/14*A*b/((b*x^2 + a)^(7/2)*a^2) - B/((b*x^2 + a)^(7/2)*a*x) -
1/2*A/((b*x^2 + a)^(7/2)*a*x^2)
```

mupad [B] time = 2.52, size = 279, normalized size = 1.27

$$\frac{\frac{C}{7\,a}+\frac{C\,\left(b\,x^2+a\right)^2}{3\,a^3}+\frac{C\,\left(b\,x^2+a\right)^3}{a^4}+\frac{C\,\left(b\,x^2+a\right)}{5\,a^2}}{\left(b\,x^2+a\right)^{7/2}}-\frac{\frac{A\,b}{7\,a}+\frac{9\,A\,b\,\left(b\,x^2+a\right)}{35\,a^2}+\frac{3\,A\,b\,\left(b\,x^2+a\right)^2}{5\,a^3}+\frac{3\,A\,b\,\left(b\,x^2+a\right)^3}{a^4}-\frac{9\,A\,b\,\left(b\,x^2+a\right)^4}{2\,a^5}}{a\,\left(b\,x^2+a\right)^{7/2}-\left(b\,x^2+a\right)^{9/2}}-\frac{\frac{B}{a^4}+\frac{128\,B\,b}{35\,a^5}}{x\,\sqrt{b\,x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x)
```

```
[Out] (C/(7*a) + (C*(a + b*x^2)^2)/(3*a^3) + (C*(a + b*x^2)^3)/a^4 + (C*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - ((A*b)/(7*a) + (9*A*b*(a + b*x^2))/(35*a^2
) + (3*A*b*(a + b*x^2)^2)/(5*a^3) + (3*A*b*(a + b*x^2)^3)/a^4 - (9*A*b*(a +
 b*x^2)^4)/(2*a^5))/(a*(a + b*x^2)^(7/2) - (a + b*x^2)^(9/2)) - (B/a^4 + (1
28*B*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - (C*atanh((a + b*x^2)^(1/2)/a^
(1/2)))/a^(9/2) + (9*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(11/2)) - (
29*B*b*x)/(35*a^4*(a + b*x^2)^(3/2)) - (13*B*b*x)/(35*a^3*(a + b*x^2)^(5/2)
) - (B*b*x)/(7*a^2*(a + b*x^2)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.58 $\int \frac{A(cx)^m}{a+bx^2}\,dx$

Optimal. Leaf size=45

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}$$

```
[Out] A*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/c/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 364}

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(A*(c*x)^m)/(a + b*x^2),x]
```

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a*c*(1 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A(cx)^m}{a+bx^2}\,dx &= A\int \frac{(cx)^m}{a+bx^2}\,dx\\ &= \frac{A(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.96

$$\frac{Ax(cx)^m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*(c*x)^m)/(a + b*x^2),x]
```

```
[Out] (A*x*(c*x)^m*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/
(a*(1 + m))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^m A}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^m*A/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m A}{bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m*A/(b*x^2 + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{A\,(cx)^m}{b\,x^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A*(c*x)^m/(b*x^2+a),x)
```

```
[Out] int(A*(c*x)^m/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$A \int \frac{(cx)^m}{bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] A*integrate((c*x)^m/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A\,(c\,x)^m}{b\,x^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*(c*x)^m)/(a + b*x^2),x)
```

```
[Out] int((A*(c*x)^m)/(a + b*x^2), x)
```

sympy [C] time = 1.97, size = 97, normalized size = 2.16

$$A \left(\frac{c^m m x x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{c^m x x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)**m/(b*x**2+a),x)
```

```
[Out] A*(c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi
)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)))
```

3.59 $\int \frac{(cx)^m(A+Bx)}{a+bx^2} \, dx$

Optimal. Leaf size=91

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}$$

```
[Out] A*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/c/(1+m)+B*(c
*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-b*x^2/a)/a/c^2/(2+m)
```

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {808, 364}

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a*c*(1 + m)) + (B*(c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)])/(a*c^2*(2 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx &= A\int \frac{(cx)^m}{a+bx^2}\,dx+\frac{B\int \frac{(cx)^{1+m}}{a+bx^2}\,dx}{c}\\ &= \frac{A(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)}+\frac{B(cx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac^2(2+m)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 0.90

$$\frac{x(cx)^m\left(A(m+2)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+B(m+1)x\,{}_2F_1\left(1,\frac{m}{2}+1;\frac{m}{2}+2;-\frac{bx^2}{a}\right)\right)}{a(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -((b*x^2)/a)
] + A*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a
*(1 + m)*(2 + m))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(Bx+A)\,(cx)^m}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx+A)\,(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(Bx+A)\,(cx)^m}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx+A)\,(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^m\,\left(A+B\,x\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x)^m*(A + B*x))/(a + b*x^2),x)
```

```
[Out] int(((c*x)^m*(A + B*x))/(a + b*x^2), x)
```

sympy [C] time = 6.40, size = 192, normalized size = 2.11

$$\frac{Ac^m m x x^m \Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Ac^m x x^m \Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Bc^m m x^2 x^m \Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(B*x+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
+ 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*c**m*m*x**2
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a*ga
mma(m/2 + 2)) + B*c**m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
+ 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2))
```

3.60 $\int \frac{(cx)^m(A+Cx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=76

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

```
[Out] C*(c*x)^(1+m)/b/c/(1+m)+(A*b-C*a)*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2
+1/2*m],-b*x^2/a)/a/b/c/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {459, 364}

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
```

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m\left(A+Cx^2\right)}{a+bx^2}\,dx&=\frac{C(cx)^{1+m}}{bc(1+m)}-\frac{(-Ab(1+m)+aC(1+m))\int \frac{(cx)^m}{a+bx^2}\,dx}{b(1+m)}\\&=\frac{C(cx)^{1+m}}{bc(1+m)}+\frac{(Ab-aC)(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abc(1+m)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 0.74

$$\frac{x(cx)^m\left((Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+aC\right)}{ab(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(a*C + (A*b - a*C)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)]))/(a*b*(1 + m))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2 + A\right)(cx)^m}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + A\right)(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(C\,x^2 + A\right)(cx)^m}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + A\right)(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\,x^2 + A\right)\,(c\,x)^m}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*x^2)*(c*x)^m)/(a + b*x^2),x)
```

```
[Out] int(((A + C*x^2)*(c*x)^m)/(a + b*x^2), x)
```

sympy [C] time = 6.47, size = 204, normalized size = 2.68

$$\frac{Ac^m mxx^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Ac^m xx^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Cc^m mx^3 x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \right.}{4a\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(C*x**2+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
+ 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + C*c**m*m*x**3
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*
a*gamma(m/2 + 5/2)) + 3*C*c**m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a,
 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

3.61 $\int \frac{(cx)^m\left(A+Bx+Cx^2\right)}{a+bx^2} dx$

Optimal. Leaf size=121

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

```
[Out] C*(c*x)^(1+m)/b/c/(1+m)+(A*b-C*a)*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2
+1/2*m],-b*x^2/a)/a/b/c/(1+m)+B*(c*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m
],-b*x^2/a)/a/c^2/(2+m)
```

Rubi [A] time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1802, 808, 364}

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
```

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m)) + (B*(c*x)^(2 +
 m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*c^2*(2 + m
))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^m \left(A+Bx+Cx^2\right)}{a+bx^2}\,dx &= \int \left(\frac{C(cx)^m}{b}+\frac{(cx)^m(Ab-aC+bBx)}{b\left(a+bx^2\right)}\right)dx \\
&= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{\int \frac{(cx)^m(Ab-aC+bBx)}{a+bx^2}\,dx}{b} \\
&= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{B\int \frac{(cx)^{1+m}}{a+bx^2}\,dx}{c}+\frac{(Ab-aC)\int \frac{(cx)^m}{a+bx^2}\,dx}{b} \\
&= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{(Ab-aC)(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abc(1+m)}+\frac{B(cx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+}{2}\right.}{ac^2(2+m)}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 99, normalized size = 0.82

$$
\frac{x(cx)^m\left((m+2)(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+bB(m+1)x\,{}_2F_1\left(1,\frac{m}{2}+1;\frac{m}{2}+2;-\frac{bx^2}{a}\right)+aC(m+2)\right)}{ab(m+1)(m+2)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
[Out] (x*(c*x)^m*(a*C*(2 + m) + b*B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m
/2, -((b*x^2)/a)] + (A*b - a*C)*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3
+ m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*(2 + m))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(Cx^2+Bx+A\right)(cx)^m}{bx^2+a},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
[Out] integral((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(Cx^2+Bx+A\right)(cx)^m}{bx^2+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$
\int \frac{\left(C\,x^2+Bx+A\right)(cx)^m}{b\,x^2+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
[Out] int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)(cx)^m}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^m\,\left(C\,x^2+B\,x+A\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x)
[Out] int(((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2), x)
```

sympy [C] time = 7.60, size = 298, normalized size = 2.46

$$\frac{Ac^m mxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Ac^m xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Bc^m mx^2x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\right.}{4a\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(C*x**2+B*x+A)/(b*x**2+a),x)
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
+ 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*c**m*m*x**2
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a*ga
mma(m/2 + 2)) + B*c**m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
+ 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2)) + C*c**m*m*x**3*x**m*lerchphi(b*x*
*2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
 + 3*C*c**m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamm
a(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

3.62 $\int x^3 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=65

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

```
[Out] 1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/9
*b*D*x^9
```

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b*
C*x^8)/8 + (b*D*x^9)/9
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)dx &= \int \left(aAx^3+aBx^4+(Ab+aC)x^5+(bB+aD)x^6+bCx^7+bDx^8\right)dx\\ &= \frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bD\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.00

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b*
C*x^8)/8 + (b*D*x^9)/9
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.37, size = 57, normalized size = 0.88

$$\frac{1}{9}\,Dbx^{9} + \frac{1}{8}\,Cbx^{8} + \frac{1}{7}\,Dax^{7} + \frac{1}{7}\,Bbx^{7} + \frac{1}{6}\,Cax^{6} + \frac{1}{6}\,Abx^{6} + \frac{1}{5}\,Bax^{5} + \frac{1}{4}\,Aax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*D*a*x^7 + 1/7*B*b*x^7 + 1/6*C*a*x^6 + 1/6*A
*b*x^6 + 1/5*B*a*x^5 + 1/4*A*a*x^4
```

maple [A] time = 0.00, size = 54, normalized size = 0.83

$$\frac{Db\,x^{9}}{9} + \frac{Cb\,x^{8}}{8} + \frac{Ba\,x^{5}}{5} + \frac{(bB + aD)\,x^{7}}{7} + \frac{Aa\,x^{4}}{4} + \frac{(Ab + aC)\,x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/9
*b*D*x^9
```

maxima [A] time = 1.34, size = 53, normalized size = 0.82

$$\frac{1}{9}\,Dbx^{9} + \frac{1}{8}\,Cbx^{8} + \frac{1}{7}\,(Da + Bb)x^{7} + \frac{1}{5}\,Bax^{5} + \frac{1}{6}\,(Ca + Ab)x^{6} + \frac{1}{4}\,Aax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1/6*(C*a +
A*b)*x^6 + 1/4*A*a*x^4
```

mupad [B] time = 1.20, size = 57, normalized size = 0.88

$$\frac{a\,x^{7}\,D}{7} + \frac{b\,x^{9}\,D}{9} + \frac{A\,a\,x^{4}}{4} + \frac{B\,a\,x^{5}}{5} + \frac{A\,b\,x^{6}}{6} + \frac{C\,a\,x^{6}}{6} + \frac{B\,b\,x^{7}}{7} + \frac{C\,b\,x^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^7*D)/7 + (b*x^9*D)/9 + (A*a*x^4)/4 + (B*a*x^5)/5 + (A*b*x^6)/6 + (C*a*
x^6)/6 + (B*b*x^7)/7 + (C*b*x^8)/8
```

sympy [A] time = 0.10, size = 60, normalized size = 0.92

$$\frac{Aax^{4}}{4} + \frac{Bax^{5}}{5} + \frac{Cbx^{8}}{8} + \frac{Dbx^{9}}{9} + x^{7}\left(\frac{Bb}{7} + \frac{Da}{7}\right) + x^{6}\left(\frac{Ab}{6} + \frac{Ca}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**4/4 + B*a*x**5/5 + C*b*x**8/8 + D*b*x**9/9 + x**7*(B*b/7 + D*a/7) +
x**6*(A*b/6 + C*a/6)
```

3.63 $\int x^2 \left(a+b x^2\right) \left(A+B x+C x^2+D x^3\right) \, dx$

Optimal. Leaf size=65

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/8
*b*D*x^8
```

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1802}

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b*
C*x^7)/7 + (b*D*x^8)/8
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)dx &= \int \left(aAx^2+aBx^3+(Ab+aC)x^4+(bB+aD)x^5+bCx^6+bDx^7\right)dx\\ &= \frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}(bB+aD)x^6+\frac{1}{7}bCx^7+\frac{1}{8}bD\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.00

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b*
C*x^7)/7 + (b*D*x^8)/8
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.38, size = 57, normalized size = 0.88

$$\frac{1}{8}Dbx^8 + \frac{1}{7}Cbx^7 + \frac{1}{6}Dax^6 + \frac{1}{6}Bbx^6 + \frac{1}{5}Cax^5 + \frac{1}{5}Abx^5 + \frac{1}{4}Bax^4 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*D*a*x^6 + 1/6*B*b*x^6 + 1/5*C*a*x^5 + 1/5*A
*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

maple [A] time = 0.00, size = 54, normalized size = 0.83

$$\frac{Db\,x^8}{8} + \frac{Cb\,x^7}{7} + \frac{Ba\,x^4}{4} + \frac{(bB + aD)\,x^6}{6} + \frac{Aa\,x^3}{3} + \frac{(Ab + aC)\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/3*A*a*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/8
*b*D*x^8
```

maxima [A] time = 1.33, size = 53, normalized size = 0.82

$$\frac{1}{8}Dbx^8 + \frac{1}{7}Cbx^7 + \frac{1}{6}(Da + Bb)x^6 + \frac{1}{4}Bax^4 + \frac{1}{5}(Ca + Ab)x^5 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1/5*(C*a +
A*b)*x^5 + 1/3*A*a*x^3
```

mupad [B] time = 1.18, size = 57, normalized size = 0.88

$$\frac{a\,x^6\,D}{6} + \frac{b\,x^8\,D}{8} + \frac{A\,a\,x^3}{3} + \frac{B\,a\,x^4}{4} + \frac{A\,b\,x^5}{5} + \frac{C\,a\,x^5}{5} + \frac{B\,b\,x^6}{6} + \frac{C\,b\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^6*D)/6 + (b*x^8*D)/8 + (A*a*x^3)/3 + (B*a*x^4)/4 + (A*b*x^5)/5 + (C*a*
x^5)/5 + (B*b*x^6)/6 + (C*b*x^7)/7
```

sympy [A] time = 0.09, size = 60, normalized size = 0.92

$$\frac{Aax^3}{3} + \frac{Bax^4}{4} + \frac{Cbx^7}{7} + \frac{Dbx^8}{8} + x^6\left(\frac{Bb}{6} + \frac{Da}{6}\right) + x^5\left(\frac{Ab}{5} + \frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + C*b*x**7/7 + D*b*x**8/8 + x**6*(B*b/6 + D*a/6) +
x**5*(A*b/5 + C*a/5)
```

3.64 $\int x \left(a+b x^2\right) \left(A+B x+C x^2+D x^3\right) \, dx$

Optimal. Leaf size=65

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b*C*x^6+1/7
*b*D*x^7
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1802}

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b*
C*x^6)/6 + (b*D*x^7)/7
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x \left(a+b x^2\right) \left(A+B x+C x^2+D x^3\right) \, dx &= \int \left(aAx+aBx^2+(Ab+aC)x^3+(bB+aD)x^4+bCx^5+bDx^6\right) \, dx\\ &= \frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}(bB+aD)x^5+\frac{1}{6}bCx^6+\frac{1}{7}bDx\end{aligned}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 1.00

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b*
C*x^6)/6 + (b*D*x^7)/7
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.32, size = 57, normalized size = 0.88

$$\frac{1}{7}Dbx^7 + \frac{1}{6}Cbx^6 + \frac{1}{5}Dax^5 + \frac{1}{5}Bbx^5 + \frac{1}{4}Cax^4 + \frac{1}{4}Abx^4 + \frac{1}{3}Bax^3 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*D*a*x^5 + 1/5*B*b*x^5 + 1/4*C*a*x^4 + 1/4*A
*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

maple [A] time = 0.00, size = 54, normalized size = 0.83

$$\frac{Db\,x^7}{7} + \frac{Cb\,x^6}{6} + \frac{Ba\,x^3}{3} + \frac{(bB + aD)\,x^5}{5} + \frac{Aa\,x^2}{2} + \frac{(Ab + aC)\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b*C*x^6+1/7
*b*D*x^7
```

maxima [A] time = 1.31, size = 53, normalized size = 0.82

$$\frac{1}{7}Dbx^7 + \frac{1}{6}Cbx^6 + \frac{1}{5}(Da + Bb)x^5 + \frac{1}{3}Bax^3 + \frac{1}{4}(Ca + Ab)x^4 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1/4*(C*a +
A*b)*x^4 + 1/2*A*a*x^2
```

mupad [B] time = 1.19, size = 57, normalized size = 0.88

$$\frac{a\,x^5\,D}{5} + \frac{b\,x^7\,D}{7} + \frac{A\,a\,x^2}{2} + \frac{B\,a\,x^3}{3} + \frac{A\,b\,x^4}{4} + \frac{C\,a\,x^4}{4} + \frac{B\,b\,x^5}{5} + \frac{C\,b\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^5*D)/5 + (b*x^7*D)/7 + (A*a*x^2)/2 + (B*a*x^3)/3 + (A*b*x^4)/4 + (C*a*
x^4)/4 + (B*b*x^5)/5 + (C*b*x^6)/6
```

sympy [A] time = 0.14, size = 60, normalized size = 0.92

$$\frac{Aax^2}{2} + \frac{Bax^3}{3} + \frac{Cbx^6}{6} + \frac{Dbx^7}{7} + x^5\left(\frac{Bb}{5} + \frac{Da}{5}\right) + x^4\left(\frac{Ab}{4} + \frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**2/2 + B*a*x**3/3 + C*b*x**6/6 + D*b*x**7/7 + x**5*(B*b/5 + D*a/5) +
x**4*(A*b/4 + C*a/4)
```

3.65 $\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=60

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*x
^6
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1810}

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5)
/5 + (b*D*x^6)/6
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \int \left(aA+aBx+(Ab+aC)x^2+(bB+aD)x^3+bCx^4+bDx^5\right)\,dx\\ &= aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.00

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5)
/5 + (b*D*x^6)/6
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.36, size = 54, normalized size = 0.90

$$\frac{1}{6}Dbx^6+\frac{1}{5}Cbx^5+\frac{1}{4}Dax^4+\frac{1}{4}Bbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Abx^3+\frac{1}{2}Bax^2+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*D*a*x^4 + 1/4*B*b*x^4 + 1/3*C*a*x^3 + 1/3*A
*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

maple [A] time = 0.00, size = 51, normalized size = 0.85

$$\frac{Db\,x^6}{6}+\frac{Cb\,x^5}{5}+\frac{Ba\,x^2}{2}+\frac{(bB+aD)\,x^4}{4}+Aax+\frac{(Ab+aC)\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] A*a*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*x
^6
```

maxima [A] time = 1.32, size = 50, normalized size = 0.83

$$\frac{1}{6}\,Dbx^6+\frac{1}{5}\,Cbx^5+\frac{1}{4}\,(Da+Bb)x^4+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
A*b)*x^3 + A*a*x
```

mupad [B] time = 1.16, size = 54, normalized size = 0.90

$$\frac{a\,x^4\,D}{4}+\frac{b\,x^6\,D}{6}+A\,a\,x+\frac{B\,a\,x^2}{2}+\frac{A\,b\,x^3}{3}+\frac{C\,a\,x^3}{3}+\frac{B\,b\,x^4}{4}+\frac{C\,b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^4*D)/4 + (b*x^6*D)/6 + A*a*x + (B*a*x^2)/2 + (A*b*x^3)/3 + (C*a*x^3)/3
 + (B*b*x^4)/4 + (C*b*x^5)/5
```

sympy [A] time = 0.12, size = 56, normalized size = 0.93

$$Aax+\frac{Bax^2}{2}+\frac{Cbx^5}{5}+\frac{Dbx^6}{6}+x^4\left(\frac{Bb}{4}+\frac{Da}{4}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x + B*a*x**2/2 + C*b*x**5/5 + D*b*x**6/6 + x**4*(B*b/4 + D*a/4) + x**3*
(A*b/3 + C*a/3)
```

3.66 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} dx$

Optimal. Leaf size=56

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+\frac{1}{3}x^3(aD+bB)+aBx+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5$$

```
[Out] a*B*x+1/2*(A*b+C*a)*x^2+1/3*(B*b+D*a)*x^3+1/4*b*C*x^4+1/5*b*D*x^5+a*A*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+\frac{1}{3}x^3(aD+bB)+aBx+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5)
/5 + a*A*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} dx &= \int \left(aB+\frac{aA}{x}+(Ab+aC)x+(bB+aD)x^2+bCx^3+bDx^4\right) dx\\ &= aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}(bB+aD)x^3+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5+aA\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 1.00

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+\frac{1}{3}x^3(aD+bB)+aBx+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5)
/5 + a*A*Log[x]
```

fricas [A] time = 0.72, size = 48, normalized size = 0.86

$$\frac{1}{5}Dbx^5+\frac{1}{4}Cbx^4+\frac{1}{3}(Da+Bb)x^3+Bax+\frac{1}{2}(Ca+Ab)x^2+Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*x
^2 + A*a*log(x)
```

giac [A] time = 0.41, size = 53, normalized size = 0.95

$$\frac{1}{5}\,Dbx^5+\frac{1}{4}\,Cbx^4+\frac{1}{3}\,Dax^3+\frac{1}{3}\,Bbx^3+\frac{1}{2}\,Cax^2+\frac{1}{2}\,Abx^2+Bax+Aa\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*D*a*x^3 + 1/3*B*b*x^3 + 1/2*C*a*x^2 + 1/2*A
*b*x^2 + B*a*x + A*a*log(abs(x))
```

maple [A] time = 0.00, size = 53, normalized size = 0.95

$$\frac{Db\,x^5}{5}+\frac{Cb\,x^4}{4}+\frac{Bb\,x^3}{3}+\frac{Da\,x^3}{3}+\frac{Ab\,x^2}{2}+\frac{Ca\,x^2}{2}+Aa\ln(x)+Bax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/5*b*D*x^5+1/4*b*C*x^4+1/3*B*b*x^3+1/3*D*x^3*a+1/2*A*x^2*b+1/2*C*x^2*a+a*B
*x+a*A*ln(x)
```

maxima [A] time = 1.35, size = 48, normalized size = 0.86

$$\frac{1}{5}\,Dbx^5+\frac{1}{4}\,Cbx^4+\frac{1}{3}\,(Da+Bb)x^3+Bax+\frac{1}{2}\,(Ca+Ab)x^2+Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*x
^2 + A*a*log(x)
```

mupad [B] time = 1.17, size = 52, normalized size = 0.93

$$\frac{a\,x^3\,D}{3}+\frac{b\,x^5\,D}{5}+B\,a\,x+\frac{A\,b\,x^2}{2}+\frac{C\,a\,x^2}{2}+\frac{B\,b\,x^3}{3}+\frac{C\,b\,x^4}{4}+A\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (a*x^3*D)/3 + (b*x^5*D)/5 + B*a*x + (A*b*x^2)/2 + (C*a*x^2)/2 + (B*b*x^3)/3
 + (C*b*x^4)/4 + A*a*log(x)
```

sympy [A] time = 0.32, size = 54, normalized size = 0.96

$$Aa\log\left(x\right)+Bax+\frac{Cbx^4}{4}+\frac{Dbx^5}{5}+x^3\left(\frac{Bb}{3}+\frac{Da}{3}\right)+x^2\left(\frac{Ab}{2}+\frac{Ca}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a*log(x) + B*a*x + C*b*x**4/4 + D*b*x**5/5 + x**3*(B*b/3 + D*a/3) + x**2*
(A*b/2 + C*a/2)
```

3.67 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2} dx$

Optimal. Leaf size=54

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

```
[Out] -a*A/x+(A*b+C*a)*x+1/2*(B*b+D*a)*x^2+1/3*b*C*x^3+1/4*b*D*x^4+a*B*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)/
4 + a*B*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx &= \int \left(Ab\left(1+\frac{aC}{Ab}\right)+\frac{aA}{x^2}+\frac{aB}{x}+(bB+aD)x+bCx^2+bDx^3\right)dx\\ &= -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 1.00

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)/
4 + a*B*Log[x]
```

fricas [A] time = 0.73, size = 55, normalized size = 1.02

$$\frac{3\,Dbx^5+4\,Cbx^4+6\,(Da+Bb)x^3+12\,Bax\log(x)+12\,(Ca+Ab)x^2-12\,Aa}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*D*b*x^5 + 4*C*b*x^4 + 6*(D*a + B*b)*x^3 + 12*B*a*x*log(x) + 12*(C*a
 + A*b)*x^2 - 12*A*a)/x
```

giac [A] time = 0.39, size = 50, normalized size = 0.93

$$\frac{1}{4}\,Dbx^4 + \frac{1}{3}\,Cbx^3 + \frac{1}{2}\,Dax^2 + \frac{1}{2}\,Bbx^2 + Cax + Abx + Ba\log\left(|x|\right) - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*D*a*x^2 + 1/2*B*b*x^2 + C*a*x + A*b*x + B*a
*log(abs(x)) - A*a/x
```

maple [A] time = 0.01, size = 50, normalized size = 0.93

$$\frac{Db\,x^4}{4} + \frac{Cb\,x^3}{3} + \frac{Bb\,x^2}{2} + \frac{Da\,x^2}{2} + Abx + Ba\ln(x) + Cax - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/4*b*D*x^4+1/3*b*C*x^3+1/2*B*x^2*b+1/2*D*x^2*a+A*b*x+a*C*x-a*A/x+a*B*ln(x)
```

maxima [A] time = 1.34, size = 48, normalized size = 0.89

$$\frac{1}{4}\,Dbx^4 + \frac{1}{3}\,Cbx^3 + \frac{1}{2}\,(Da + Bb)x^2 + Ba\log(x) + (Ca + Ab)x - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*(D*a + B*b)*x^2 + B*a*log(x) + (C*a + A*b)*
x - A*a/x
```

mupad [B] time = 1.14, size = 49, normalized size = 0.91

$$\frac{a\,x^2\,D}{2} + \frac{b\,x^4\,D}{4} + A\,b\,x + C\,a\,x - \frac{A\,a}{x} + \frac{B\,b\,x^2}{2} + \frac{C\,b\,x^3}{3} + B\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] (a*x^2*D)/2 + (b*x^4*D)/4 + A*b*x + C*a*x - (A*a)/x + (B*b*x^2)/2 + (C*b*x^
3)/3 + B*a*log(x)
```

sympy [A] time = 0.28, size = 49, normalized size = 0.91

$$-\frac{Aa}{x} + Ba\log\left(x\right) + \frac{Cbx^3}{3} + \frac{Dbx^4}{4} + x^2\left(\frac{Bb}{2} + \frac{Da}{2}\right) + x\left(Ab + Ca\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + C*b*x**3/3 + D*b*x**4/4 + x**2*(B*b/2 + D*a/2) + x*(A
*b + C*a)
```

3.68 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$

Optimal. Leaf size=54

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}+x(aD+bB)-\frac{aB}{x}+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3$$

```
[Out] -1/2*a*A/x^2-a*B/x+(B*b+D*a)*x+1/2*b*C*x^2+1/3*b*D*x^3+(A*b+C*a)*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}+x(aD+bB)-\frac{aB}{x}+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) - (a*B)/x + (b*B + a*D)*x + (b*C*x^2)/2 + (b*D*x^3)/3 + (A*b
 + a*C)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx &= \int \left(bB\left(1+\frac{aD}{bB}\right)+\frac{aA}{x^3}+\frac{aB}{x^2}+\frac{Ab+aC}{x}+bCx+bDx^2\right)dx\\ &= -\frac{aA}{2x^2}-\frac{aB}{x}+(bB+aD)x+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3+(Ab+aC)\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.94

$$\log(x)(aC+Ab)-\frac{a\left(A+2Bx-2Dx^3\right)}{2x^2}+\frac{1}{6}bx\left(6B+3Cx+2Dx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] (b*x*(6*B + 3*C*x + 2*D*x^2))/6 - (a*(A + 2*B*x - 2*D*x^3))/(2*x^2) + (A*b
+ a*C)*Log[x]
```

fricas [A] time = 0.63, size = 55, normalized size = 1.02

$$\frac{2\,Dbx^5+3\,Cbx^4+6\,(Da+Bb)x^3+6\,(Ca+Ab)x^2\log(x)-6\,Bax-3\,Aa}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*D*b*x^5 + 3*C*b*x^4 + 6*(D*a + B*b)*x^3 + 6*(C*a + A*b)*x^2*log(x) -
 6*B*a*x - 3*A*a)/x^2
```

giac [**A**] time = 0.38, size = 48, normalized size = 0.89

$$\frac{1}{3}\,Dbx^3 + \frac{1}{2}\,Cbx^2 + Dax + Bbx + (Ca + Ab)\log(|x|) - \frac{2\,Bax + Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + D*a*x + B*b*x + (C*a + A*b)*log(abs(x)) - 1/2*(
2*B*a*x + A*a)/x^2
```

maple [**A**] time = 0.01, size = 48, normalized size = 0.89

$$\frac{Db\,x^3}{3} + \frac{Cb\,x^2}{2} + Ab\ln(x) + Bbx + Ca\ln(x) + Dax - \frac{Ba}{x} - \frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

```
[Out] 1/3*b*D*x^3+1/2*b*C*x^2+B*b*x+a*D*x-1/2*a*A/x^2-a*B/x+A*ln(x)*b+C*ln(x)*a
```

maxima [**A**] time = 1.32, size = 48, normalized size = 0.89

$$\frac{1}{3}\,Dbx^3 + \frac{1}{2}\,Cbx^2 + (Da + Bb)x + (Ca + Ab)\log(x) - \frac{2\,Bax + Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + (D*a + B*b)*x + (C*a + A*b)*log(x) - 1/2*(2*B*a
*x + A*a)/x^2
```

mupad [**B**] time = 1.14, size = 47, normalized size = 0.87

$$\frac{b\,x^3\,D}{3} + B\,b\,x - \frac{A\,a}{2\,x^2} - \frac{B\,a}{x} + \frac{C\,b\,x^2}{2} + A\,b\,\ln(x) + C\,a\,\ln(x) + a\,x\,D$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

```
[Out] (b*x^3*D)/3 + B*b*x - (A*a)/(2*x^2) - (B*a)/x + (C*b*x^2)/2 + A*b*log(x) +
C*a*log(x) + a*x*D
```

sympy [**A**] time = 0.51, size = 51, normalized size = 0.94

$$\frac{Cbx^2}{2} + \frac{Dbx^3}{3} + x\,(Bb + Da) + (Ab + Ca)\log{(x)} + \frac{-Aa - 2Bax}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b*x**2/2 + D*b*x**3/3 + x*(B*b + D*a) + (A*b + C*a)*log(x) + (-A*a - 2*B*
a*x)/(2*x**2)
```

3.69 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4} \, dx$

Optimal. Leaf size=54

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

```
[Out] -1/3*a*A/x^3-1/2*a*B/x^2+(-A*b-C*a)/x+b*C*x+1/2*b*D*x^2+(B*b+D*a)*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + b*C*x + (b*D*x^2)/2 + (b*B
 + a*D)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4} \, dx &= \int \left(bC+\frac{aA}{x^4}+\frac{aB}{x^3}+\frac{Ab+aC}{x^2}+\frac{bB+aD}{x}+bDx\right) dx\\ &= -\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 1.02

$$\frac{-aC-Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 - (a*B)/(2*x^2) + (-(A*b) - a*C)/x + b*C*x + (b*D*x^2)/2 + (
b*B + a*D)*Log[x]
```

fricas [A] time = 0.59, size = 55, normalized size = 1.02

$$\frac{3\,Dbx^5+6\,Cbx^4+6\,(Da+Bb)x^3\log(x)-3\,Bax-6\,(Ca+Ab)x^2-2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*D*b*x^5 + 6*C*b*x^4 + 6*(D*a + B*b)*x^3*log(x) - 3*B*a*x - 6*(C*a +
A*b)*x^2 - 2*A*a)/x^3
```

giac [A] time = 0.43, size = 50, normalized size = 0.93

$$\frac{1}{2}\,Dbx^2 + Cbx + (Da + Bb)\log\left(|x|\right) - \frac{3\,Bax + 6\,(Ca + Ab)x^2 + 2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(abs(x)) - 1/6*(3*B*a*x + 6*(C*a + A*b
)*x^2 + 2*A*a)/x^3
```

maple [A] time = 0.00, size = 51, normalized size = 0.94

$$\frac{Db\,x^2}{2} + Bb\ln(x) + Cbx + Da\ln(x) - \frac{Ab}{x} - \frac{Ca}{x} - \frac{Ba}{2x^2} - \frac{Aa}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/2*b*D*x^2+b*C*x-1/3*A*a/x^3-1/2*B*a/x^2-1/x*A*b-1/x*a*C+B*b*ln(x)+D*ln(x)
*a
```

maxima [A] time = 1.33, size = 49, normalized size = 0.91

$$\frac{1}{2}\,Dbx^2 + Cbx + (Da + Bb)\log(x) - \frac{3\,Bax + 6\,(Ca + Ab)x^2 + 2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(x) - 1/6*(3*B*a*x + 6*(C*a + A*b)*x^2
 + 2*A*a)/x^3
```

mupad [B] time = 1.15, size = 50, normalized size = 0.93

$$\frac{b\,x^2\,D}{2} + a\,\ln(x)\,D + C\,b\,x - \frac{A\,a}{3\,x^3} - \frac{A\,b}{x} - \frac{B\,a}{2\,x^2} - \frac{C\,a}{x} + B\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

```
[Out] (b*x^2*D)/2 + a*log(x)*D + C*b*x - (A*a)/(3*x^3) - (A*b)/x - (B*a)/(2*x^2)
- (C*a)/x + B*b*log(x)
```

sympy [A] time = 1.01, size = 54, normalized size = 1.00

$$Cbx + \frac{Dbx^2}{2} + (Bb + Da)\log\left(x\right) + \frac{-2Aa - 3Bax + x^2\left(-6Ab - 6Ca\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b*x + D*b*x**2/2 + (B*b + D*a)*log(x) + (-2*A*a - 3*B*a*x + x**2*(-6*A*b
- 6*C*a))/(6*x**3)
```

3.70 $\int x^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=109

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

```
[Out] 1/4*a^2*A*x^4+1/5*a^2*B*x^5+1/6*a*(2*A*b+C*a)*x^6+1/7*a*(2*B*b+D*a)*x^7+1/8
*b*(A*b+2*C*a)*x^8+1/9*b*(B*b+2*D*a)*x^9+1/10*b^2*C*x^10+1/11*b^2*D*x^11
```

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^4)/4 + (a^2*B*x^5)/5 + (a*(2*A*b + a*C)*x^6)/6 + (a*(2*b*B + a*D)*
x^7)/7 + (b*(A*b + 2*a*C)*x^8)/8 + (b*(b*B + 2*a*D)*x^9)/9 + (b^2*C*x^10)/1
0 + (b^2*D*x^11)/11
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx = \int \left(a^2Ax^3+a^2Bx^4+a(2Ab+aC)x^5+a(2bB+aD)x^6+b(Ab+2a$$
$$= \frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{6}a(2Ab+aC)x^6+\frac{1}{7}a(2bB+aD)x^7+\frac{1}{8}b(Ab$$

Mathematica [A] time = 0.06, size = 98, normalized size = 0.90

$$a^2\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}+\frac{1}{42}x^6(7C+6Dx)\right)+\frac{1}{252}abx^6(84A+x(72B+7x(9C+8Dx)))+\frac{b^2x^8\left(495A+4x\left(110B+99Cx+9\right.\right.}{3960}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^2*((A*x^4)/4 + (B*x^5)/5 + (x^6*(7*C + 6*D*x))/42) + (b^2*x^8*(495*A + 4*
x*(110*B + 99*C*x + 90*D*x^2)))/3960 + (a*b*x^6*(84*A + x*(72*B + 7*x*(9*C
+ 8*D*x))))/252
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.35, size = 105, normalized size = 0.96

$$\frac{1}{11}\,Db^2x^{11}+\frac{1}{10}\,Cb^2x^{10}+\frac{2}{9}\,Dabx^9+\frac{1}{9}\,Bb^2x^9+\frac{1}{4}\,Cabx^8+\frac{1}{8}\,Ab^2x^8+\frac{1}{7}\,Da^2x^7+\frac{2}{7}\,Babx^7+\frac{1}{6}\,Ca^2x^6+\frac{1}{3}\,Aabx^6+\frac{1}{5}\,Ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 2/9*D*a*b*x^9 + 1/9*B*b^2*x^9 + 1/4*C*a
*b*x^8 + 1/8*A*b^2*x^8 + 1/7*D*a^2*x^7 + 2/7*B*a*b*x^7 + 1/6*C*a^2*x^6 + 1/
3*A*a*b*x^6 + 1/5*B*a^2*x^5 + 1/4*A*a^2*x^4
```

maple [A] time = 0.00, size = 102, normalized size = 0.94

$$\frac{Db^2x^{11}}{11}+\frac{C\,b^2x^{10}}{10}+\frac{\left(b^2B+2abD\right)x^9}{9}+\frac{B\,a^2x^5}{5}+\frac{\left(b^2A+2abC\right)x^8}{8}+\frac{A\,a^2x^4}{4}+\frac{\left(2abB+a^2D\right)x^7}{7}+\frac{\left(2Aab+a^2C\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/11*b^2*D*x^11+1/10*b^2*C*x^10+1/9*(B*b^2+2*D*a*b)*x^9+1/8*(A*b^2+2*C*a*b)
*x^8+1/7*(2*B*a*b+D*a^2)*x^7+1/6*(2*A*a*b+C*a^2)*x^6+1/5*a^2*B*x^5+1/4*a^2*
A*x^4
```

maxima [A] time = 1.36, size = 101, normalized size = 0.93

$$\frac{1}{11}\,Db^2x^{11}+\frac{1}{10}\,Cb^2x^{10}+\frac{1}{9}\left(2\,Dab+Bb^2\right)x^9+\frac{1}{8}\left(2\,Cab+Ab^2\right)x^8+\frac{1}{5}\,Ba^2x^5+\frac{1}{7}\left(Da^2+2\,Bab\right)x^7+\frac{1}{4}\,Aa^2x^4+\frac{1}{6}\,(C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1/8*(2*C*a*
b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*x^7 + 1/4*A*a^2*x^4
+ 1/6*(C*a^2 + 2*A*a*b)*x^6
```

mupad [B] time = 1.13, size = 108, normalized size = 0.99

$$\frac{a^2\,x^7\,D}{7}+\frac{b^2\,x^{11}\,D}{11}+\frac{A\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24}+\frac{B\,x^5\,\left(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4\right)}{315}+\frac{C\,x^6\,\left(10\,a^2+15\,a\,b\,x^2\right.}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^7*D)/7 + (b^2*x^11*D)/11 + (A*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/2
4 + (B*x^5*(63*a^2 + 35*b^2*x^4 + 90*a*b*x^2))/315 + (C*x^6*(10*a^2 + 6*b^2
*x^4 + 15*a*b*x^2))/60 + (2*a*b*x^9*D)/9
```

sympy [A] time = 0.14, size = 110, normalized size = 1.01

$$\frac{Aa^2x^4}{4}+\frac{Ba^2x^5}{5}+\frac{Cb^2x^{10}}{10}+\frac{Db^2x^{11}}{11}+x^9\left(\frac{Bb^2}{9}+\frac{2Dab}{9}\right)+x^8\left(\frac{Ab^2}{8}+\frac{Cab}{4}\right)+x^7\left(\frac{2Bab}{7}+\frac{Da^2}{7}\right)+x^6\left(\frac{Aab}{3}+\frac{Ca^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**4/4 + B*a**2*x**5/5 + C*b**2*x**10/10 + D*b**2*x**11/11 + x**9*(B
*b**2/9 + 2*D*a*b/9) + x**8*(A*b**2/8 + C*a*b/4) + x**7*(2*B*a*b/7 + D*a**2
/7) + x**6*(A*a*b/3 + C*a**2/6)
```

3.71 $\int x^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=109

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

```
[Out] 1/3*a^2*A*x^3+1/4*a^2*B*x^4+1/5*a*(2*A*b+C*a)*x^5+1/6*a*(2*B*b+D*a)*x^6+1/7
*b*(A*b+2*C*a)*x^7+1/8*b*(B*b+2*D*a)*x^8+1/9*b^2*C*x^9+1/10*b^2*D*x^10
```

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*(2*b*B + a*D)*
x^6)/6 + (b*(A*b + 2*a*C)*x^7)/7 + (b*(b*B + 2*a*D)*x^8)/8 + (b^2*C*x^9)/9
+ (b^2*D*x^10)/10
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx &= \int \left(a^2Ax^2+a^2Bx^3+a(2Ab+aC)x^4+a(2bB+aD)x^5+b(Ab+2a\right.\\ &= \frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{6}a(2bB+aD)x^6+\frac{1}{7}b(Ab\end{aligned}$$

Mathematica [A] time = 0.07, size = 92, normalized size = 0.84

$$\frac{42a^2x^3(20A+x(15B+2x(6C+5Dx)))+6abx^5(168A+5x(28B+3x(8C+7Dx)))+b^2x^7(360A+7x(45B+4x}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 6*a*b*x^5*(168*A + 5*x*
(28*B + 3*x*(8*C + 7*D*x))) + b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*
x))))/2520
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.37, size = 105, normalized size = 0.96

$$\frac{1}{10}\,Db^2x^{10}+\frac{1}{9}\,Cb^2x^9+\frac{1}{4}\,Dabx^8+\frac{1}{8}\,Bb^2x^8+\frac{2}{7}\,Cabx^7+\frac{1}{7}\,Ab^2x^7+\frac{1}{6}\,Da^2x^6+\frac{1}{3}\,Babx^6+\frac{1}{5}\,Ca^2x^5+\frac{2}{5}\,Aabx^5+\frac{1}{4}\,Ba^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/4*D*a*b*x^8 + 1/8*B*b^2*x^8 + 2/7*C*a*b
*x^7 + 1/7*A*b^2*x^7 + 1/6*D*a^2*x^6 + 1/3*B*a*b*x^6 + 1/5*C*a^2*x^5 + 2/5*
A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

maple [A] time = 0.00, size = 102, normalized size = 0.94

$$\frac{Db^2x^{10}}{10}+\frac{C\,b^2x^9}{9}+\frac{\left(b^2B+2abD\right)x^8}{8}+\frac{B\,a^2x^4}{4}+\frac{\left(b^2A+2abC\right)x^7}{7}+\frac{A\,a^2x^3}{3}+\frac{\left(2abB+a^2D\right)x^6}{6}+\frac{\left(2Aab+a^2C\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/10*b^2*D*x^10+1/9*b^2*C*x^9+1/8*(B*b^2+2*D*a*b)*x^8+1/7*(A*b^2+2*C*a*b)*x
^7+1/6*(2*B*a*b+D*a^2)*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*A*a^2*
x^3
```

maxima [A] time = 1.30, size = 101, normalized size = 0.93

$$\frac{1}{10}\,Db^2x^{10}+\frac{1}{9}\,Cb^2x^9+\frac{1}{8}\,\left(2\,Dab+Bb^2\right)x^8+\frac{1}{7}\,\left(2\,Cab+Ab^2\right)x^7+\frac{1}{4}\,Ba^2x^4+\frac{1}{6}\,\left(Da^2+2\,Bab\right)x^6+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,\left(Ca\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7*(2*C*a*b
+ A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^6 + 1/3*A*a^2*x^3 +
1/5*(C*a^2 + 2*A*a*b)*x^5
```

mupad [B] time = 1.11, size = 108, normalized size = 0.99

$$\frac{a^2\,x^6\,D}{6}+\frac{b^2\,x^{10}\,D}{10}+\frac{A\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105}+\frac{B\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24}+\frac{C\,x^5\,\left(63\,a^2+90\,a\,b\,x\right.}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^6*D)/6 + (b^2*x^10*D)/10 + (A*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2)
)/105 + (B*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/24 + (C*x^5*(63*a^2 + 35*b^
2*x^4 + 90*a*b*x^2))/315 + (a*b*x^8*D)/4
```

sympy [A] time = 0.14, size = 110, normalized size = 1.01

$$\frac{Aa^2x^3}{3}+\frac{Ba^2x^4}{4}+\frac{Cb^2x^9}{9}+\frac{Db^2x^{10}}{10}+x^8\left(\frac{Bb^2}{8}+\frac{Dab}{4}\right)+x^7\left(\frac{Ab^2}{7}+\frac{2Cab}{7}\right)+x^6\left(\frac{Bab}{3}+\frac{Da^2}{6}\right)+x^5\left(\frac{2Aab}{5}+\frac{Ca^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + C*b**2*x**9/9 + D*b**2*x**10/10 + x**8*(B*b
**2/8 + D*a*b/4) + x**7*(A*b**2/7 + 2*C*a*b/7) + x**6*(B*a*b/3 + D*a**2/6)
+ x**5*(2*A*a*b/5 + C*a**2/5)
```

3.72 $\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=104

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{A\left(a+bx^2\right)^3}{6b}+\frac{1}{7}bx^7(2aD+bB)+\frac{1}{5}ax^5(aD+2bB)+\frac{1}{3}abCx^6+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9$$

```
[Out] 1/3*a^2*B*x^3+1/4*a^2*C*x^4+1/5*a*(2*B*b+D*a)*x^5+1/3*a*b*C*x^6+1/7*b*(B*b+
2*D*a)*x^7+1/8*b^2*C*x^8+1/9*b^2*D*x^9+1/6*A*(b*x^2+a)^3/b
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1582, 1810}

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{A\left(a+bx^2\right)^3}{6b}+\frac{1}{7}bx^7(2aD+bB)+\frac{1}{5}ax^5(aD+2bB)+\frac{1}{3}abCx^6+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*B*x^3)/3 + (a^2*C*x^4)/4 + (a*(2*b*B + a*D)*x^5)/5 + (a*b*C*x^6)/3 + (
b*(b*B + 2*a*D)*x^7)/7 + (b^2*C*x^8)/8 + (b^2*D*x^9)/9 + (A*(a + b*x^2)^3)/
(6*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)dx &= \frac{A\left(a+bx^2\right)^3}{6b}+\int\left(a+bx^2\right)^2\left(-Ax+x\left(A+Bx+Cx^2+Dx^3\right)\right)dx\\ &= \frac{A\left(a+bx^2\right)^3}{6b}+\int\left(a^2Bx^2+a^2Cx^3+a(2bB+aD)x^4+2abCx^5+b(\right.\\ &= \frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{1}{5}a(2bB+aD)x^5+\frac{1}{3}abCx^6+\frac{1}{7}b(bB+2aD)x^7\end{aligned}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 0.88

$$\frac{42a^2x^2(30A+x(20B+3x(5C+4Dx)))+12abx^4(105A+2x(42B+5x(7C+6Dx)))+5b^2x^6(84A+x(72B+7x}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) + 12*a*b*x^4*(105*A + 2*x
*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x
))))/2520
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.36, size = 105, normalized size = 1.01

$$\frac{1}{9}Db^2x^9+\frac{1}{8}Cb^2x^8+\frac{2}{7}Dabx^7+\frac{1}{7}Bb^2x^7+\frac{1}{3}Cabx^6+\frac{1}{6}Ab^2x^6+\frac{1}{5}Da^2x^5+\frac{2}{5}Babx^5+\frac{1}{4}Ca^2x^4+\frac{1}{2}Aabx^4+\frac{1}{3}Ba^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 2/7*D*a*b*x^7 + 1/7*B*b^2*x^7 + 1/3*C*a*b*x
^6 + 1/6*A*b^2*x^6 + 1/5*D*a^2*x^5 + 2/5*B*a*b*x^5 + 1/4*C*a^2*x^4 + 1/2*A*
a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

maple [A] time = 0.00, size = 102, normalized size = 0.98

$$\frac{Db^2x^9}{9}+\frac{C\,b^2x^8}{8}+\frac{\left(b^2B+2abD\right)x^7}{7}+\frac{B\,a^2x^3}{3}+\frac{\left(b^2A+2abC\right)x^6}{6}+\frac{A\,a^2x^2}{2}+\frac{\left(2abB+a^2D\right)x^5}{5}+\frac{\left(2Aab+a^2C\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/9*b^2*D*x^9+1/8*b^2*C*x^8+1/7*(B*b^2+2*D*a*b)*x^7+1/6*(A*b^2+2*C*a*b)*x^6
+1/5*(2*B*a*b+D*a^2)*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*x^
2
```

maxima [A] time = 1.39, size = 101, normalized size = 0.97

$$\frac{1}{9}Db^2x^9+\frac{1}{8}Cb^2x^8+\frac{1}{7}\left(2\,Dab+Bb^2\right)x^7+\frac{1}{6}\left(2\,Cab+Ab^2\right)x^6+\frac{1}{3}Ba^2x^3+\frac{1}{5}\left(Da^2+2\,Bab\right)x^5+\frac{1}{2}Aa^2x^2+\frac{1}{4}\left(Ca^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(2*C*a*b +
A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5 + 1/2*A*a^2*x^2 + 1/
4*(C*a^2 + 2*A*a*b)*x^4
```

mupad [B] time = 1.11, size = 107, normalized size = 1.03

$$\frac{a^2\,x^5\,D}{5}+\frac{b^2\,x^9\,D}{9}+\frac{A\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+b^2\,x^4\right)}{6}+\frac{B\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105}+\frac{C\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^5*D)/5 + (b^2*x^9*D)/9 + (A*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (
B*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2))/105 + (C*x^4*(6*a^2 + 3*b^2*x^4 +
 8*a*b*x^2))/24 + (2*a*b*x^7*D)/7
```

sympy [A] time = 0.09, size = 110, normalized size = 1.06

$$\frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{Cb^2x^8}{8}+\frac{Db^2x^9}{9}+x^7\left(\frac{Bb^2}{7}+\frac{2Dab}{7}\right)+x^6\left(\frac{Ab^2}{6}+\frac{Cab}{3}\right)+x^5\left(\frac{2Bab}{5}+\frac{Da^2}{5}\right)+x^4\left(\frac{Aab}{2}+\frac{Ca^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + C*b**2*x**8/8 + D*b**2*x**9/9 + x**7*(B*b**
2/7 + 2*D*a*b/7) + x**6*(A*b**2/6 + C*a*b/3) + x**5*(2*B*a*b/5 + D*a**2/5)
+ x**4*(A*a*b/2 + C*a**2/4)
```

3.73 $\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=99

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

```
[Out] a^2*A*x+1/3*a*(2*A*b+C*a)*x^3+1/4*a^2*D*x^4+1/5*b*(A*b+2*C*a)*x^5+1/3*a*b*D
*x^6+1/7*b^2*C*x^7+1/8*b^2*D*x^8+1/6*B*(b*x^2+a)^3/b
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1582, 1810}

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a*C)*x^5)/5
 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(a + b*x^2)^3)/(6*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{B\left(a+bx^2\right)^3}{6b}+\int \left(a+bx^2\right)^2\left(A+Cx^2+Dx^3\right)\,dx\\ &= \frac{B\left(a+bx^2\right)^3}{6b}+\int \left(a^2A+a(2Ab+aC)x^2+a^2Dx^3+b(Ab+2aC)\right.\\ &= a^2Ax+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{4}a^2Dx^4+\frac{1}{5}b(Ab+2aC)x^5+\frac{1}{3}abDx^6\end{aligned}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 0.89

$$\frac{1}{840}\left(70a^2x(12A+x(6B+x(4C+3Dx)))+28abx^3(20A+x(15B+2x(6C+5Dx)))+b^2x^5(168A+5x(28B+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (70*a^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 28*a*b*x^3*(20*A + x*(15*B +
 2*x*(6*C + 5*D*x))) + b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/84
0
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.38, size = 102, normalized size = 1.03

$$\frac{1}{8}Db^2x^8+\frac{1}{7}Cb^2x^7+\frac{1}{3}Dabx^6+\frac{1}{6}Bb^2x^6+\frac{2}{5}Cabx^5+\frac{1}{5}Ab^2x^5+\frac{1}{4}Da^2x^4+\frac{1}{2}Babx^4+\frac{1}{3}Ca^2x^3+\frac{2}{3}Aabx^3+\frac{1}{2}Ba^2x^2+A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/3*D*a*b*x^6 + 1/6*B*b^2*x^6 + 2/5*C*a*b*x
^5 + 1/5*A*b^2*x^5 + 1/4*D*a^2*x^4 + 1/2*B*a*b*x^4 + 1/3*C*a^2*x^3 + 2/3*A*
a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

maple [A] time = 0.00, size = 99, normalized size = 1.00

$$\frac{Db^2x^8}{8}+\frac{C\,b^2x^7}{7}+\frac{\left(b^2B+2abD\right)x^6}{6}+\frac{B\,a^2x^2}{2}+\frac{\left(b^2A+2abC\right)x^5}{5}+A\,a^2x+\frac{\left(2abB+a^2D\right)x^4}{4}+\frac{\left(2Aab+a^2C\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/8*b^2*D*x^8+1/7*b^2*C*x^7+1/6*(B*b^2+2*D*a*b)*x^6+1/5*(A*b^2+2*C*a*b)*x^5
+1/4*(2*B*a*b+D*a^2)*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

maxima [A] time = 1.35, size = 98, normalized size = 0.99

$$\frac{1}{8}Db^2x^8+\frac{1}{7}Cb^2x^7+\frac{1}{6}\left(2\,Dab+Bb^2\right)x^6+\frac{1}{5}\left(2\,Cab+Ab^2\right)x^5+\frac{1}{2}Ba^2x^2+\frac{1}{4}\left(Da^2+2\,Bab\right)x^4+Aa^2x+\frac{1}{3}\left(Ca^2+2\,Aa\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C*a
^2 + 2*A*a*b)*x^3
```

mupad [B] time = 1.11, size = 105, normalized size = 1.06

$$\frac{A\,x\,\left(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4\right)}{15}+\frac{a^2\,x^4\,D}{4}+\frac{b^2\,x^8\,D}{8}+\frac{B\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+b^2\,x^4\right)}{6}+\frac{C\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b\right.}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (a^2*x^4*D)/4 + (b^2*x^8*D)/8
+ (B*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (C*x^3*(35*a^2 + 15*b^2*x^4 + 4
2*a*b*x^2))/105 + (a*b*x^6*D)/3
```

sympy [A] time = 0.09, size = 107, normalized size = 1.08

$$Aa^2x + \frac{Ba^2x^2}{2} + \frac{Cb^2x^7}{7} + \frac{Db^2x^8}{8} + x^6\left(\frac{Bb^2}{6} + \frac{Dab}{3}\right) + x^5\left(\frac{Ab^2}{5} + \frac{2Cab}{5}\right) + x^4\left(\frac{Bab}{2} + \frac{Da^2}{4}\right) + x^3\left(\frac{2Aab}{3} + \frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + C*b**2*x**7/7 + D*b**2*x**8/8 + x**6*(B*b**2/6 +
 D*a*b/3) + x**5*(A*b**2/5 + 2*C*a*b/5) + x**4*(B*a*b/2 + D*a**2/4) + x**3*
(2*A*a*b/3 + C*a**2/3)
```

3.74 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx$

Optimal. Leaf size=92

$$a^2A\log(x)+a^2Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

```
[Out] a^2*B*x+a*A*b*x^2+1/3*a*(2*B*b+D*a)*x^3+1/4*A*b^2*x^4+1/5*b*(B*b+2*D*a)*x^5
+1/7*b^2*D*x^7+1/6*C*(b*x^2+a)^3/b+a^2*A*ln(x)
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1583, 1802}

$$a^2A\log(x)+a^2Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a^2*B*x + a*A*b*x^2 + (a*(2*b*B + a*D)*x^3)/3 + (A*b^2*x^4)/4 + (b*(b*B + 2
*a*D)*x^5)/5 + (b^2*D*x^7)/7 + (C*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx &= \frac{C\left(a+bx^2\right)^3}{6b}+\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Dx^3\right)}{x}\,dx\\ &= \frac{C\left(a+bx^2\right)^3}{6b}+\int \left(a^2B+\frac{a^2A}{x}+2aAbx+a(2bB+aD)x^2+Ab^2x^3+\right.\\ &= a^2Bx+aAbx^2+\frac{1}{3}a(2bB+aD)x^3+\frac{1}{4}Ab^2x^4+\frac{1}{5}b(bB+2aD)x^5+\frac{1}{7}b\end{aligned}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 0.96

$$\frac{1}{420}x\left(70a^2(6B+x(3C+2Dx))+14abx(30A+x(20B+3x(5C+4Dx)))+b^2x^3(105A+2x(42B+5x(7C+6Dx\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(70*a^2*(6*B + x*(3*C + 2*D*x)) + 14*a*b*x*(30*A + x*(20*B + 3*x*(5*C +
4*D*x))) + b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x)))))/420 + a^2*A*L
og[x]
```

fricas [A] time = 0.53, size = 96, normalized size = 1.04

$$\frac{1}{7}Db^2x^7+\frac{1}{6}Cb^2x^6+\frac{1}{5}\left(2\,Dab+Bb^2\right)x^5+\frac{1}{4}\left(2\,Cab+Ab^2\right)x^4+Ba^2x+\frac{1}{3}\left(Da^2+2\,Bab\right)x^3+Aa^2\log(x)+\frac{1}{2}\left(Ca^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*a^
2 + 2*A*a*b)*x^2
```

giac [A] time = 0.46, size = 100, normalized size = 1.09

$$\frac{1}{7}Db^2x^7+\frac{1}{6}Cb^2x^6+\frac{2}{5}Dabx^5+\frac{1}{5}Bb^2x^5+\frac{1}{2}Cabx^4+\frac{1}{4}Ab^2x^4+\frac{1}{3}Da^2x^3+\frac{2}{3}Babx^3+\frac{1}{2}Ca^2x^2+Aabx^2+Ba^2x+Aa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 2/5*D*a*b*x^5 + 1/5*B*b^2*x^5 + 1/2*C*a*b*x
^4 + 1/4*A*b^2*x^4 + 1/3*D*a^2*x^3 + 2/3*B*a*b*x^3 + 1/2*C*a^2*x^2 + A*a*b*
x^2 + B*a^2*x + A*a^2*log(abs(x))
```

maple [A] time = 0.00, size = 100, normalized size = 1.09

$$\frac{Db^2x^7}{7}+\frac{C\,b^2x^6}{6}+\frac{B\,b^2x^5}{5}+\frac{2Dab\,x^5}{5}+\frac{A\,b^2x^4}{4}+\frac{Cab\,x^4}{2}+\frac{2Bab\,x^3}{3}+\frac{Da^2x^3}{3}+Aab\,x^2+\frac{C\,a^2x^2}{2}+A\,a^2\ln(x)+B\,a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/7*b^2*D*x^7+1/6*C*b^2*x^6+1/5*B*b^2*x^5+2/5*D*x^5*a*b+1/4*A*b^2*x^4+1/2*C
*x^4*a*b+2/3*B*x^3*a*b+1/3*D*x^3*a^2+A*a*b*x^2+1/2*C*x^2*a^2+B*a^2*x+a^2*A*
ln(x)
```

maxima [A] time = 1.38, size = 96, normalized size = 1.04

$$\frac{1}{7}Db^2x^7+\frac{1}{6}Cb^2x^6+\frac{1}{5}\left(2\,Dab+Bb^2\right)x^5+\frac{1}{4}\left(2\,Cab+Ab^2\right)x^4+Ba^2x+\frac{1}{3}\left(Da^2+2\,Bab\right)x^3+Aa^2\log(x)+\frac{1}{2}\left(Ca^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*a^
2 + 2*A*a*b)*x^2
```

mupad [B] time = 1.11, size = 103, normalized size = 1.12

$$\frac{A\,\left(4\,a^2\,\ln(x)+b^2\,x^4+4\,a\,b\,x^2\right)}{4}+\frac{B\,x\,\left(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4\right)}{15}+\frac{a^2\,x^3\,D}{3}+\frac{b^2\,x^7\,D}{7}+\frac{C\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (A*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + (B*x*(15*a^2 + 3*b^2*x^4 + 10*
a*b*x^2))/15 + (a^2*x^3*D)/3 + (b^2*x^7*D)/7 + (C*x^2*(3*a^2 + b^2*x^4 + 3*
a*b*x^2))/6 + (2*a*b*x^5*D)/5
```

sympy [A] time = 0.32, size = 104, normalized size = 1.13

$$Aa^2 \log(x) + Ba^2x + \frac{Cb^2x^6}{6} + \frac{Db^2x^7}{7} + x^5\left(\frac{Bb^2}{5} + \frac{2Dab}{5}\right) + x^4\left(\frac{Ab^2}{4} + \frac{Cab}{2}\right) + x^3\left(\frac{2Bab}{3} + \frac{Da^2}{3}\right) + x^2\left(Aab + \frac{Ca^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*a**2*x + C*b**2*x**6/6 + D*b**2*x**7/7 + x**5*(B*b**2/5 +
 2*D*a*b/5) + x**4*(A*b**2/4 + C*a*b/2) + x**3*(2*B*a*b/3 + D*a**2/3) + x**
2*(A*a*b + C*a**2/2)
```

3.75 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2} dx$

Optimal. Leaf size=90

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{3}bx^3(2aC+Ab)+ax(aC+2Ab)+abBx^2+\frac{D\left(a+bx^2\right)^3}{6b}+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5$$

```
[Out] -a^2*A/x+a*(2*A*b+C*a)*x+a*b*B*x^2+1/3*b*(A*b+2*C*a)*x^3+1/4*b^2*B*x^4+1/5*
b^2*C*x^5+1/6*D*(b*x^2+a)^3/b+a^2*B*ln(x)
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1583, 1628}

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{3}bx^3(2aC+Ab)+ax(aC+2Ab)+abBx^2+\frac{D\left(a+bx^2\right)^3}{6b}+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + (b*(A*b + 2*a*C)*x^3)/3 + (b
^2*B*x^4)/4 + (b^2*C*x^5)/5 + (D*(a + b*x^2)^3)/(6*b) + a^2*B*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx &= \frac{D\left(a+bx^2\right)^3}{6b}+\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2\right)}{x^2}\,dx\\ &= \frac{D\left(a+bx^2\right)^3}{6b}+\int \left(a(2Ab+aC)+\frac{a^2A}{x^2}+\frac{a^2B}{x}+2abBx+b(Ab+\right.\\ &= -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}b(Ab+2aC)x^3+\frac{1}{4}b^2Bx^4+\frac{1}{5}\end{aligned}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 0.98

$$a^2\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right)+a^2B\log(x)+\frac{1}{6}abx(12A+x(6B+x(4C+3Dx)))+\frac{1}{60}b^2x^3(20A+x(15B+2x(6C+5Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^2*(-(A/x) + C*x + (D*x^2)/2) + (a*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))
/6 + (b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*B*Log[x]
```

fricas [A] time = 0.48, size = 103, normalized size = 1.14

$$\frac{10\,Db^2x^7+12\,Cb^2x^6+15\left(2\,Dab+Bb^2\right)x^5+20\left(2\,Cab+Ab^2\right)x^4+60\,Ba^2x\log(x)+30\left(Da^2+2\,Bab\right)x^3-60\,A}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*D*b^2*x^7 + 12*C*b^2*x^6 + 15*(2*D*a*b + B*b^2)*x^5 + 20*(2*C*a*b
+ A*b^2)*x^4 + 60*B*a^2*x*log(x) + 30*(D*a^2 + 2*B*a*b)*x^3 - 60*A*a^2 + 60
*(C*a^2 + 2*A*a*b)*x^2)/x
```

giac [A] time = 0.38, size = 98, normalized size = 1.09

$$\frac{1}{6}\,Db^2x^6+\frac{1}{5}\,Cb^2x^5+\frac{1}{2}\,Dabx^4+\frac{1}{4}\,Bb^2x^4+\frac{2}{3}\,Cabx^3+\frac{1}{3}\,Ab^2x^3+\frac{1}{2}\,Da^2x^2+Babx^2+Ca^2x+2\,Aabx+Ba^2\log\left(|x|\right)-\frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/2*D*a*b*x^4 + 1/4*B*b^2*x^4 + 2/3*C*a*b*x
^3 + 1/3*A*b^2*x^3 + 1/2*D*a^2*x^2 + B*a*b*x^2 + C*a^2*x + 2*A*a*b*x + B*a^
2*log(abs(x)) - A*a^2/x
```

maple [A] time = 0.01, size = 98, normalized size = 1.09

$$\frac{Db^2x^6}{6}+\frac{C\,b^2x^5}{5}+\frac{B\,b^2x^4}{4}+\frac{Dab\,x^4}{2}+\frac{A\,b^2x^3}{3}+\frac{2Cab\,x^3}{3}+Bab\,x^2+\frac{Da^2x^2}{2}+2Aabx+B\,a^2\ln(x)+C\,a^2x-\frac{A\,a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/6*D*b^2*x^6+1/5*b^2*C*x^5+1/4*B*b^2*x^4+1/2*D*x^4*a*b+1/3*A*x^3*b^2+2/3*C
*x^3*a*b+B*a*b*x^2+1/2*D*x^2*a^2+2*A*a*b*x+a^2*C*x-A*a^2/x+a^2*B*ln(x)
```

maxima [A] time = 1.32, size = 96, normalized size = 1.07

$$\frac{1}{6}\,Db^2x^6+\frac{1}{5}\,Cb^2x^5+\frac{1}{4}\left(2\,Dab+Bb^2\right)x^4+\frac{1}{3}\left(2\,Cab+Ab^2\right)x^3+Ba^2\log(x)+\frac{1}{2}\left(Da^2+2\,Bab\right)x^2-\frac{Aa^2}{x}+\left(Ca^2+2\,Aa\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/4*(2*D*a*b + B*b^2)*x^4 + 1/3*(2*C*a*b +
A*b^2)*x^3 + B*a^2*log(x) + 1/2*(D*a^2 + 2*B*a*b)*x^2 - A*a^2/x + (C*a^2 +
2*A*a*b)*x
```

mupad [B] time = 1.11, size = 92, normalized size = 1.02

$$\frac{B\,\left(4\,a^2\,\ln(x)+b^2\,x^4+4\,a\,b\,x^2\right)}{4}+\frac{{\left(b\,x^2+a\right)}^3\,D}{6\,b}+\frac{C\,x\,\left(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4\right)}{15}+\frac{A\,\left(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4\right)}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] (B*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + ((a + b*x^2)^3*D)/(6*b) + (C*x
*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (A*(b^2*x^4 - 3*a^2 + 6*a*b*x^2))/
(3*x)
```

sympy [A] time = 0.35, size = 99, normalized size = 1.10

$$-\frac{Aa^2}{x}+Ba^2\log(x)+\frac{Cb^2x^5}{5}+\frac{Db^2x^6}{6}+x^4\left(\frac{Bb^2}{4}+\frac{Dab}{2}\right)+x^3\left(\frac{Ab^2}{3}+\frac{2Cab}{3}\right)+x^2\left(Bab+\frac{Da^2}{2}\right)+x\left(2Aab+Ca^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + C*b**2*x**5/5 + D*b**2*x**6/6 + x**4*(B*b**2/4
+ D*a*b/2) + x**3*(A*b**2/3 + 2*C*a*b/3) + x**2*(B*a*b + D*a**2/2) + x*(2*A
*a*b + C*a**2)
```

3.76 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x^3}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{2}bx^2(2aC+Ab)+a\log(x)(aC+2Ab)+\frac{1}{3}bx^3(2aD+bB)+ax(aD+2bB)+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5$$

```
[Out] -1/2*a^2*A/x^2-a^2*B/x+a*(2*B*b+D*a)*x+1/2*b*(A*b+2*C*a)*x^2+1/3*b*(B*b+2*D
*a)*x^3+1/4*b^2*C*x^4+1/5*b^2*D*x^5+a*(2*A*b+C*a)*ln(x)
```

Rubi [A] time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{2}bx^2(2aC+Ab)+a\log(x)(aC+2Ab)+\frac{1}{3}bx^3(2aD+bB)+ax(aD+2bB)+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + a*(2*b*B + a*D)*x + (b*(A*b + 2*a*C)*x^2)/2
+ (b*(b*B + 2*a*D)*x^3)/3 + (b^2*C*x^4)/4 + (b^2*D*x^5)/5 + a*(2*A*b + a*C)
*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x^3}\,dx &= \int \left(a(2bB+aD)+\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+\frac{a(2Ab+aC)}{x}+b(Ab+2aC)x+b(b\right.\\ &= -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+a(2bB+aD)x+\frac{1}{2}b(Ab+2aC)x^2+\frac{1}{3}b(bB+2aD)x^3\end{aligned}$$

Mathematica [A] time = 0.04, size = 87, normalized size = 0.89

$$-\frac{a^2\left(A+2Bx-2Dx^3\right)}{2x^2}+a\log(x)(aC+2Ab)+\frac{1}{3}abx(6B+x(3C+2Dx))+\frac{1}{60}b^2x^2(30A+x(20B+3x(5C+4Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a^2*(A + 2*B*x - 2*D*x^3))/x^2 + (a*b*x*(6*B + x*(3*C + 2*D*x)))/3 +
(b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/60 + a*(2*A*b + a*C)*Log[x]
```

fricas [A] time = 0.50, size = 103, normalized size = 1.05

$$\frac{12\,Db^2x^7+15\,Cb^2x^6+20\left(2\,Dab+Bb^2\right)x^5+30\left(2\,Cab+Ab^2\right)x^4-60\,Ba^2x+60\left(Da^2+2\,Bab\right)x^3+60\left(Ca^2+2\right.}{60\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/60*(12*D*b^2*x^7 + 15*C*b^2*x^6 + 20*(2*D*a*b + B*b^2)*x^5 + 30*(2*C*a*b
+ A*b^2)*x^4 - 60*B*a^2*x + 60*(D*a^2 + 2*B*a*b)*x^3 + 60*(C*a^2 + 2*A*a*b)
*x^2*log(x) - 30*A*a^2)/x^2
```

giac [A] time = 0.34, size = 97, normalized size = 0.99

$$\frac{1}{5}Db^2x^5+\frac{1}{4}Cb^2x^4+\frac{2}{3}Dabx^3+\frac{1}{3}Bb^2x^3+Cabx^2+\frac{1}{2}Ab^2x^2+Da^2x+2Babx+\left(Ca^2+2Aab\right)\log\left(|x|\right)-\frac{2Ba^2x+A}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 2/3*D*a*b*x^3 + 1/3*B*b^2*x^3 + C*a*b*x^2 +
 1/2*A*b^2*x^2 + D*a^2*x + 2*B*a*b*x + (C*a^2 + 2*A*a*b)*log(abs(x)) - 1/2*
(2*B*a^2*x + A*a^2)/x^2
```

maple [A] time = 0.01, size = 97, normalized size = 0.99

$$\frac{Db^2x^5}{5}+\frac{Cb^2x^4}{4}+\frac{Bb^2x^3}{3}+\frac{2Dabx^3}{3}+\frac{Ab^2x^2}{2}+Cabx^2+2Aab\ln(x)+2Babx+Ca^2\ln(x)+Da^2x-\frac{Ba^2}{x}-\frac{Aa^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

```
[Out] 1/5*b^2*D*x^5+1/4*b^2*C*x^4+1/3*B*x^3*b^2+2/3*D*x^3*a*b+1/2*A*b^2*x^2+C*x^2
*a*b+2*B*x*a*b+a^2*D*x-1/2*a^2*A/x^2-a^2*B/x+2*A*ln(x)*a*b+C*ln(x)*a^2
```

maxima [A] time = 1.32, size = 96, normalized size = 0.98

$$\frac{1}{5}Db^2x^5+\frac{1}{4}Cb^2x^4+\frac{1}{3}\left(2Dab+Bb^2\right)x^3+\frac{1}{2}\left(2Cab+Ab^2\right)x^2+\left(Da^2+2Bab\right)x+\left(Ca^2+2Aab\right)\log(x)-\frac{2Ba^2x+}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 1/3*(2*D*a*b + B*b^2)*x^3 + 1/2*(2*C*a*b +
A*b^2)*x^2 + (D*a^2 + 2*B*a*b)*x + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B*a^2*
x + A*a^2)/x^2
```

mupad [B] time = 1.11, size = 103, normalized size = 1.05

$$\frac{C\left(4a^2\ln(x)+b^2x^4+4abx^2\right)}{4}+a^2xD+\frac{b^2x^5D}{5}+\frac{A\left(b^2x^4-a^2+4abx^2\ln(x)\right)}{2x^2}+\frac{B\left(-3a^2+6abx^2+b^2x^4\right.}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

```
[Out] (C*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + a^2*x*D + (b^2*x^5*D)/5 + (A*(
b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (B*(b^2*x^4 - 3*a^2 + 6*a*b*x^
2))/(3*x) + (2*a*b*x^3*D)/3
```

sympy [A] time = 0.58, size = 100, normalized size = 1.02

$$\frac{Cb^2x^4}{4}+\frac{Db^2x^5}{5}+a\left(2Ab+Ca\right)\log\left(x\right)+x^3\left(\frac{Bb^2}{3}+\frac{2Dab}{3}\right)+x^2\left(\frac{Ab^2}{2}+Cab\right)+x\left(2Bab+Da^2\right)+\frac{-Aa^2-2Ba^2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b**2*x**4/4 + D*b**2*x**5/5 + a*(2*A*b + C*a)*log(x) + x**3*(B*b**2/3 + 2
*D*a*b/3) + x**2*(A*b**2/2 + C*a*b) + x*(2*B*a*b + D*a**2) + (-A*a**2 - 2*B
*a**2*x)/(2*x**2)
```

3.77 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4} dx$

Optimal. Leaf size=98

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

```
[Out] -1/3*a^2*A/x^3-1/2*a^2*B/x^2-a*(2*A*b+C*a)/x+b*(A*b+2*C*a)*x+1/2*b*(B*b+2*D
*a)*x^2+1/3*b^2*C*x^3+1/4*b^2*D*x^4+a*(2*B*b+D*a)*ln(x)
```

Rubi [A] time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + b*(A*b + 2*a*C)*
x + (b*(b*B + 2*a*D)*x^2)/2 + (b^2*C*x^3)/3 + (b^2*D*x^4)/4 + a*(2*b*B + a*
D)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}dx = \int \left(b(Ab+2aC)+\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+\frac{a(2Ab+aC)}{x^2}+\frac{a(2bB+aD)}{x}+b\right.$$
$$= -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+b(Ab+2aC)x+\frac{1}{2}b(bB+2aD)x^2+$$

Mathematica [A] time = 0.06, size = 83, normalized size = 0.85

$$-\frac{a^2(2A+3x(B+2Cx))}{6x^3}-\frac{2aAb}{x}+a\log(x)(aD+2bB)+abx(2C+Dx)+\frac{1}{12}b^2x\left(12A+x\left(6B+4Cx+3Dx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] (-2*a*A*b)/x + a*b*x*(2*C + D*x) - (a^2*(2*A + 3*x*(B + 2*C*x)))/(6*x^3) +
(b^2*x*(12*A + x*(6*B + 4*C*x + 3*D*x^2)))/12 + a*(2*b*B + a*D)*Log[x]
```

fricas [A] time = 0.74, size = 103, normalized size = 1.05

$$\frac{3Db^2x^7+4Cb^2x^6+6\left(2Dab+Bb^2\right)x^5+12\left(2Cab+Ab^2\right)x^4+12\left(Da^2+2Bab\right)x^3\log(x)-6Ba^2x-4Aa^2}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*D*b^2*x^7 + 4*C*b^2*x^6 + 6*(2*D*a*b + B*b^2)*x^5 + 12*(2*C*a*b + A
*b^2)*x^4 + 12*(D*a^2 + 2*B*a*b)*x^3*log(x) - 6*B*a^2*x - 4*A*a^2 - 12*(C*a
^2 + 2*A*a*b)*x^2)/x^3
```

giac [A] time = 0.43, size = 97, normalized size = 0.99

$$\frac{1}{4}Db^2x^4+\frac{1}{3}Cb^2x^3+Dabx^2+\frac{1}{2}Bb^2x^2+2\,Cabx+Ab^2x+\left(Da^2+2\,Bab\right)\log\left(|x|\right)-\frac{3\,Ba^2x+2\,Aa^2+6\left(Ca^2+2\,Aab\right)\text{:}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + D*a*b*x^2 + 1/2*B*b^2*x^2 + 2*C*a*b*x + A*b
^2*x + (D*a^2 + 2*B*a*b)*log(abs(x)) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2
+ 2*A*a*b)*x^2)/x^3
```

maple [A] time = 0.01, size = 97, normalized size = 0.99

$$\frac{Db^2x^4}{4}+\frac{C\,b^2x^3}{3}+\frac{B\,b^2x^2}{2}+Dab\,x^2+A\,b^2x+2Bab\ln(x)+2Cabx+Da^2\ln(x)-\frac{2Aab}{x}-\frac{C\,a^2}{x}-\frac{B\,a^2}{2x^2}-\frac{A\,a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/4*b^2*D*x^4+1/3*b^2*C*x^3+1/2*B*b^2*x^2+D*x^2*a*b+A*x*b^2+2*a*b*C*x-1/3*a
^2*A/x^3-1/2*B*a^2/x^2-2*a/x*A*b-a^2/x*C+2*B*a*b*ln(x)+D*ln(x)*a^2
```

maxima [A] time = 1.37, size = 97, normalized size = 0.99

$$\frac{1}{4}Db^2x^4+\frac{1}{3}Cb^2x^3+\frac{1}{2}\left(2\,Dab+Bb^2\right)x^2+\left(2\,Cab+Ab^2\right)x+\left(Da^2+2\,Bab\right)\log(x)-\frac{3\,Ba^2x+2\,Aa^2+6\left(Ca^2+2\,Aa\right.}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + 1/2*(2*D*a*b + B*b^2)*x^2 + (2*C*a*b + A*b^
2)*x + (D*a^2 + 2*B*a*b)*log(x) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2 + 2*A
*a*b)*x^2)/x^3
```

mupad [B] time = 1.28, size = 106, normalized size = 1.08

$$\frac{b^2\,x^4\,D}{4}+\frac{a^2\,\ln\left(x^2\right)\,D}{2}-\frac{A\,\left(a^2+6\,a\,b\,x^2-3\,b^2\,x^4\right)}{3\,x^3}+\frac{B\,\left(b^2\,x^4-a^2+4\,a\,b\,x^2\,\ln(x)\right)}{2\,x^2}+\frac{C\,\left(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4\right)}{3\,x}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

```
[Out] (b^2*x^4*D)/4 + (a^2*log(x^2)*D)/2 - (A*(a^2 - 3*b^2*x^4 + 6*a*b*x^2))/(3*x
^3) + (B*(b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (C*(b^2*x^4 - 3*a^2
+ 6*a*b*x^2))/(3*x) + a*b*x^2*D
```

sympy [A] time = 1.46, size = 100, normalized size = 1.02

$$\frac{Cb^2x^3}{3}+\frac{Db^2x^4}{4}+a\left(2Bb+Da\right)\log\left(x\right)+x^2\left(\frac{Bb^2}{2}+Dab\right)+x\left(Ab^2+2Cab\right)+\frac{-2Aa^2-3Ba^2x+x^2\left(-12Aab-6Ca^2\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**2*x**3/3 + D*b**2*x**4/4 + a*(2*B*b + D*a)*log(x) + x**2*(B*b**2/2 + D
*a*b) + x*(A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x**2*(-12*A*a*b -
6*C*a**2))/(6*x**3)
```

3.78 $\int x^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=149

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+$$

```
[Out] 1/4*a^3*A*x^4+1/5*a^3*B*x^5+1/6*a^2*(3*A*b+C*a)*x^6+1/7*a^2*(3*B*b+D*a)*x^7
+3/8*a*b*(A*b+C*a)*x^8+1/3*a*b*(B*b+D*a)*x^9+1/10*b^2*(A*b+3*C*a)*x^10+1/11
*b^2*(B*b+3*D*a)*x^11+1/12*b^3*C*x^12+1/13*b^3*D*x^13
```

Rubi [A] time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{4}a^3Ax^4+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{5}a^3Bx^5+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B + a
*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(A*
b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b^3*
D*x^13)/13
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx = \int \left(a^3Ax^3+a^3Bx^4+a^2(3Ab+aC)x^5+a^2(3bB+aD)x^6+3ab(Ab\right.$$
$$=\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2(3Ab+aC)x^6+\frac{1}{7}a^2(3bB+aD)x^7+\frac{3}{8}ab($$

Mathematica [A] time = 0.03, size = 149, normalized size = 1.00

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B + a
*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(A*
b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b^3*
D*x^13)/13
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.46, size = 153, normalized size = 1.03

$$\frac{1}{13}Db^3x^{13}+\frac{1}{12}Cb^3x^{12}+\frac{3}{11}Dab^2x^{11}+\frac{1}{11}Bb^3x^{11}+\frac{3}{10}Cab^2x^{10}+\frac{1}{10}Ab^3x^{10}+\frac{1}{3}Da^2bx^9+\frac{1}{3}Bab^2x^9+\frac{3}{8}Ca^2bx^8+\frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 3/11*D*a*b^2*x^11 + 1/11*B*b^3*x^11 + 3
/10*C*a*b^2*x^10 + 1/10*A*b^3*x^10 + 1/3*D*a^2*b*x^9 + 1/3*B*a*b^2*x^9 + 3/
8*C*a^2*b*x^8 + 3/8*A*a*b^2*x^8 + 1/7*D*a^3*x^7 + 3/7*B*a^2*b*x^7 + 1/6*C*a
^3*x^6 + 1/2*A*a^2*b*x^6 + 1/5*B*a^3*x^5 + 1/4*A*a^3*x^4
```

maple [A] time = 0.00, size = 150, normalized size = 1.01

$$\frac{Db^3x^{13}}{13}+\frac{C\,b^3x^{12}}{12}+\frac{\left(b^3B+3a\,b^2D\right)x^{11}}{11}+\frac{\left(A\,b^3+3a\,b^2C\right)x^{10}}{10}+\frac{B\,a^3x^5}{5}+\frac{\left(3a\,b^2B+3a^2bD\right)x^9}{9}+\frac{A\,a^3x^4}{4}+\frac{\left(3a\,b^2A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/13*b^3*D*x^13+1/12*b^3*C*x^12+1/11*(B*b^3+3*D*a*b^2)*x^11+1/10*(A*b^3+3*C
*a*b^2)*x^10+1/9*(3*B*a*b^2+3*D*a^2*b)*x^9+1/8*(3*A*a*b^2+3*C*a^2*b)*x^8+1/
7*(3*B*a^2*b+D*a^3)*x^7+1/6*(3*A*a^2*b+C*a^3)*x^6+1/5*a^3*B*x^5+1/4*a^3*A*x
^4
```

maxima [A] time = 1.33, size = 145, normalized size = 0.97

$$\frac{1}{13}Db^3x^{13}+\frac{1}{12}Cb^3x^{12}+\frac{1}{11}\left(3\,Dab^2+Bb^3\right)x^{11}+\frac{1}{10}\left(3\,Cab^2+Ab^3\right)x^{10}+\frac{1}{3}\left(Da^2b+Bab^2\right)x^9+\frac{1}{5}Ba^3x^5+\frac{3}{8}\left(Ca^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11 + 1/10*(3
*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 + 1/5*B*a^3*x^5 + 3/8*
(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7*(D*a^3 + 3*B*a^2*b)*x^7 + 1/6
*(C*a^3 + 3*A*a^2*b)*x^6
```

mupad [B] time = 1.30, size = 153, normalized size = 1.03

$$\frac{A\,a^3\,x^4}{4}+\frac{B\,a^3\,x^5}{5}+\frac{A\,b^3\,x^{10}}{10}+\frac{C\,a^3\,x^6}{6}+\frac{B\,b^3\,x^{11}}{11}+\frac{C\,b^3\,x^{12}}{12}+\frac{a^3\,x^7\,D}{7}+\frac{b^3\,x^{13}\,D}{13}+\frac{a^2\,b\,x^9\,D}{3}+\frac{3\,a\,b^2\,x^{11}\,D}{11}+\frac{A\,a^2\,b\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^4)/4 + (B*a^3*x^5)/5 + (A*b^3*x^10)/10 + (C*a^3*x^6)/6 + (B*b^3*x^
11)/11 + (C*b^3*x^12)/12 + (a^3*x^7*D)/7 + (b^3*x^13*D)/13 + (a^2*b*x^9*D)/
3 + (3*a*b^2*x^11*D)/11 + (A*a^2*b*x^6)/2 + (3*A*a*b^2*x^8)/8 + (3*B*a^2*b*
x^7)/7 + (B*a*b^2*x^9)/3 + (3*C*a^2*b*x^8)/8 + (3*C*a*b^2*x^10)/10
```

sympy [A] time = 0.17, size = 163, normalized size = 1.09

$$\frac{Aa^3x^4}{4}+\frac{Ba^3x^5}{5}+\frac{Cb^3x^{12}}{12}+\frac{Db^3x^{13}}{13}+x^{11}\left(\frac{Bb^3}{11}+\frac{3Dab^2}{11}\right)+x^{10}\left(\frac{Ab^3}{10}+\frac{3Cab^2}{10}\right)+x^9\left(\frac{Bab^2}{3}+\frac{Da^2b}{3}\right)+x^8\left(\frac{3Aab^2}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**4/4 + B*a**3*x**5/5 + C*b**3*x**12/12 + D*b**3*x**13/13 + x**11*(
B*b**3/11 + 3*D*a*b**2/11) + x**10*(A*b**3/10 + 3*C*a*b**2/10) + x**9*(B*a*
b**2/3 + D*a**2*b/3) + x**8*(3*A*a*b**2/8 + 3*C*a**2*b/8) + x**7*(3*B*a**2*
b/7 + D*a**3/7) + x**6*(A*a**2*b/2 + C*a**3/6)
```

3.79 $\int x^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=149

$$\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+bB)$$

```
[Out] 1/3*a^3*A*x^3+1/4*a^3*B*x^4+1/5*a^2*(3*A*b+C*a)*x^5+1/6*a^2*(3*B*b+D*a)*x^6
+3/7*a*b*(A*b+C*a)*x^7+3/8*a*b*(B*b+D*a)*x^8+1/9*b^2*(A*b+3*C*a)*x^9+1/10*b
^2*(B*b+3*D*a)*x^10+1/11*b^3*C*x^11+1/12*b^3*D*x^12
```

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1802}

$$\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{3}a^3Ax^3+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{4}a^3Bx^4+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+bB)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^3)/3 + (a^3*B*x^4)/4 + (a^2*(3*A*b + a*C)*x^5)/5 + (a^2*(3*b*B + a
*D)*x^6)/6 + (3*a*b*(A*b + a*C)*x^7)/7 + (3*a*b*(b*B + a*D)*x^8)/8 + (b^2*(
A*b + 3*a*C)*x^9)/9 + (b^2*(b*B + 3*a*D)*x^10)/10 + (b^3*C*x^11)/11 + (b^3*
D*x^12)/12
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)dx = \int \left(a^3Ax^2+a^3Bx^3+a^2(3Ab+aC)x^4+a^2(3bB+aD)x^5+3ab(\right.$$
$$= \frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2(3Ab+aC)x^5+\frac{1}{6}a^2(3bB+aD)x^6+\frac{3}{7}$$

Mathematica [A] time = 0.06, size = 125, normalized size = 0.84

$$\frac{462a^3x^3(20A+x(15B+2x(6C+5Dx)))+99a^2bx^5(168A+5x(28B+3x(8C+7Dx)))+33ab^2x^7(360A+7x}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (14*b^3*x^9*(220*A + 3*x*(66*B + 60*C*x + 55*D*x^2)) + 462*a^3*x^3*(20*A +
x*(15*B + 2*x*(6*C + 5*D*x))) + 99*a^2*b*x^5*(168*A + 5*x*(28*B + 3*x*(8*C
+ 7*D*x))) + 33*a*b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/27720
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.46, size = 153, normalized size = 1.03

$$\frac{1}{12}Db^3x^{12}+\frac{1}{11}Cb^3x^{11}+\frac{3}{10}Dab^2x^{10}+\frac{1}{10}Bb^3x^{10}+\frac{1}{3}Cab^2x^9+\frac{1}{9}Ab^3x^9+\frac{3}{8}Da^2bx^8+\frac{3}{8}Bab^2x^8+\frac{3}{7}Ca^2bx^7+\frac{3}{7}Aab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 3/10*D*a*b^2*x^10 + 1/10*B*b^3*x^10 + 1
/3*C*a*b^2*x^9 + 1/9*A*b^3*x^9 + 3/8*D*a^2*b*x^8 + 3/8*B*a*b^2*x^8 + 3/7*C*
a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 1/6*D*a^3*x^6 + 1/2*B*a^2*b*x^6 + 1/5*C*a^3*x
^5 + 3/5*A*a^2*b*x^5 + 1/4*B*a^3*x^4 + 1/3*A*a^3*x^3
```

maple [A] time = 0.00, size = 150, normalized size = 1.01

$$\frac{Db^3x^{12}}{12}+\frac{Cb^3x^{11}}{11}+\frac{\left(b^3B+3ab^2D\right)x^{10}}{10}+\frac{\left(Ab^3+3ab^2C\right)x^9}{9}+\frac{Ba^3x^4}{4}+\frac{\left(3ab^2B+3a^2bD\right)x^8}{8}+\frac{Aa^3x^3}{3}+\frac{\left(3ab^2A+3\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/12*b^3*D*x^12+1/11*b^3*C*x^11+1/10*(B*b^3+3*D*a*b^2)*x^10+1/9*(A*b^3+3*C*
a*b^2)*x^9+1/8*(3*B*a*b^2+3*D*a^2*b)*x^8+1/7*(3*A*a*b^2+3*C*a^2*b)*x^7+1/6*
(3*B*a^2*b+D*a^3)*x^6+1/5*(3*A*a^2*b+C*a^3)*x^5+1/4*a^3*B*x^4+1/3*a^3*A*x^3
```

maxima [A] time = 1.36, size = 145, normalized size = 0.97

$$\frac{1}{12}Db^3x^{12}+\frac{1}{11}Cb^3x^{11}+\frac{1}{10}\left(3Dab^2+Bb^3\right)x^{10}+\frac{1}{9}\left(3Cab^2+Ab^3\right)x^9+\frac{3}{8}\left(Da^2b+Bab^2\right)x^8+\frac{1}{4}Ba^3x^4+\frac{3}{7}\left(Ca^2b+A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10 + 1/9*(3*
C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/4*B*a^3*x^4 + 3/7*(C
*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(D*a^3 + 3*B*a^2*b)*x^6 + 1/5*(
C*a^3 + 3*A*a^2*b)*x^5
```

mupad [B] time = 1.28, size = 153, normalized size = 1.03

$$\frac{Aa^3x^3}{3}+\frac{Ba^3x^4}{4}+\frac{Ab^3x^9}{9}+\frac{Ca^3x^5}{5}+\frac{Bb^3x^{10}}{10}+\frac{Cb^3x^{11}}{11}+\frac{a^3x^6D}{6}+\frac{b^3x^{12}D}{12}+\frac{3a^2bx^8D}{8}+\frac{3ab^2x^{10}D}{10}+\frac{3Aa^2bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^3)/3 + (B*a^3*x^4)/4 + (A*b^3*x^9)/9 + (C*a^3*x^5)/5 + (B*b^3*x^10
)/10 + (C*b^3*x^11)/11 + (a^3*x^6*D)/6 + (b^3*x^12*D)/12 + (3*a^2*b*x^8*D)/
8 + (3*a*b^2*x^10*D)/10 + (3*A*a^2*b*x^5)/5 + (3*A*a*b^2*x^7)/7 + (B*a^2*b*
x^6)/2 + (3*B*a*b^2*x^8)/8 + (3*C*a^2*b*x^7)/7 + (C*a*b^2*x^9)/3
```

sympy [A] time = 0.14, size = 165, normalized size = 1.11

$$\frac{Aa^3x^3}{3}+\frac{Ba^3x^4}{4}+\frac{Cb^3x^{11}}{11}+\frac{Db^3x^{12}}{12}+x^{10}\left(\frac{Bb^3}{10}+\frac{3Dab^2}{10}\right)+x^9\left(\frac{Ab^3}{9}+\frac{Cab^2}{3}\right)+x^8\left(\frac{3Bab^2}{8}+\frac{3Da^2b}{8}\right)+x^7\left(\frac{3Aab^2}{7}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**3/3 + B*a**3*x**4/4 + C*b**3*x**11/11 + D*b**3*x**12/12 + x**10*(
B*b**3/10 + 3*D*a*b**2/10) + x**9*(A*b**3/9 + C*a*b**2/3) + x**8*(3*B*a*b**
2/8 + 3*D*a**2*b/8) + x**7*(3*A*a*b**2/7 + 3*C*a**2*b/7) + x**6*(B*a**2*b/2
 + D*a**3/6) + x**5*(3*A*a**2*b/5 + C*a**3/5)
```

3.80 $\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)dx$

Optimal. Leaf size=138

$$\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2x^5(aD+3bB)+\frac{1}{2}a^2bCx^6+\frac{A\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^2x^9(3aD+bB)+\frac{3}{8}ab^2Cx^8+\frac{3}{7}abx^7(aD+bB)+\frac{1}{10}b$$

```
[Out] 1/3*a^3*B*x^3+1/4*a^3*C*x^4+1/5*a^2*(3*B*b+D*a)*x^5+1/2*a^2*b*C*x^6+3/7*a*b
*(B*b+D*a)*x^7+3/8*a*b^2*C*x^8+1/9*b^2*(B*b+3*D*a)*x^9+1/10*b^3*C*x^10+1/11
*b^3*D*x^11+1/8*A*(b*x^2+a)^4/b
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1582, 1810}

$$\frac{1}{5}a^2x^5(aD+3bB)+\frac{1}{2}a^2bCx^6+\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{A\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^2x^9(3aD+bB)+\frac{3}{8}ab^2Cx^8+\frac{3}{7}abx^7(aD+bB)+\frac{1}{10}b$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*B*x^3)/3 + (a^3*C*x^4)/4 + (a^2*(3*b*B + a*D)*x^5)/5 + (a^2*b*C*x^6)/2
 + (3*a*b*(b*B + a*D)*x^7)/7 + (3*a*b^2*C*x^8)/8 + (b^2*(b*B + 3*a*D)*x^9)/
9 + (b^3*C*x^10)/10 + (b^3*D*x^11)/11 + (A*(a + b*x^2)^4)/(8*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)dx &= \frac{A\left(a+bx^2\right)^4}{8b}+\int\left(a+bx^2\right)^3\left(-Ax+x\left(A+Bx+Cx^2+Dx^3\right)\right)dx\\ &= \frac{A\left(a+bx^2\right)^4}{8b}+\int\left(a^3Bx^2+a^3Cx^3+a^2(3bB+aD)x^4+3a^2bCx^5+\right.\\ &= \frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2(3bB+aD)x^5+\frac{1}{2}a^2bCx^6+\frac{3}{7}ab(bB+aD)\end{aligned}$$

Mathematica [A] time = 0.06, size = 124, normalized size = 0.90

$$\frac{462a^3x^2(30A+x(20B+3x(5C+4Dx)))+198a^2bx^4(105A+2x(42B+5x(7C+6Dx)))+165ab^2x^6(84A+x(72}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (7*b^3*x^8*(495*A + 4*x*(110*B + 99*C*x + 90*D*x^2)) + 462*a^3*x^2*(30*A +
x*(20*B + 3*x*(5*C + 4*D*x))) + 198*a^2*b*x^4*(105*A + 2*x*(42*B + 5*x*(7*C
 + 6*D*x))) + 165*a*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x))))/27720
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.36, size = 153, normalized size = 1.11

$$\frac{1}{11}\,Db^3x^{11}+\frac{1}{10}\,Cb^3x^{10}+\frac{1}{3}\,Dab^2x^9+\frac{1}{9}\,Bb^3x^9+\frac{3}{8}\,Cab^2x^8+\frac{1}{8}\,Ab^3x^8+\frac{3}{7}\,Da^2bx^7+\frac{3}{7}\,Bab^2x^7+\frac{1}{2}\,Ca^2bx^6+\frac{1}{2}\,Aab^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/3*D*a*b^2*x^9 + 1/9*B*b^3*x^9 + 3/8*C
*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3/7*D*a^2*b*x^7 + 3/7*B*a*b^2*x^7 + 1/2*C*a^2*
b*x^6 + 1/2*A*a*b^2*x^6 + 1/5*D*a^3*x^5 + 3/5*B*a^2*b*x^5 + 1/4*C*a^3*x^4 +
 3/4*A*a^2*b*x^4 + 1/3*B*a^3*x^3 + 1/2*A*a^3*x^2
```

maple [A] time = 0.00, size = 150, normalized size = 1.09

$$\frac{Db^3x^{11}}{11}+\frac{C\,b^3x^{10}}{10}+\frac{\left(b^3B+3a\,b^2D\right)x^9}{9}+\frac{\left(A\,b^3+3a\,b^2C\right)x^8}{8}+\frac{B\,a^3x^3}{3}+\frac{\left(3a\,b^2B+3a^2bD\right)x^7}{7}+\frac{A\,a^3x^2}{2}+\frac{\left(3a\,b^2A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/11*b^3*D*x^11+1/10*b^3*C*x^10+1/9*(B*b^3+3*D*a*b^2)*x^9+1/8*(A*b^3+3*C*a*
b^2)*x^8+1/7*(3*B*a*b^2+3*D*a^2*b)*x^7+1/6*(3*A*a*b^2+3*C*a^2*b)*x^6+1/5*(3
*B*a^2*b+D*a^3)*x^5+1/4*(3*A*a^2*b+C*a^3)*x^4+1/3*a^3*B*x^3+1/2*a^3*A*x^2
```

maxima [A] time = 1.34, size = 145, normalized size = 1.05

$$\frac{1}{11}\,Db^3x^{11}+\frac{1}{10}\,Cb^3x^{10}+\frac{1}{9}\left(3\,Dab^2+Bb^3\right)x^9+\frac{1}{8}\left(3\,Cab^2+Ab^3\right)x^8+\frac{3}{7}\left(Da^2b+Bab^2\right)x^7+\frac{1}{3}\,Ba^3x^3+\frac{1}{2}\left(Ca^2b+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 + 1/8*(3*C*
a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*B*a^3*x^3 + 1/2*(C*a
^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*a^3 + 3*B*a^2*b)*x^5 + 1/4*(C*
a^3 + 3*A*a^2*b)*x^4
```

mupad [B] time = 1.28, size = 153, normalized size = 1.11

$$\frac{A\,a^3\,x^2}{2}+\frac{B\,a^3\,x^3}{3}+\frac{A\,b^3\,x^8}{8}+\frac{C\,a^3\,x^4}{4}+\frac{B\,b^3\,x^9}{9}+\frac{C\,b^3\,x^{10}}{10}+\frac{a^3\,x^5\,D}{5}+\frac{b^3\,x^{11}\,D}{11}+\frac{3\,a^2\,b\,x^7\,D}{7}+\frac{a\,b^2\,x^9\,D}{3}+\frac{3\,A\,a^2\,b\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^2)/2 + (B*a^3*x^3)/3 + (A*b^3*x^8)/8 + (C*a^3*x^4)/4 + (B*b^3*x^9)
/9 + (C*b^3*x^10)/10 + (a^3*x^5*D)/5 + (b^3*x^11*D)/11 + (3*a^2*b*x^7*D)/7
+ (a*b^2*x^9*D)/3 + (3*A*a^2*b*x^4)/4 + (A*a*b^2*x^6)/2 + (3*B*a^2*b*x^5)/5
 + (3*B*a*b^2*x^7)/7 + (C*a^2*b*x^6)/2 + (3*C*a*b^2*x^8)/8
```

sympy [A] time = 0.14, size = 163, normalized size = 1.18

$$\frac{Aa^3x^2}{2}+\frac{Ba^3x^3}{3}+\frac{Cb^3x^{10}}{10}+\frac{Db^3x^{11}}{11}+x^9\left(\frac{Bb^3}{9}+\frac{Dab^2}{3}\right)+x^8\left(\frac{Ab^3}{8}+\frac{3Cab^2}{8}\right)+x^7\left(\frac{3Bab^2}{7}+\frac{3Da^2b}{7}\right)+x^6\left(\frac{Aab^2}{2}+\frac{C\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**2/2 + B*a**3*x**3/3 + C*b**3*x**10/10 + D*b**3*x**11/11 + x**9*(B
*b**3/9 + D*a*b**2/3) + x**8*(A*b**3/8 + 3*C*a*b**2/8) + x**7*(3*B*a*b**2/7
 + 3*D*a**2*b/7) + x**6*(A*a*b**2/2 + C*a**2*b/2) + x**5*(3*B*a**2*b/5 + D*
a**3/5) + x**4*(3*A*a**2*b/4 + C*a**3/4)
```

3.81 $\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal. Leaf size=133

$$a^3Ax+\frac{1}{4}a^3Dx^4+\frac{1}{3}a^2x^3(aC+3Ab)+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b$$

```
[Out] a^3*A*x+1/3*a^2*(3*A*b+C*a)*x^3+1/4*a^3*D*x^4+3/5*a*b*(A*b+C*a)*x^5+1/2*a^2
*b*D*x^6+1/7*b^2*(A*b+3*C*a)*x^7+3/8*a*b^2*D*x^8+1/9*b^3*C*x^9+1/10*b^3*D*x
^10+1/8*B*(b*x^2+a)^4/b
```

Rubi [A] time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1582, 1810}

$$\frac{1}{3}a^2x^3(aC+3Ab)+a^3Ax+\frac{1}{2}a^2bDx^6+\frac{1}{4}a^3Dx^4+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b + a*C)*x^
5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3*a*b^2*D*x^8)/8 + (b
^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2)^4)/(8*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx &= \frac{B\left(a+bx^2\right)^4}{8b}+\int \left(a+bx^2\right)^3 \left(A+Cx^2+Dx^3\right) \, dx\\
&= \frac{B\left(a+bx^2\right)^4}{8b}+\int \left(a^3A+a^2(3Ab+aC)x^2+a^3Dx^3+3ab(Ab+aC\right.\\
&= a^3Ax+\frac{1}{3}a^2(3Ab+aC)x^3+\frac{1}{4}a^3Dx^4+\frac{3}{5}ab(Ab+aC)x^5+\frac{1}{2}a^2bD
\end{aligned}$$

Mathematica [A] time = 0.05, size = 121, normalized size = 0.91

$$\frac{210a^3x(12A+x(6B+x(4C+3Dx)))+126a^2bx^3(20A+x(15B+2x(6C+5Dx)))+9ab^2x^5(168A+5x(28B+}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (210*a^3*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 126*a^2*b*x^3*(20*A + x*(15
*B + 2*x*(6*C + 5*D*x))) + 9*a*b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*
x))) + b^3*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/2520
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.40, size = 149, normalized size = 1.12

$$\frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{3}{8}\,Dab^2x^8+\frac{1}{8}\,Bb^3x^8+\frac{3}{7}\,Cab^2x^7+\frac{1}{7}\,Ab^3x^7+\frac{1}{2}\,Da^2bx^6+\frac{1}{2}\,Bab^2x^6+\frac{3}{5}\,Ca^2bx^5+\frac{3}{5}\,Aab^2x^5+\frac{1}{4}\,D$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 3/8*D*a*b^2*x^8 + 1/8*B*b^3*x^8 + 3/7*C*a
*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*D*a^2*b*x^6 + 1/2*B*a*b^2*x^6 + 3/5*C*a^2*b*
x^5 + 3/5*A*a*b^2*x^5 + 1/4*D*a^3*x^4 + 3/4*B*a^2*b*x^4 + 1/3*C*a^3*x^3 + A
*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

maple [A] time = 0.00, size = 147, normalized size = 1.11

$$\frac{Db^3x^{10}}{10}+\frac{C\,b^3x^9}{9}+\frac{\left(b^3B+3a\,b^2D\right)x^8}{8}+\frac{\left(A\,b^3+3a\,b^2C\right)x^7}{7}+\frac{B\,a^3x^2}{2}+\frac{\left(3a\,b^2B+3a^2bD\right)x^6}{6}+A\,a^3x+\frac{\left(3a\,b^2A+3a^2b\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/10*b^3*D*x^10+1/9*b^3*C*x^9+1/8*(B*b^3+3*D*a*b^2)*x^8+1/7*(A*b^3+3*C*a*b^
2)*x^7+1/6*(3*B*a*b^2+3*D*a^2*b)*x^6+1/5*(3*A*a*b^2+3*C*a^2*b)*x^5+1/4*(3*B
*a^2*b+D*a^3)*x^4+1/3*(3*A*a^2*b+C*a^3)*x^3+1/2*a^3*B*x^2+a^3*A*x
```

maxima [A] time = 1.34, size = 142, normalized size = 1.07

$$\frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\left(3\,Dab^2+Bb^3\right)x^8+\frac{1}{7}\left(3\,Cab^2+Ab^3\right)x^7+\frac{1}{2}\left(Da^2b+Bab^2\right)x^6+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\left(Ca^2b+Aab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a*
b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a^2
*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 + 3*
A*a^2*b)*x^3
```

mupad [B] time = 1.26, size = 149, normalized size = 1.12

$$\frac{B\,a^3\,x^2}{2}+\frac{A\,b^3\,x^7}{7}+\frac{C\,a^3\,x^3}{3}+\frac{B\,b^3\,x^8}{8}+\frac{C\,b^3\,x^9}{9}+\frac{a^3\,x^4\,D}{4}+\frac{b^3\,x^{10}\,D}{10}+A\,a^3\,x+\frac{a^2\,b\,x^6\,D}{2}+\frac{3\,a\,b^2\,x^8\,D}{8}+A\,a^2\,b\,x^3+\frac{3\,A\,a}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (B*a^3*x^2)/2 + (A*b^3*x^7)/7 + (C*a^3*x^3)/3 + (B*b^3*x^8)/8 + (C*b^3*x^9)
/9 + (a^3*x^4*D)/4 + (b^3*x^10*D)/10 + A*a^3*x + (a^2*b*x^6*D)/2 + (3*a*b^2
*x^8*D)/8 + A*a^2*b*x^3 + (3*A*a*b^2*x^5)/5 + (3*B*a^2*b*x^4)/4 + (B*a*b^2*
x^6)/2 + (3*C*a^2*b*x^5)/5 + (3*C*a*b^2*x^7)/7
```

sympy [A] time = 0.13, size = 158, normalized size = 1.19

$$Aa^3x+\frac{Ba^3x^2}{2}+\frac{Cb^3x^9}{9}+\frac{Db^3x^{10}}{10}+x^8\left(\frac{Bb^3}{8}+\frac{3Dab^2}{8}\right)+x^7\left(\frac{Ab^3}{7}+\frac{3Cab^2}{7}\right)+x^6\left(\frac{Bab^2}{2}+\frac{Da^2b}{2}\right)+x^5\left(\frac{3Aab^2}{5}+\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x + B*a**3*x**2/2 + C*b**3*x**9/9 + D*b**3*x**10/10 + x**8*(B*b**3/8
 + 3*D*a*b**2/8) + x**7*(A*b**3/7 + 3*C*a*b**2/7) + x**6*(B*a*b**2/2 + D*a*
*2*b/2) + x**5*(3*A*a*b**2/5 + 3*C*a**2*b/5) + x**4*(3*B*a**2*b/4 + D*a**3/
4) + x**3*(A*a**2*b + C*a**3/3)
```

3.82 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x}\,dx$

Optimal. Leaf size=129

$$a^3A\log(x)+a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2x^3(aD+3bB)+\frac{3}{4}aAb^2x^4+\frac{1}{7}b^2x^7(3aD+bB)+\frac{3}{5}abx^5(aD+bB)+\frac{C(a+bx^2)^4}{8b}+\frac{1}{6}A$$

```
[Out] a^3*B*x+3/2*a^2*A*b*x^2+1/3*a^2*(3*B*b+D*a)*x^3+3/4*a*A*b^2*x^4+3/5*a*b*(B*
b+D*a)*x^5+1/6*A*b^3*x^6+1/7*b^2*(B*b+3*D*a)*x^7+1/9*b^3*D*x^9+1/8*C*(b*x^2
+a)^4/b+a^3*A*ln(x)
```

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1583, 1802}

$$\frac{3}{2}a^2Abx^2+a^3A\log(x)+\frac{1}{3}a^2x^3(aD+3bB)+a^3Bx+\frac{3}{4}aAb^2x^4+\frac{1}{7}b^2x^7(3aD+bB)+\frac{3}{5}abx^5(aD+bB)+\frac{C(a+bx^2)^4}{8b}+\frac{1}{6}A$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a^3*B*x + (3*a^2*A*b*x^2)/2 + (a^2*(3*b*B + a*D)*x^3)/3 + (3*a*A*b^2*x^4)/4
 + (3*a*b*(b*B + a*D)*x^5)/5 + (A*b^3*x^6)/6 + (b^2*(b*B + 3*a*D)*x^7)/7 +
(b^3*D*x^9)/9 + (C*(a + b*x^2)^4)/(8*b) + a^3*A*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x}\,dx &= \frac{C(a+bx^2)^4}{8b}+\int \frac{(a+bx^2)^3(A+Bx+Dx^3)}{x}\,dx\\ &= \frac{C(a+bx^2)^4}{8b}+\int \left(a^3B+\frac{a^3A}{x}+3a^2Abx+a^2(3bB+aD)x^2+3aAb^2\right.\\ &= a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2(3bB+aD)x^3+\frac{3}{4}aAb^2x^4+\frac{3}{5}ab(bB+aD)x^5\end{aligned}$$

Mathematica [A] time = 0.07, size = 121, normalized size = 0.94

$$a^3A\log(x)+\frac{x\left(420a^3(6B+x(3C+2Dx))+126a^2bx(30A+x(20B+3x(5C+4Dx)))+18ab^2x^3(105A+2x(42B\right.}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(420*a^3*(6*B + x*(3*C + 2*D*x)) + 126*a^2*b*x*(30*A + x*(20*B + 3*x*(5*
C + 4*D*x))) + 18*a*b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^
3*x^5*(84*A + x*(72*B + 7*x*(9*C + 8*D*x)))))/2520 + a^3*A*Log[x]
```

fricas [A] time = 0.71, size = 140, normalized size = 1.09

$$\frac{1}{9}Db^3x^9+\frac{1}{8}Cb^3x^8+\frac{1}{7}\left(3Dab^2+Bb^3\right)x^7+\frac{1}{6}\left(3Cab^2+Ab^3\right)x^6+\frac{3}{5}\left(Da^2b+Bab^2\right)x^5+Ba^3x+\frac{3}{4}\left(Ca^2b+Aab^2\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b^
2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A*a
*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*A*a
^2*b)*x^2
```

giac [A] time = 0.33, size = 148, normalized size = 1.15

$$\frac{1}{9}Db^3x^9+\frac{1}{8}Cb^3x^8+\frac{3}{7}Dab^2x^7+\frac{1}{7}Bb^3x^7+\frac{1}{2}Cab^2x^6+\frac{1}{6}Ab^3x^6+\frac{3}{5}Da^2bx^5+\frac{3}{5}Bab^2x^5+\frac{3}{4}Ca^2bx^4+\frac{3}{4}Aab^2x^4+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 3/7*D*a*b^2*x^7 + 1/7*B*b^3*x^7 + 1/2*C*a*b
^2*x^6 + 1/6*A*b^3*x^6 + 3/5*D*a^2*b*x^5 + 3/5*B*a*b^2*x^5 + 3/4*C*a^2*b*x^
4 + 3/4*A*a*b^2*x^4 + 1/3*D*a^3*x^3 + B*a^2*b*x^3 + 1/2*C*a^3*x^2 + 3/2*A*a
^2*b*x^2 + B*a^3*x + A*a^3*log(abs(x))
```

maple [A] time = 0.00, size = 148, normalized size = 1.15

$$\frac{Db^3x^9}{9}+\frac{Cb^3x^8}{8}+\frac{Bb^3x^7}{7}+\frac{3Dab^2x^7}{7}+\frac{Ab^3x^6}{6}+\frac{Cab^2x^6}{2}+\frac{3Bab^2x^5}{5}+\frac{3Da^2bx^5}{5}+\frac{3Aab^2x^4}{4}+\frac{3Ca^2bx^4}{4}+Ba^2bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/9*b^3*D*x^9+1/8*C*b^3*x^8+1/7*B*x^7*b^3+3/7*D*x^7*a*b^2+1/6*A*b^3*x^6+1/2
*C*x^6*a*b^2+3/5*B*x^5*a*b^2+3/5*D*x^5*a^2*b+3/4*a*A*b^2*x^4+3/4*C*x^4*a^2*
b+B*x^3*a^2*b+1/3*D*x^3*a^3+3/2*a^2*A*b*x^2+1/2*C*x^2*a^3+a^3*B*x+a^3*A*ln(
x)
```

maxima [A] time = 1.32, size = 140, normalized size = 1.09

$$\frac{1}{9}Db^3x^9+\frac{1}{8}Cb^3x^8+\frac{1}{7}\left(3Dab^2+Bb^3\right)x^7+\frac{1}{6}\left(3Cab^2+Ab^3\right)x^6+\frac{3}{5}\left(Da^2b+Bab^2\right)x^5+Ba^3x+\frac{3}{4}\left(Ca^2b+Aab^2\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b^
2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A*a
*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*A*a
^2*b)*x^2
```

mupad [B] time = 1.26, size = 147, normalized size = 1.14

$$\frac{A\,b^3\,x^6}{6}+\frac{C\,a^3\,x^2}{2}+\frac{B\,b^3\,x^7}{7}+\frac{C\,b^3\,x^8}{8}+A\,a^3\,\ln(x)+\frac{a^3\,x^3\,D}{3}+\frac{b^3\,x^9\,D}{9}+B\,a^3\,x+\frac{3\,a^2\,b\,x^5\,D}{5}+\frac{3\,a\,b^2\,x^7\,D}{7}+\frac{3\,A\,a^2\,b\,x^2}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (A*b^3*x^6)/6 + (C*a^3*x^2)/2 + (B*b^3*x^7)/7 + (C*b^3*x^8)/8 + A*a^3*log(x
) + (a^3*x^3*D)/3 + (b^3*x^9*D)/9 + B*a^3*x + (3*a^2*b*x^5*D)/5 + (3*a*b^2*
x^7*D)/7 + (3*A*a^2*b*x^2)/2 + (3*A*a*b^2*x^4)/4 + B*a^2*b*x^3 + (3*B*a*b^2
*x^5)/5 + (3*C*a^2*b*x^4)/4 + (C*a*b^2*x^6)/2
```

sympy [A] time = 0.40, size = 158, normalized size = 1.22

$$Aa^3\log{(x)}+Ba^3x+\frac{Cb^3x^8}{8}+\frac{Db^3x^9}{9}+x^7\left(\frac{Bb^3}{7}+\frac{3Dab^2}{7}\right)+x^6\left(\frac{Ab^3}{6}+\frac{Cab^2}{2}\right)+x^5\left(\frac{3Bab^2}{5}+\frac{3Da^2b}{5}\right)+x^4\left(\frac{3Aab^2}{4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**3*log(x) + B*a**3*x + C*b**3*x**8/8 + D*b**3*x**9/9 + x**7*(B*b**3/7 +
 3*D*a*b**2/7) + x**6*(A*b**3/6 + C*a*b**2/2) + x**5*(3*B*a*b**2/5 + 3*D*a*
*2*b/5) + x**4*(3*A*a*b**2/4 + 3*C*a**2*b/4) + x**3*(B*a**2*b + D*a**3/3) +
 x**2*(3*A*a**2*b/2 + C*a**3/2)
```

3.83 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{a^3A}{x}+a^3B\log(x)+a^2x(aC+3Ab)+\frac{3}{2}a^2bBx^2+\frac{1}{5}b^2x^5(3aC+Ab)+abx^3(aC+Ab)+\frac{3}{4}ab^2Bx^4+\frac{D(a+bx^2)^4}{8b}+\frac{1}{6}b^3$$

```
[Out] -a^3*A/x+a^2*(3*A*b+C*a)*x+3/2*a^2*b*B*x^2+a*b*(A*b+C*a)*x^3+3/4*a*b^2*B*x^
4+1/5*b^2*(A*b+3*C*a)*x^5+1/6*b^3*B*x^6+1/7*b^3*C*x^7+1/8*D*(b*x^2+a)^4/b+a
^3*B*ln(x)
```

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1583, 1628}

$$a^2x(aC+3Ab)-\frac{a^3A}{x}+\frac{3}{2}a^2bBx^2+a^3B\log(x)+\frac{1}{5}b^2x^5(3aC+Ab)+abx^3(aC+Ab)+\frac{3}{4}ab^2Bx^4+\frac{D(a+bx^2)^4}{8b}+\frac{1}{6}b^3B$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*C)*x + (3*a^2*b*B*x^2)/2 + a*b*(A*b + a*C)*x^
3 + (3*a*b^2*B*x^4)/4 + (b^2*(A*b + 3*a*C)*x^5)/5 + (b^3*B*x^6)/6 + (b^3*C*
x^7)/7 + (D*(a + b*x^2)^4)/(8*b) + a^3*B*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx &= \frac{D(a+bx^2)^4}{8b}+\int \frac{(a+bx^2)^3(A+Bx+Cx^2)}{x^2}\,dx\\ &= \frac{D(a+bx^2)^4}{8b}+\int \left(a^2(3Ab+aC)+\frac{a^3A}{x^2}+\frac{a^3B}{x}+3a^2bBx+3ab(A\\ &= -\frac{a^3A}{x}+a^2(3Ab+aC)x+\frac{3}{2}a^2bBx^2+ab(Ab+aC)x^3+\frac{3}{4}ab^2Bx^4+\end{aligned}$$

Mathematica [A] time = 0.08, size = 123, normalized size = 0.99

$$a^3\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right)+a^3B\log(x)+\frac{1}{4}a^2bx(12A+x(6B+x(4C+3Dx)))+\frac{1}{20}ab^2x^3(20A+x(15B+2x(6C+5Dx)))+$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^3*(-(A/x) + C*x + (D*x^2)/2) + (a^2*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x))
))/4 + (a*b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/20 + (b^3*x^5*(168
*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/840 + a^3*B*Log[x]
```

fricas [A] time = 0.64, size = 147, normalized size = 1.19

$$\frac{105\,Db^3x^9+120\,Cb^3x^8+140\left(3\,Dab^2+Bb^3\right)x^7+168\left(3\,Cab^2+Ab^3\right)x^6+630\left(Da^2b+Bab^2\right)x^5+840\,Ba^3x\log}{840\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*D*b^3*x^9 + 120*C*b^3*x^8 + 140*(3*D*a*b^2 + B*b^3)*x^7 + 168*(3
*C*a*b^2 + A*b^3)*x^6 + 630*(D*a^2*b + B*a*b^2)*x^5 + 840*B*a^3*x*log(x) +
840*(C*a^2*b + A*a*b^2)*x^4 - 840*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 + 840
*(C*a^3 + 3*A*a^2*b)*x^2)/x
```

giac [A] time = 0.41, size = 145, normalized size = 1.17

$$\frac{1}{8}\,Db^3x^8+\frac{1}{7}\,Cb^3x^7+\frac{1}{2}\,Dab^2x^6+\frac{1}{6}\,Bb^3x^6+\frac{3}{5}\,Cab^2x^5+\frac{1}{5}\,Ab^3x^5+\frac{3}{4}\,Da^2bx^4+\frac{3}{4}\,Bab^2x^4+Ca^2bx^3+Aab^2x^3+\frac{1}{2}\,Da^3x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/2*D*a*b^2*x^6 + 1/6*B*b^3*x^6 + 3/5*C*a*b
^2*x^5 + 1/5*A*b^3*x^5 + 3/4*D*a^2*b*x^4 + 3/4*B*a*b^2*x^4 + C*a^2*b*x^3 +
A*a*b^2*x^3 + 1/2*D*a^3*x^2 + 3/2*B*a^2*b*x^2 + C*a^3*x + 3*A*a^2*b*x + B*a
^3*log(abs(x)) - A*a^3/x
```

maple [A] time = 0.01, size = 145, normalized size = 1.17

$$\frac{Db^3x^8}{8}+\frac{C\,b^3x^7}{7}+\frac{B\,b^3x^6}{6}+\frac{Da\,b^2x^6}{2}+\frac{A\,b^3x^5}{5}+\frac{3Ca\,b^2x^5}{5}+\frac{3Ba\,b^2x^4}{4}+\frac{3Da^2b\,x^4}{4}+Aa\,b^2x^3+C\,a^2b\,x^3+\frac{3B\,a^2b\,x^2}{2}+\frac{Da^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/8*D*b^3*x^8+1/7*b^3*C*x^7+1/6*B*b^3*x^6+1/2*D*x^6*a*b^2+1/5*A*x^5*b^3+3/5
*C*x^5*a*b^2+3/4*B*a*b^2*x^4+3/4*D*x^4*a^2*b+A*x^3*a*b^2+C*x^3*a^2*b+3/2*B*
a^2*b*x^2+1/2*D*x^2*a^3+3*A*a^2*b*x+a^3*C*x-a^3*A/x+a^3*B*ln(x)
```

maxima [A] time = 1.35, size = 139, normalized size = 1.12

$$\frac{1}{8}\,Db^3x^8+\frac{1}{7}\,Cb^3x^7+\frac{1}{6}\left(3\,Dab^2+Bb^3\right)x^6+\frac{1}{5}\left(3\,Cab^2+Ab^3\right)x^5+\frac{3}{4}\left(Da^2b+Bab^2\right)x^4+Ba^3\log(x)+\left(Ca^2b+Aab^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/6*(3*D*a*b^2 + B*b^3)*x^6 + 1/5*(3*C*a*b^
2 + A*b^3)*x^5 + 3/4*(D*a^2*b + B*a*b^2)*x^4 + B*a^3*log(x) + (C*a^2*b + A*
a*b^2)*x^3 - A*a^3/x + 1/2*(D*a^3 + 3*B*a^2*b)*x^2 + (C*a^3 + 3*A*a^2*b)*x
```

mupad [B] time = 1.18, size = 121, normalized size = 0.98

$$\frac{\left(b\,x^2+a\right)^4\,D}{8\,b}-\frac{A\,a^3}{x}+\frac{A\,b^3\,x^5}{5}+\frac{B\,b^3\,x^6}{6}+\frac{C\,b^3\,x^7}{7}+B\,a^3\,\ln(x)+C\,a^3\,x+3\,A\,a^2\,b\,x+A\,a\,b^2\,x^3+\frac{3\,B\,a^2\,b\,x^2}{2}+\frac{3\,B\,a\,b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] ((a + b*x^2)^4*D)/(8*b) - (A*a^3)/x + (A*b^3*x^5)/5 + (B*b^3*x^6)/6 + (C*b^
3*x^7)/7 + B*a^3*log(x) + C*a^3*x + 3*A*a^2*b*x + A*a*b^2*x^3 + (3*B*a^2*b*
x^2)/2 + (3*B*a*b^2*x^4)/4 + C*a^2*b*x^3 + (3*C*a*b^2*x^5)/5
```

sympy [A] time = 0.46, size = 150, normalized size = 1.21

$$-\frac{Aa^3}{x}+Ba^3\log(x)+\frac{Cb^3x^7}{7}+\frac{Db^3x^8}{8}+x^6\left(\frac{Bb^3}{6}+\frac{Dab^2}{2}\right)+x^5\left(\frac{Ab^3}{5}+\frac{3Cab^2}{5}\right)+x^4\left(\frac{3Bab^2}{4}+\frac{3Da^2b}{4}\right)+x^3\left(Aab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**3/x + B*a**3*log(x) + C*b**3*x**7/7 + D*b**3*x**8/8 + x**6*(B*b**3/6
+ D*a*b**2/2) + x**5*(A*b**3/5 + 3*C*a*b**2/5) + x**4*(3*B*a*b**2/4 + 3*D*a
**2*b/4) + x**3*(A*a*b**2 + C*a**2*b) + x**2*(3*B*a**2*b/2 + D*a**3/2) + x*
(3*A*a**2*b + C*a**3)
```

3.84 $\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3} dx$

Optimal. Leaf size=135

$$-\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2\log(x)(aC+3Ab)+a^2x(aD+3bB)+\frac{1}{4}b^2x^4(3aC+Ab)+\frac{3}{2}abx^2(aC+Ab)+\frac{1}{5}b^2x^5(3aD+bB)+abx^3(aD$$

```
[Out] -1/2*a^3*A/x^2-a^3*B/x+a^2*(3*B*b+D*a)*x+3/2*a*b*(A*b+C*a)*x^2+a*b*(B*b+D*a
)*x^3+1/4*b^2*(A*b+3*C*a)*x^4+1/5*b^2*(B*b+3*D*a)*x^5+1/6*b^3*C*x^6+1/7*b^3
*D*x^7+a^2*(3*A*b+C*a)*ln(x)
```

Rubi [A] time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$a^2\log(x)(aC+3Ab)-\frac{a^3A}{2x^2}+a^2x(aD+3bB)-\frac{a^3B}{x}+\frac{1}{4}b^2x^4(3aC+Ab)+\frac{3}{2}abx^2(aC+Ab)+\frac{1}{5}b^2x^5(3aD+bB)+abx^3(aD+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^3*A)/(2*x^2) - (a^3*B)/x + a^2*(3*b*B + a*D)*x + (3*a*b*(A*b + a*C)*x^2
)/2 + a*b*(b*B + a*D)*x^3 + (b^2*(A*b + 3*a*C)*x^4)/4 + (b^2*(b*B + 3*a*D)*
x^5)/5 + (b^3*C*x^6)/6 + (b^3*D*x^7)/7 + a^2*(3*A*b + a*C)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \int \left(a^2(3bB+aD)+\frac{a^3A}{x^3}+\frac{a^3B}{x^2}+\frac{a^2(3Ab+aC)}{x}+3ab(Ab+aC)x+\right.$$
$$= -\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2(3bB+aD)x+\frac{3}{2}ab(Ab+aC)x^2+ab(bB+aD)x^3+$$

Mathematica [A] time = 0.06, size = 124, normalized size = 0.92

$$-\frac{a^3(A+2Bx-2Dx^3)}{2x^2}+a^2\log(x)(aC+3Ab)+\frac{1}{2}a^2bx(6B+x(3C+2Dx))+\frac{1}{20}ab^2x^2(30A+x(20B+3x(5C+4Dx)))+$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a^3*(A + 2*B*x - 2*D*x^3))/x^2 + (a^2*b*x*(6*B + x*(3*C + 2*D*x)))/2
+ (a*b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/20 + (b^3*x^4*(105*A +
2*x*(42*B + 5*x*(7*C + 6*D*x))))/420 + a^2*(3*A*b + a*C)*Log[x]
```

fricas [A] time = 0.54, size = 147, normalized size = 1.09

$$\frac{60Db^3x^9+70Cb^3x^8+84(3Dab^2+Bb^3)x^7+105(3Cab^2+Ab^3)x^6+420(Da^2b+Bab^2)x^5-420Ba^3x+630(}{420x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/420*(60*D*b^3*x^9 + 70*C*b^3*x^8 + 84*(3*D*a*b^2 + B*b^3)*x^7 + 105*(3*C*
a*b^2 + A*b^3)*x^6 + 420*(D*a^2*b + B*a*b^2)*x^5 - 420*B*a^3*x + 630*(C*a^2
*b + A*a*b^2)*x^4 - 210*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 + 420*(C*a^3 +
3*A*a^2*b)*x^2*log(x))/x^2
```

giac [A] time = 0.41, size = 144, normalized size = 1.07

$$\frac{1}{7}Db^3x^7+\frac{1}{6}Cb^3x^6+\frac{3}{5}Dab^2x^5+\frac{1}{5}Bb^3x^5+\frac{3}{4}Cab^2x^4+\frac{1}{4}Ab^3x^4+Da^2bx^3+Bab^2x^3+\frac{3}{2}Ca^2bx^2+\frac{3}{2}Aab^2x^2+Da^3x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 3/5*D*a*b^2*x^5 + 1/5*B*b^3*x^5 + 3/4*C*a*b
^2*x^4 + 1/4*A*b^3*x^4 + D*a^2*b*x^3 + B*a*b^2*x^3 + 3/2*C*a^2*b*x^2 + 3/2*
A*a*b^2*x^2 + D*a^3*x + 3*B*a^2*b*x + (C*a^3 + 3*A*a^2*b)*log(abs(x)) - 1/2
*(2*B*a^3*x + A*a^3)/x^2
```

maple [A] time = 0.01, size = 144, normalized size = 1.07

$$\frac{Db^3x^7}{7}+\frac{Cb^3x^6}{6}+\frac{Bb^3x^5}{5}+\frac{3Dab^2x^5}{5}+\frac{Ab^3x^4}{4}+\frac{3Cab^2x^4}{4}+Bab^2x^3+Da^2bx^3+\frac{3Aab^2x^2}{2}+\frac{3Ca^2bx^2}{2}+3Aa^2b\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

```
[Out] 1/7*b^3*D*x^7+1/6*b^3*C*x^6+1/5*B*x^5*b^3+3/5*D*x^5*a*b^2+1/4*A*b^3*x^4+3/4
*C*x^4*a*b^2+B*a*b^2*x^3+D*x^3*a^2*b+3/2*A*a*b^2*x^2+3/2*C*x^2*a^2*b+3*B*a^
2*b*x+a^3*D*x-1/2*a^3*A/x^2-a^3*B/x+3*A*ln(x)*a^2*b+C*ln(x)*a^3
```

maxima [A] time = 1.36, size = 139, normalized size = 1.03

$$\frac{1}{7}Db^3x^7+\frac{1}{6}Cb^3x^6+\frac{1}{5}\left(3Dab^2+Bb^3\right)x^5+\frac{1}{4}\left(3Cab^2+Ab^3\right)x^4+\left(Da^2b+Bab^2\right)x^3+\frac{3}{2}\left(Ca^2b+Aab^2\right)x^2+\left(Da^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 1/5*(3*D*a*b^2 + B*b^3)*x^5 + 1/4*(3*C*a*b^
2 + A*b^3)*x^4 + (D*a^2*b + B*a*b^2)*x^3 + 3/2*(C*a^2*b + A*a*b^2)*x^2 + (D
*a^3 + 3*B*a^2*b)*x + (C*a^3 + 3*A*a^2*b)*log(x) - 1/2*(2*B*a^3*x + A*a^3)/
x^2
```

mupad [B] time = 1.26, size = 143, normalized size = 1.06

$$\frac{A\,b^3\,x^4}{4}-\frac{B\,a^3}{x}-\frac{A\,a^3}{2\,x^2}+\frac{B\,b^3\,x^5}{5}+\frac{C\,b^3\,x^6}{6}+C\,a^3\,\ln(x)+a^3\,x\,D+\frac{b^3\,x^7\,D}{7}+a^2\,b\,x^3\,D+\frac{3\,a\,b^2\,x^5\,D}{5}+3\,B\,a^2\,b\,x+\frac{3\,A\,a\,b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

```
[Out] (A*b^3*x^4)/4 - (B*a^3)/x - (A*a^3)/(2*x^2) + (B*b^3*x^5)/5 + (C*b^3*x^6)/6
 + C*a^3*log(x) + a^3*x*D + (b^3*x^7*D)/7 + a^2*b*x^3*D + (3*a*b^2*x^5*D)/5
 + 3*B*a^2*b*x + (3*A*a*b^2*x^2)/2 + B*a*b^2*x^3 + (3*C*a^2*b*x^2)/2 + (3*C
*a*b^2*x^4)/4 + 3*A*a^2*b*log(x)
```

<u>**sympy**</u> **[A]** time = 0.67, size = 151, normalized size = 1.12

$$\frac{Cb^3x^6}{6}+\frac{Db^3x^7}{7}+a^2\left(3Ab+Ca\right)\log\left(x\right)+x^5\left(\frac{Bb^3}{5}+\frac{3Dab^2}{5}\right)+x^4\left(\frac{Ab^3}{4}+\frac{3Cab^2}{4}\right)+x^3\left(Bab^2+Da^2b\right)+x^2\left(\frac{3Aab^2}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b**3*x**6/6 + D*b**3*x**7/7 + a**2*(3*A*b + C*a)*log(x) + x**5*(B*b**3/5
+ 3*D*a*b**2/5) + x**4*(A*b**3/4 + 3*C*a*b**2/4) + x**3*(B*a*b**2 + D*a**2*
b) + x**2*(3*A*a*b**2/2 + 3*C*a**2*b/2) + x*(3*B*a**2*b + D*a**3) + (-A*a**
3 - 2*B*a**3*x)/(2*x**2)
```

3.85 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4} dx$

Optimal. Leaf size=139

$$-\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(aC+3Ab)}{x}+a^2\log(x)(aD+3bB)+\frac{1}{3}b^2x^3(3aC+Ab)+3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB)+\frac{3}{2}abx^2($$

```
[Out] -1/3*a^3*A/x^3-1/2*a^3*B/x^2-a^2*(3*A*b+C*a)/x+3*a*b*(A*b+C*a)*x+3/2*a*b*(B
*b+D*a)*x^2+1/3*b^2*(A*b+3*C*a)*x^3+1/4*b^2*(B*b+3*D*a)*x^4+1/5*b^3*C*x^5+1
/6*b^3*D*x^6+a^2*(3*B*b+D*a)*ln(x)
```

Rubi [A] time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$-\frac{a^2(aC+3Ab)}{x}-\frac{a^3A}{3x^3}+a^2\log(x)(aD+3bB)-\frac{a^3B}{2x^2}+\frac{1}{3}b^2x^3(3aC+Ab)+3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB)+\frac{3}{2}abx^2($$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a^3*A)/(3*x^3) - (a^3*B)/(2*x^2) - (a^2*(3*A*b + a*C))/x + 3*a*b*(A*b + a
*C)*x + (3*a*b*(b*B + a*D)*x^2)/2 + (b^2*(A*b + 3*a*C)*x^3)/3 + (b^2*(b*B +
 3*a*D)*x^4)/4 + (b^3*C*x^5)/5 + (b^3*D*x^6)/6 + a^2*(3*b*B + a*D)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}dx &= \int \left(3ab(Ab+aC)+\frac{a^3A}{x^4}+\frac{a^3B}{x^3}+\frac{a^2(3Ab+aC)}{x^2}+\frac{a^2(3bB+aD)}{x}\right.\\ &= -\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(3Ab+aC)}{x}+3ab(Ab+aC)x+\frac{3}{2}ab(bB+aD)x^2\end{aligned}$$

Mathematica [A] time = 0.05, size = 124, normalized size = 0.89

$$-\frac{a^3(2A+3x(B+2Cx))}{6x^3}+\frac{3a^2b\left(x^2(2C+Dx)-2A\right)}{2x}+a^2\log(x)(aD+3bB)+\frac{1}{4}ab^2x(12A+x(6B+x(4C+3Dx)))+$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/6*(a^3*(2*A + 3*x*(B + 2*C*x)))/x^3 + (3*a^2*b*(-2*A + x^2*(2*C + D*x)))
/(2*x) + (a*b^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))/4 + (b^3*x^3*(20*A +
x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*(3*b*B + a*D)*Log[x]
```

fricas [A] time = 0.82, size = 147, normalized size = 1.06

$$\frac{10\,Db^3x^9+12\,Cb^3x^8+15\left(3\,Dab^2+Bb^3\right)x^7+20\left(3\,Cab^2+Ab^3\right)x^6+90\left(Da^2b+Bab^2\right)x^5-30\,Ba^3x+180\,(}{60\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/60*(10*D*b^3*x^9 + 12*C*b^3*x^8 + 15*(3*D*a*b^2 + B*b^3)*x^7 + 20*(3*C*a*
b^2 + A*b^3)*x^6 + 90*(D*a^2*b + B*a*b^2)*x^5 - 30*B*a^3*x + 180*(C*a^2*b +
 A*a*b^2)*x^4 + 60*(D*a^3 + 3*B*a^2*b)*x^3*log(x) - 20*A*a^3 - 60*(C*a^3 +
3*A*a^2*b)*x^2)/x^3
```

giac [A] time = 0.40, size = 146, normalized size = 1.05

$$\frac{1}{6}Db^3x^6+\frac{1}{5}Cb^3x^5+\frac{3}{4}Dab^2x^4+\frac{1}{4}Bb^3x^4+Cab^2x^3+\frac{1}{3}Ab^3x^3+\frac{3}{2}Da^2bx^2+\frac{3}{2}Bab^2x^2+3Ca^2bx+3Aab^2x+\left(Da^3+3B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 3/4*D*a*b^2*x^4 + 1/4*B*b^3*x^4 + C*a*b^2*x
^3 + 1/3*A*b^3*x^3 + 3/2*D*a^2*b*x^2 + 3/2*B*a*b^2*x^2 + 3*C*a^2*b*x + 3*A*
a*b^2*x + (D*a^3 + 3*B*a^2*b)*log(abs(x)) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C
*a^3 + 3*A*a^2*b)*x^2)/x^3
```

maple [A] time = 0.01, size = 146, normalized size = 1.05

$$\frac{Db^3x^6}{6}+\frac{Cb^3x^5}{5}+\frac{Bb^3x^4}{4}+\frac{3Dab^2x^4}{4}+\frac{Ab^3x^3}{3}+Cab^2x^3+\frac{3Bab^2x^2}{2}+\frac{3Da^2bx^2}{2}+3Aab^2x+3Ba^2b\ln(x)+3Ca^2bx+D$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/6*b^3*D*x^6+1/5*b^3*C*x^5+1/4*B*x^4*b^3+3/4*D*x^4*a*b^2+1/3*A*b^3*x^3+C*x
^3*a*b^2+3/2*B*x^2*a*b^2+3/2*D*x^2*a^2*b+3*A*a*b^2*x+3*a^2*b*C*x-1/3*a^3*A/
x^3-1/2*a^3*B/x^2-3*a^2/x*A*b-a^3/x*C+3*B*ln(x)*a^2*b+D*ln(x)*a^3
```

maxima [A] time = 1.32, size = 142, normalized size = 1.02

$$\frac{1}{6}Db^3x^6+\frac{1}{5}Cb^3x^5+\frac{1}{4}\left(3Dab^2+Bb^3\right)x^4+\frac{1}{3}\left(3Cab^2+Ab^3\right)x^3+\frac{3}{2}\left(Da^2b+Bab^2\right)x^2+3\left(Ca^2b+Aab^2\right)x+\left(Da^3+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 1/4*(3*D*a*b^2 + B*b^3)*x^4 + 1/3*(3*C*a*b^
2 + A*b^3)*x^3 + 3/2*(D*a^2*b + B*a*b^2)*x^2 + 3*(C*a^2*b + A*a*b^2)*x + (D
*a^3 + 3*B*a^2*b)*log(x) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C*a^3 + 3*A*a^2*b)
*x^2)/x^3
```

mupad [B] time = 1.37, size = 148, normalized size = 1.06

$$\frac{B\,b^3\,x^4}{4}-\frac{C\,a^3}{x}-\frac{B\,a^3}{2\,x^2}+\frac{C\,b^3\,x^5}{5}+\frac{b^3\,x^6\,D}{6}-\frac{A\,\left(a^3+9\,a^2\,b\,x^2-9\,a\,b^2\,x^4-b^3\,x^6\right)}{3\,x^3}+\frac{a^3\,\ln\left(x^2\right)\,D}{2}+\frac{3\,a^2\,b\,x^2\,D}{2}+3\,C\,a^2\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

```
[Out] (B*b^3*x^4)/4 - (C*a^3)/x - (B*a^3)/(2*x^2) + (C*b^3*x^5)/5 + (b^3*x^6*D)/6
 - (A*(a^3 - b^3*x^6 + 9*a^2*b*x^2 - 9*a*b^2*x^4))/(3*x^3) + (a^3*log(x^2)*
D)/2 + (3*a^2*b*x^2*D)/2 + 3*C*a^2*b*x + (3*a*b^2*x^4*D)/4 + (3*B*a*b^2*x^2
)/2 + C*a*b^2*x^3 + 3*B*a^2*b*log(x)
```

sympy [**A**] time = 1.09, size = 155, normalized size = 1.12

$$\frac{Cb^3x^5}{5}+\frac{Db^3x^6}{6}+a^2\left(3Bb+Da\right)\log\left(x\right)+x^4\left(\frac{Bb^3}{4}+\frac{3Dab^2}{4}\right)+x^3\left(\frac{Ab^3}{3}+Cab^2\right)+x^2\left(\frac{3Bab^2}{2}+\frac{3Da^2b}{2}\right)+x\left(3Aal$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**3*x**5/5 + D*b**3*x**6/6 + a**2*(3*B*b + D*a)*log(x) + x**4*(B*b**3/4
+ 3*D*a*b**2/4) + x**3*(A*b**3/3 + C*a*b**2) + x**2*(3*B*a*b**2/2 + 3*D*a**
2*b/2) + x*(3*A*a*b**2 + 3*C*a**2*b) + (-2*A*a**3 - 3*B*a**3*x + x**2*(-18*
A*a**2*b - 6*C*a**3))/(6*x**3)
```

3.86 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

Optimal. Leaf size=151

$$\frac{a^{3/2}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log\left(a+bx^2\right)}{2b^4}-\frac{ax(Ab-aC)}{b^3}+\frac{x^3(Ab-aC)}{3b^2}-\frac{ax^2(bB-aD)}{2b^3}+\frac{x^4(bB-aD)}{4b^2}$$

```
[Out] -a*(A*b-C*a)*x/b^3-1/2*a*(B*b-D*a)*x^2/b^3+1/3*(A*b-C*a)*x^3/b^2+1/4*(B*b-D
*a)*x^4/b^2+1/5*C*x^5/b+1/6*D*x^6/b+a^(3/2)*(A*b-C*a)*arctan(x*b^(1/2)/a^(1
/2))/b^(7/2)+1/2*a^2*(B*b-D*a)*ln(b*x^2+a)/b^4
```

Rubi [A] time = 0.14, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1802, 635, 205, 260}

$$\frac{a^{3/2}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log\left(a+bx^2\right)}{2b^4}+\frac{x^3(Ab-aC)}{3b^2}-\frac{ax(Ab-aC)}{b^3}+\frac{x^4(bB-aD)}{4b^2}-\frac{ax^2(bB-aD)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a*(A*b - a*C)*x)/b^3) - (a*(b*B - a*D)*x^2)/(2*b^3) + ((A*b - a*C)*x^3)/
(3*b^2) + ((b*B - a*D)*x^4)/(4*b^2) + (C*x^5)/(5*b) + (D*x^6)/(6*b) + (a^(3
/2)*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2) + (a^2*(b*B - a*D)*Log
[a + b*x^2])/(2*b^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = \int \left(-\frac{a(Ab-aC)}{b^3} - \frac{a(bB-aD)x}{b^3} + \frac{(Ab-aC)x^2}{b^2} + \frac{(bB-aD)x^3}{b^2} + \frac{Cx^4}{b} + \right.$$

$$= -\frac{a(Ab-aC)x}{b^3} - \frac{a(bB-aD)x^2}{2b^3} + \frac{(Ab-aC)x^3}{3b^2} + \frac{(bB-aD)x^4}{4b^2} + \frac{Cx^5}{5b} + \frac{D}{6}$$

$$= -\frac{a(Ab-aC)x}{b^3} - \frac{a(bB-aD)x^2}{2b^3} + \frac{(Ab-aC)x^3}{3b^2} + \frac{(bB-aD)x^4}{4b^2} + \frac{Cx^5}{5b} + \frac{D}{6}$$

$$= -\frac{a(Ab-aC)x}{b^3} - \frac{a(bB-aD)x^2}{2b^3} + \frac{(Ab-aC)x^3}{3b^2} + \frac{(bB-aD)x^4}{4b^2} + \frac{Cx^5}{5b} + \frac{D}{6}$$

Mathematica [A] time = 0.08, size = 130, normalized size = 0.86

$$\frac{-60a^{3/2}\sqrt{b}\,(aC-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) + bx\left(30a^2(2C+Dx) - 5ab(12A+x(6B+x(4C+3Dx))) + b^2x^2(20A+x(\right.}{60b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(30*a^2*(2*C + D*x) - 5*a*b*(12*A + x*(6*B + x*(4*C + 3*D*x))) + b^2*x
^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x)))) - 60*a^(3/2)*Sqrt[b]*(-(A*b) + a*
C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 30*a^2*(-(b*B) + a*D)*Log[a + b*x^2])/(60*
b^4)
```

fricas [A] time = 0.71, size = 332, normalized size = 2.20

$$\left[\frac{10\,Db^3x^6 + 12\,Cb^3x^5 - 15\left(Dab^2 - Bb^3\right)x^4 - 20\left(Cab^2 - Ab^3\right)x^3 + 30\left(Da^2b - Bab^2\right)x^2 - 30\left(Ca^2b - Aab^2\right)\sqrt{}}{60\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/60*(10*D*b^3*x^6 + 12*C*b^3*x^5 - 15*(D*a*b^2 - B*b^3)*x^4 - 20*(C*a*b^2
 - A*b^3)*x^3 + 30*(D*a^2*b - B*a*b^2)*x^2 - 30*(C*a^2*b - A*a*b^2)*sqrt(-a
/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 60*(C*a^2*b - A*a*b^2
)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^2 + a))/b^4, 1/60*(10*D*b^3*x^6 + 12*C*b
^3*x^5 - 15*(D*a*b^2 - B*b^3)*x^4 - 20*(C*a*b^2 - A*b^3)*x^3 + 30*(D*a^2*b
- B*a*b^2)*x^2 - 60*(C*a^2*b - A*a*b^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) +
 60*(C*a^2*b - A*a*b^2)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^2 + a))/b^4]
```

giac [A] time = 0.43, size = 161, normalized size = 1.07

$$-\frac{\left(Ca^3 - Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} - \frac{\left(Da^3 - Ba^2b\right)\log\left(bx^2+a\right)}{2\,b^4} + \frac{10\,Db^5x^6 + 12\,Cb^5x^5 - 15\,Dab^4x^4 + 15\,Bb^5x^4 - 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(D*a^3 - B*a
^2*b)*log(b*x^2 + a)/b^4 + 1/60*(10*D*b^5*x^6 + 12*C*b^5*x^5 - 15*D*a*b^4*x
```

```
^4 + 15*B*b^5*x^4 - 20*C*a*b^4*x^3 + 20*A*b^5*x^3 + 30*D*a^2*b^3*x^2 - 30*B
*a*b^4*x^2 + 60*C*a^2*b^3*x - 60*A*a*b^4*x)/b^6
```

maple [A] time = 0.01, size = 176, normalized size = 1.17

$$\frac{Dx^6}{6b}+\frac{C\,x^5}{5b}+\frac{B\,x^4}{4b}-\frac{Da\,x^4}{4b^2}+\frac{A\,x^3}{3b}-\frac{Ca\,x^3}{3b^2}+\frac{A\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}-\frac{Ba\,x^2}{2b^2}-\frac{C\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3}+\frac{Da^2x^2}{2b^3}-\frac{Aax}{b^2}+\frac{B\,a^2\ln\left(\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/6*D*x^6/b+1/5*C*x^5/b+1/4/b*B*x^4-1/4/b^2*D*x^4*a+1/3/b*A*x^3-1/3/b^2*C*x
^3*a-1/2/b^2*B*x^2*a+1/2/b^3*D*x^2*a^2-1/b^2*A*a*x+1/b^3*a^2*C*x+1/2*a^2/b^
3*ln(b*x^2+a)*B-1/2*a^3/b^4*ln(b*x^2+a)*D+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b
)^(1/2)*b*x)*A-a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C
```

maxima [A] time = 2.96, size = 145, normalized size = 0.96

$$-\frac{\left(Ca^3-Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{10\,Db^2x^6+12\,Cb^2x^5-15\left(Dab-Bb^2\right)x^4-20\left(Cab-Ab^2\right)x^3+30\left(Da^2-Bab\right)x^2}{60\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/60*(10*D*b^2*x
^6 + 12*C*b^2*x^5 - 15*(D*a*b - B*b^2)*x^4 - 20*(C*a*b - A*b^2)*x^3 + 30*(D
*a^2 - B*a*b)*x^2 + 60*(C*a^2 - A*a*b)*x)/b^3 - 1/2*(D*a^3 - B*a^2*b)*log(b
*x^2 + a)/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

sympy [B] time = 1.43, size = 316, normalized size = 2.09

$$\frac{Cx^5}{5b}+\frac{Dx^6}{6b}+x^4\left(\frac{B}{4b}-\frac{Da}{4b^2}\right)+x^3\left(\frac{A}{3b}-\frac{Ca}{3b^2}\right)+x^2\left(-\frac{Ba}{2b^2}+\frac{Da^2}{2b^3}\right)+x\left(-\frac{Aa}{b^2}+\frac{Ca^2}{b^3}\right)+\left(-\frac{a^2\left(-Bb+Da\right)}{2b^4}-\frac{\sqrt{-a^3b^9}\,(-}{2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**5/(5*b) + D*x**6/(6*b) + x**4*(B/(4*b) - D*a/(4*b**2)) + x**3*(A/(3*b)
 - C*a/(3*b**2)) + x**2*(-B*a/(2*b**2) + D*a**2/(2*b**3)) + x*(-A*a/b**2 +
C*a**2/b**3) + (-a**2*(-B*b + D*a)/(2*b**4) - sqrt(-a**3*b**9)*(-A*b + C*a)
/(2*b**8))*log(x + (B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b**4)
 - sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b)) + (-a**
2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8))*log(x + (
B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3*b**9)*
(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b))
```

3.87 $\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$

Optimal. Leaf size=130

$$\frac{a^{3/2}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{a(Ab-aC)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(Ab-aC)}{2b^2}-\frac{ax(bB-aD)}{b^3}+\frac{x^3(bB-aD)}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}$$

```
[Out] -a*(B*b-D*a)*x/b^3+1/2*(A*b-C*a)*x^2/b^2+1/3*(B*b-D*a)*x^3/b^2+1/4*C*x^4/b+
1/5*D*x^5/b+a^(3/2)*(B*b-D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)-1/2*a*(A*b-
C*a)*ln(b*x^2+a)/b^3
```

Rubi [A] time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$\frac{a^{3/2}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x^2(Ab-aC)}{2b^2}-\frac{a(Ab-aC)\log\left(a+bx^2\right)}{2b^3}+\frac{x^3(bB-aD)}{3b^2}-\frac{ax(bB-aD)}{b^3}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a*(b*B - a*D)*x)/b^3) + ((A*b - a*C)*x^2)/(2*b^2) + ((b*B - a*D)*x^3)/(3
*b^2) + (C*x^4)/(4*b) + (D*x^5)/(5*b) + (a^(3/2)*(b*B - a*D)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/b^(7/2) - (a*(A*b - a*C)*Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3 \left(A + Bx + Cx^2 + Dx^3\right)}{a + bx^2} dx = \int \left(-\frac{a(bB - aD)}{b^3} + \frac{(Ab - aC)x}{b^2} + \frac{(bB - aD)x^2}{b^2} + \frac{Cx^3}{b} + \frac{Dx^4}{b} + \frac{a^2(bB - aD)}{b^3 (}\right.$$

$$= -\frac{a(bB - aD)x}{b^3} + \frac{(Ab - aC)x^2}{2b^2} + \frac{(bB - aD)x^3}{3b^2} + \frac{Cx^4}{4b} + \frac{Dx^5}{5b} + \frac{\int \frac{a^2(bB-aD)-ab(}{a+bx^2}}{b^3}$$

$$= -\frac{a(bB - aD)x}{b^3} + \frac{(Ab - aC)x^2}{2b^2} + \frac{(bB - aD)x^3}{3b^2} + \frac{Cx^4}{4b} + \frac{Dx^5}{5b} - \frac{(a(Ab - aC))}{b^2}$$

$$= -\frac{a(bB - aD)x}{b^3} + \frac{(Ab - aC)x^2}{2b^2} + \frac{(bB - aD)x^3}{3b^2} + \frac{Cx^4}{4b} + \frac{Dx^5}{5b} + \frac{a^{3/2}(bB - aD)}{b^{7/}}$$

Mathematica [A] time = 0.10, size = 114, normalized size = 0.88

$$\frac{x \left(60a^2D - 10ab(6B + x(3C + 2Dx)) + b^2x(30A + x(20B + 3x(5C + 4Dx)))\right) + 30a(aC - Ab) \log \left(a + bx^2\right)}{60b^3} - \frac{a^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a^(3/2)*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)) + (x*(60*a^
2*D - 10*a*b*(6*B + x*(3*C + 2*D*x)) + b^2*x*(30*A + x*(20*B + 3*x*(5*C + 4
*D*x)))) + 30*a*(-(A*b) + a*C)*Log[a + b*x^2])/(60*b^3)
```

fricas [A] time = 0.61, size = 270, normalized size = 2.08

$$\left[\frac{12\, Db^2x^5 + 15\, Cb^2x^4 - 20\left(Dab - Bb^2\right)x^3 - 30\left(Cab - Ab^2\right)x^2 + 30\left(Da^2 - Bab\right)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 60\,(}{60\, b^3} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C*a*b - A
*b^2)*x^2 + 30*(D*a^2 - B*a*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a
)/(b*x^2 + a)) + 60*(D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a))/
b^3, 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C*a*b
 - A*b^2)*x^2 - 60*(D*a^2 - B*a*b)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 60*(
D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a))/b^3]
```

giac [A] time = 0.35, size = 137, normalized size = 1.05

$$\frac{\left(Ca^2 - Aab\right) \log \left(bx^2 + a\right)}{2\, b^3} - \frac{\left(Da^3 - Ba^2b\right) \arctan \left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^3} + \frac{12\, Db^4x^5 + 15\, Cb^4x^4 - 20\, Dab^3x^3 + 20\, Bb^4x^3 - 30\, Cal}{60\, b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^4*x^5 + 15*C*b^4*x^4 - 20*D*a*b^3*x^3
+ 20*B*b^4*x^3 - 30*C*a*b^3*x^2 + 30*A*b^4*x^2 + 60*D*a^2*b^2*x - 60*B*a*b^
3*x)/b^5
```

maple [A] time = 0.01, size = 152, normalized size = 1.17

$$\frac{Dx^5}{5b}+\frac{C\,x^4}{4b}+\frac{B\,x^3}{3b}-\frac{Da\,x^3}{3b^2}+\frac{A\,x^2}{2b}+\frac{B\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}-\frac{Ca\,x^2}{2b^2}-\frac{Da^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3}-\frac{Aa\ln\left(b\,x^2+a\right)}{2b^2}-\frac{Bax}{b^2}+\frac{C\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/5*D*x^5/b+1/4*C*x^4/b+1/3*B/b*x^3-1/3/b^2*D*x^3*a+1/2/b*A*x^2-1/2/b^2*C*x
^2*a-B*a/b^2*x+1/b^3*a^2*D*x-1/2*a/b^2*ln(b*x^2+a)*A+1/2*a^2/b^3*ln(b*x^2+a
)*C+1/(a*b)^(1/2)*B*a^2/b^2*arctan(1/(a*b)^(1/2)*b*x)-a^3/b^3/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x)*D
```

maxima [A] time = 2.93, size = 127, normalized size = 0.98

$$\frac{\left(Ca^2-Aab\right)\log\left(bx^2+a\right)}{2\,b^3}-\frac{\left(Da^3-Ba^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{12\,Db^2x^5+15\,Cb^2x^4-20\left(Dab-Bb^2\right)x^3-30\left(Cab\right.}{60\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b
^2)*x^3 - 30*(C*a*b - A*b^2)*x^2 + 60*(D*a^2 - B*a*b)*x)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

sympy [B] time = 1.35, size = 274, normalized size = 2.11

$$\frac{Cx^4}{4b}+\frac{Dx^5}{5b}+x^3\left(\frac{B}{3b}-\frac{Da}{3b^2}\right)+x^2\left(\frac{A}{2b}-\frac{Ca}{2b^2}\right)+x\left(-\frac{Ba}{b^2}+\frac{Da^2}{b^3}\right)+\left(\frac{a\left(-Ab+Ca\right)}{2b^3}-\frac{\sqrt{-a^3b^7}\,\left(-Bb+Da\right)}{2b^7}\right)\log\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**4/(4*b) + D*x**5/(5*b) + x**3*(B/(3*b) - D*a/(3*b**2)) + x**2*(A/(2*b)
 - C*a/(2*b**2)) + x*(-B*a/b**2 + D*a**2/b**3) + (a*(-A*b + C*a)/(2*b**3) -
 sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*
(a*(-A*b + C*a)/(2*b**3) - sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B*a*b
 + D*a**2)) + (a*(-A*b + C*a)/(2*b**3) + sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b
**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*(a*(-A*b + C*a)/(2*b**3) + sqrt(-a*
*3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B*a*b + D*a**2))
```

3.88 $\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$

Optimal. Leaf size=111

$$-\frac{\sqrt{a}\,(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aC)}{b^2}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(bB-aD)}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}$$

```
[Out] (A*b-C*a)*x/b^2+1/2*(B*b-D*a)*x^2/b^2+1/3*C*x^3/b+1/4*D*x^4/b-1/2*a*(B*b-D*
a)*ln(b*x^2+a)/b^3-(A*b-C*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A] time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1802, 635, 205, 260}

$$\frac{x(Ab-aC)}{b^2}-\frac{\sqrt{a}\,(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x^2(bB-aD)}{2b^2}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*C)*x)/b^2 + ((b*B - a*D)*x^2)/(2*b^2) + (C*x^3)/(3*b) + (D*x^4)/(
4*b) - (Sqrt[a]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) - (a*(b*B
- a*D)*Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = \int \left(\frac{Ab-aC}{b^2}+\frac{(bB-aD)x}{b^2}+\frac{Cx^2}{b}+\frac{Dx^3}{b}-\frac{a(Ab-aC)+a(bB-aD)x}{b^2\left(a+bx^2\right)}\right) d$$

$$= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\int \frac{a(Ab-aC)+a(bB-aD)x}{a+bx^2}\,dx}{b^2}$$

$$= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{(a(Ab-aC))\int \frac{1}{a+bx^2}\,dx}{b^2}-\frac{(a(}{}$$

$$= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\sqrt{a}\,(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}-$$

Mathematica [**A**] time = 0.06, size = 95, normalized size = 0.86

$$\frac{bx\left(-6a(2C+Dx)+12Ab+bx\left(6B+4Cx+3Dx^2\right)\right)+12\sqrt{a}\,\sqrt{b}\,(aC-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+6a(aD-bB)\log\left(a\right.}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(12*A*b - 6*a*(2*C + D*x) + b*x*(6*B + 4*C*x + 3*D*x^2)) + 12*Sqrt[a]*
Sqrt[b]*(-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-(b*B) + a*D)*Log
[a + b*x^2])/(12*b^3)
```

fricas [**A**] time = 0.69, size = 238, normalized size = 2.14

$$\left[\frac{3\,Db^2x^4+4\,Cb^2x^3-6\left(Dab-Bb^2\right)x^2-6\left(Cab-Ab^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-12\left(Cab-Ab^2\right)x+6\left(Da^2\right.}{12\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*D*b^2*x^4 + 4*C*b^2*x^3 - 6*(D*a*b - B*b^2)*x^2 - 6*(C*a*b - A*b^2
)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 12*(C*a*b -
A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/b^3, 1/12*(3*D*b^2*x^4 + 4*C*b
^2*x^3 - 6*(D*a*b - B*b^2)*x^2 + 12*(C*a*b - A*b^2)*sqrt(a/b)*arctan(b*x*sq
rt(a/b)/a) - 12*(C*a*b - A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/b^3]
```

giac [**A**] time = 0.40, size = 112, normalized size = 1.01

$$\frac{\left(Ca^2-Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{\left(Da^2-Bab\right)\log\left(bx^2+a\right)}{2\,b^3}+\frac{3\,Db^3x^4+4\,Cb^3x^3-6\,Dab^2x^2+6\,Bb^3x^2-12\,Cab^2x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*a^2 - B*a*b)
*log(b*x^2 + a)/b^3 + 1/12*(3*D*b^3*x^4 + 4*C*b^3*x^3 - 6*D*a*b^2*x^2 + 6*B
*b^3*x^2 - 12*C*a*b^2*x + 12*A*b^3*x)/b^4
```

maple [A] time = 0.00, size = 128, normalized size = 1.15

$$\frac{Dx^4}{4b}+\frac{C\,x^3}{3b}-\frac{Aa\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{B\,x^2}{2b}+\frac{C\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}-\frac{Da\,x^2}{2b^2}+\frac{Ax}{b}-\frac{Ba\ln\left(b\,x^2+a\right)}{2b^2}-\frac{Cax}{b^2}+\frac{Da^2\ln\left(b\,x^2+a\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/4*D*x^4/b+1/3*C*x^3/b+1/2*B/b*x^2-1/2/b^2*D*x^2*a+A/b*x-1/b^2*a*C*x-1/2*B
*a/b^2*ln(b*x^2+a)+1/2*a^2/b^3*ln(b*x^2+a)*D-1/(a*b)^(1/2)*A*a/b*arctan(1/(
a*b)^(1/2)*b*x)+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C
```

maxima [A] time = 3.00, size = 98, normalized size = 0.88

$$\frac{\left(Ca^2-Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{3\,Dbx^4+4\,Cbx^3-6\,(Da-Bb)x^2-12\,(Ca-Ab)x}{12\,b^2}+\frac{\left(Da^2-Bab\right)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/12*(3*D*b*x^4 + 4
*C*b*x^3 - 6*(D*a - B*b)*x^2 - 12*(C*a - A*b)*x)/b^2 + 1/2*(D*a^2 - B*a*b)*
log(b*x^2 + a)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

sympy [B] time = 1.65, size = 245, normalized size = 2.21

$$\frac{Cx^3}{3b}+\frac{Dx^4}{4b}+x^2\left(\frac{B}{2b}-\frac{Da}{2b^2}\right)+x\left(\frac{A}{b}-\frac{Ca}{b^2}\right)+\left(\frac{a\,(-Bb+Da)}{2b^3}-\frac{\sqrt{-ab^7}\,\,(-Ab+Ca)}{2b^6}\right)\log\left(x+\frac{Bab-Da^2+2b^3\left(\frac{a(-B}{2}\right.}{-Ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**3/(3*b) + D*x**4/(4*b) + x**2*(B/(2*b) - D*a/(2*b**2)) + x*(A/b - C*a/
b**2) + (a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6))*log
(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*
b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b)) + (a*(-B*b + D*a)/(2*b**3) + sqrt(-a
*b**7)*(-A*b + C*a)/(2*b**6))*log(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D
*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b))
```

3.89 $\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2} \, dx$

Optimal. Leaf size=92

$$\frac{(Ab-aC)\log\left(a+bx^2\right)}{2b^2}-\frac{\sqrt{a}\,(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(bB-aD)}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

```
[Out] (B*b-D*a)*x/b^2+1/2*C*x^2/b+1/3*D*x^3/b+1/2*(A*b-C*a)*ln(b*x^2+a)/b^2-(B*b-
D*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1802, 635, 205, 260}

$$\frac{(Ab-aC)\log\left(a+bx^2\right)}{2b^2}+\frac{x(bB-aD)}{b^2}-\frac{\sqrt{a}\,(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((b*B - a*D)*x)/b^2 + (C*x^2)/(2*b) + (D*x^3)/(3*b) - (Sqrt[a]*(b*B - a*D)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + ((A*b - a*C)*Log[a + b*x^2])/(2*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = \int \left(\frac{bB-aD}{b^2}+\frac{Cx}{b}+\frac{Dx^2}{b}-\frac{a(bB-aD)-b(Ab-aC)x}{b^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\int \frac{a(bB-aD)-b(Ab-aC)x}{a+bx^2}\,dx}{b^2}$$

$$= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}+\frac{(Ab-aC)\int \frac{x}{a+bx^2}\,dx}{b}-\frac{(a(bB-aD))\int \frac{1}{a+bx^2}\,dx}{b^2}$$

$$= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\sqrt{a}\,(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{(Ab-aC)\log\left(a+\right.}{2b^2}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 0.88

$$\frac{3(Ab-aC)\log\left(a+bx^2\right)+x(-6aD+6bB+bx(3C+2Dx))}{6b^2}+\frac{\sqrt{a}\,(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + (x*(6*b*B -
6*a*D + b*x*(3*C + 2*D*x)) + 3*(A*b - a*C)*Log[a + b*x^2])/(6*b^2)
```

fricas [A] time = 0.72, size = 180, normalized size = 1.96

$$\left[\frac{2\,Dbx^3+3\,Cbx^2+3\,(Da-Bb)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(Da-Bb)x-3\,(Ca-Ab)\log\left(bx^2+a\right)}{6\,b^2},\ \frac{2\,Dbx^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*D*b*x^3 + 3*C*b*x^2 + 3*(D*a - B*b)*sqrt(-a/b)*log((b*x^2 + 2*b*x*s
qrt(-a/b) - a)/(b*x^2 + a)) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 + a
))/b^2, 1/6*(2*D*b*x^3 + 3*C*b*x^2 + 6*(D*a - B*b)*sqrt(a/b)*arctan(b*x*sqr
t(a/b)/a) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 + a))/b^2]
```

giac [A] time = 0.42, size = 88, normalized size = 0.96

$$-\frac{(Ca-Ab)\log\left(bx^2+a\right)}{2\,b^2}+\frac{\left(Da^2-Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\,Db^2x^3+3\,Cb^2x^2-6\,Dabx+6\,Bb^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^2) + 1/6*(2*D*b^2*x^3 + 3*C*b^2*x^2 - 6*D*a*b*x + 6*B*b^2*x)/
b^3
```

maple [A] time = 0.01, size = 106, normalized size = 1.15

$$\frac{Dx^3}{3b}-\frac{Ba\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{C\,x^2}{2b}+\frac{Da^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}+\frac{A\ln\left(b\,x^2+a\right)}{2b}+\frac{Bx}{b}-\frac{Ca\ln\left(b\,x^2+a\right)}{2b^2}-\frac{Dax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/3*D*x^3/b+1/2*C*x^2/b+B/b*x-1/b^2*a*D*x+1/2*A/b*ln(b*x^2+a)-1/2/b^2*ln(b*
x^2+a)*a*C-1/(a*b)^(1/2)*B*a/b*arctan(1/(a*b)^(1/2)*b*x)+1/b^2/(a*b)^(1/2)*
arctan(1/(a*b)^(1/2)*b*x)*a^2*D
```

maxima [A] time = 3.01, size = 82, normalized size = 0.89

$$-\frac{(Ca-Ab)\log\left(bx^2+a\right)}{2\,b^2}+\frac{\left(Da^2-Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\,Dbx^3+3\,Cbx^2-6\,(Da-Bb)x}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^2) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(D*a - B*b)*x)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

sympy [B] time = 0.99, size = 211, normalized size = 2.29

$$\frac{Cx^2}{2b}+\frac{Dx^3}{3b}+x\left(\frac{B}{b}-\frac{Da}{b^2}\right)+\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}\,(-Bb+Da)}{2b^5}\right)\log\left(x+\frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}\,(-I}{2b^5}\right.}{-Bb+Da}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**2/(2*b) + D*x**3/(3*b) + x*(B/b - D*a/b**2) + (-(-A*b + C*a)/(2*b**2)
- sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b + C*a + 2*b**2*(-(-A*b
 + C*a)/(2*b**2) - sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5)))/(-B*b + D*a)) + (-
(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b
+ C*a + 2*b**2*(-(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5
)))/(-B*b + D*a))
```

3.90 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} dx$

Optimal. Leaf size=73

$$\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

```
[Out] C*x/b+1/2*D*x^2/b+1/2*(B*b-D*a)*ln(b*x^2+a)/b^2+(A*b-C*a)*arctan(x*b^(1/2)/
a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1810, 635, 205, 260}

$$\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] (C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a
]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \int \left(\frac{C}{b}+\frac{Dx}{b}+\frac{Ab-aC+(bB-aD)x}{b\left(a+bx^2\right)}\right)dx$$
$$= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{\int \frac{Ab-aC+(bB-aD)x}{a+bx^2}\,dx}{b}$$
$$= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\int \frac{1}{a+bx^2}\,dx}{b}+\frac{(bB-aD)\int \frac{x}{a+bx^2}\,dx}{b}$$
$$= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.93

$$\frac{\frac{2\sqrt{b}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-aD)\log\left(a+bx^2\right)+bx(2C+Dx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] (b*x*(2*C + D*x) + (2*Sqrt[b]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt
[a] + (b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

fricas [A] time = 0.68, size = 157, normalized size = 2.15

$$\left[\frac{Dabx^2+2\,Cabx+(Ca-Ab)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-\left(Da^2-Bab\right)\log\left(bx^2+a\right)}{2\,ab^2},\ \frac{Dabx^2+2\,Cabx-2\,(C\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(D*a*b*x^2 + 2*C*a*b*x + (C*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)) - (D*a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2), 1/2*(D*a
*b*x^2 + 2*C*a*b*x - 2*(C*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*a^2
 - B*a*b)*log(b*x^2 + a))/(a*b^2)]
```

giac [A] time = 0.46, size = 66, normalized size = 0.90

$$-\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2}+\frac{Dbx^2+2\,Cbx}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*(D*a - B*b)*log(b*x^
2 + a)/b^2 + 1/2*(D*b*x^2 + 2*C*b*x)/b^2
```

maple [A] time = 0.00, size = 83, normalized size = 1.14

$$\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{Ca\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{Dx^2}{2b}+\frac{B\ln\left(b\,x^2+a\right)}{2b}+\frac{Cx}{b}-\frac{Da\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/2*D*x^2/b+C*x/b+1/2*B/b*ln(b*x^2+a)-1/2/b^2*ln(b*x^2+a)*a*D+1/(a*b)^(1/2)
*A*arctan(1/(a*b)^(1/2)*b*x)-1/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*C
```

maxima [A] time = 2.93, size = 64, normalized size = 0.88

$$-\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{Dx^2+2\,Cx}{2\,b}-\frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*(D*x^2 + 2*C*x)/b -
1/2*(D*a - B*b)*log(b*x^2 + a)/b^2
```

mupad [B] time = 1.42, size = 79, normalized size = 1.08

$$\frac{B\,\ln\left(b\,x^2+a\right)}{2\,b}-\frac{\left(a\,\ln\left(b\,x^2+a\right)-b\,x^2\right)\,D}{2\,b^2}+\frac{C\,x}{b}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2),x)
```

```
[Out] (B*log(a + b*x^2))/(2*b) - ((a*log(a + b*x^2) - b*x^2)*D)/(2*b^2) + (C*x)/b
 + (A*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2)) - (C*a^(1/2)*atan((b^(1/
2)*x)/a^(1/2)))/b^(3/2)
```

sympy [B] time = 0.88, size = 219, normalized size = 3.00

$$\frac{Cx}{b}+\frac{Dx^2}{2b}+\left(-\frac{-Bb+Da}{2b^2}-\frac{\sqrt{-ab^5}\,(-Ab+Ca)}{2ab^4}\right)\log\left(x+\frac{Bab-Da^2-2ab^2\left(-\frac{-Bb+Da}{2b^2}-\frac{\sqrt{-ab^5}\,(-Ab+Ca)}{2ab^4}\right)}{-Ab^2+Cab}\right)+\left(-\frac{-Bb}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x/b + D*x**2/(2*b) + (-(-B*b + D*a)/(2*b**2) - sqrt(-a*b**5)*(-A*b + C*a)
/(2*a*b**4))*log(x + (B*a*b - D*a**2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) - s
qrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))/(-A*b**2 + C*a*b)) + (-(-B*b + D*a)/
(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4))*log(x + (B*a*b - D*a**2 -
 2*a*b**2*(-(-B*b + D*a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))
/(-A*b**2 + C*a*b))
```

3.91 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx$

Optimal. Leaf size=72

$$-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{Dx}{b}$$

```
[Out] D*x/b+A*ln(x)/a-1/2*(A*b-C*a)*ln(b*x^2+a)/a/b+(B*b-D*a)*arctan(x*b^(1/2)/a^
(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{Dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] (D*x)/b + ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (A*
Log[x])/a - ((A*b - a*C)*Log[a + b*x^2])/(2*a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)}\,dx = \int \left(\frac{D}{b}+\frac{A}{ax}+\frac{a(bB-aD)-b(Ab-aC)x}{ab\left(a+bx^2\right)}\right)dx$$
$$= \frac{Dx}{b}+\frac{A\log(x)}{a}+\frac{\int \frac{a(bB-aD)-b(Ab-aC)x}{a+bx^2}\,dx}{ab}$$
$$= \frac{Dx}{b}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\int \frac{x}{a+bx^2}\,dx}{a}+\frac{(bB-aD)\int \frac{1}{a+bx^2}\,dx}{b}$$
$$= \frac{Dx}{b}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 1.01

$$\frac{(aC-Ab)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}-\frac{(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{Dx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] (D*x)/b - ((-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) +
(A*Log[x])/a + ((-(A*b) + a*C)*Log[a + b*x^2])/(2*a*b)
```

fricas [A] time = 0.68, size = 158, normalized size = 2.19

$$\left[\frac{2\,Dabx+2\,Ab^2\log(x)-(Da-Bb)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+\left(Cab-Ab^2\right)\log\left(bx^2+a\right)}{2\,ab^2},\ \frac{2\,Dabx+2\,Ab^2\log(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*D*a*b*x + 2*A*b^2*log(x) - (D*a - B*b)*sqrt(-a*b)*log((b*x^2 + 2*sq
rt(-a*b)*x - a)/(b*x^2 + a)) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2), 1/2
*(2*D*a*b*x + 2*A*b^2*log(x) - 2*(D*a - B*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2)]
```

giac [A] time = 0.45, size = 66, normalized size = 0.92

$$\frac{Dx}{b}+\frac{A\log\left(|x|\right)}{a}-\frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{(Ca-Ab)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] D*x/b + A*log(abs(x))/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) +
 1/2*(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

maple [A] time = 0.01, size = 80, normalized size = 1.11

$$\frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{Da\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{A\ln(x)}{a}-\frac{A\ln\left(b\,x^2+a\right)}{2a}+\frac{C\ln\left(b\,x^2+a\right)}{2b}+\frac{Dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x)
```

```
[Out] D*x/b-1/2*A/a*ln(b*x^2+a)+1/2/b*ln(b*x^2+a)*C+1/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)*B-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D+A/a*ln(x)
```

maxima [A] time = 2.93, size = 65, normalized size = 0.90

$$\frac{Dx}{b} + \frac{A\log(x)}{a} - \frac{(Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b} + \frac{(Ca - Ab)\log\left(bx^2 + a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] D*x/b + A*log(x)/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*
(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,x + C\,x^2 + x^3\,D}{x\,\left(b\,x^2 + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.92 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=76

$$-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

```
[Out] -A/a/x+B*ln(x)/a-1/2*(B*b-D*a)*ln(b*x^2+a)/a/b-(A*b-C*a)*arctan(x*b^(1/2)/a
^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) - ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) +
(B*Log[x])/a - ((b*B - a*D)*Log[a + b*x^2])/(2*a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx = \int \left(\frac{A}{ax^2}+\frac{B}{ax}+\frac{-Ab+aC-(bB-aD)x}{a\left(a+bx^2\right)}\right)dx$$
$$= -\frac{A}{ax}+\frac{B\log(x)}{a}+\frac{\int \frac{-Ab+aC-(bB-aD)x}{a+bx^2}\,dx}{a}$$
$$= -\frac{A}{ax}+\frac{B\log(x)}{a}+\frac{(-Ab+aC)\int \frac{1}{a+bx^2}\,dx}{a}+\frac{(-bB+aD)\int \frac{x}{a+bx^2}\,dx}{a}$$
$$= -\frac{A}{ax}-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.99

$$\frac{(aC-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}+\frac{(aD-bB)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) + ((-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
 + (B*Log[x])/a + ((-(b*B) + a*D)*Log[a + b*x^2])/(2*a*b)
```

fricas [A] time = 0.82, size = 165, normalized size = 2.17

$$\left[\frac{2\,Babx\log(x)+(Ca-Ab)\sqrt{-ab}\,x\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,Aab+\left(Da^2-Bab\right)x\log\left(bx^2+a\right)}{2\,a^2bx}, \frac{2\,Babx\log(x)\ \cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b*x*log(x) + (C*a - A*b)*sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)
*x - a)/(b*x^2 + a)) - 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b*x
), 1/2*(2*B*a*b*x*log(x) + 2*(C*a - A*b)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a)
- 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b*x)]
```

giac [A] time = 0.38, size = 68, normalized size = 0.89

$$\frac{B\log\left(|x|\right)}{a}+\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,ab}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] B*log(abs(x))/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D*
a - B*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

maple [A] time = 0.01, size = 83, normalized size = 1.09

$$-\frac{Ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{C\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{B\ln(x)}{a}-\frac{B\ln\left(b\,x^2+a\right)}{2a}+\frac{D\ln\left(b\,x^2+a\right)}{2b}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x)
```

```
[Out] -1/2*B/a*ln(b*x^2+a)+1/2/b*ln(b*x^2+a)*D-1/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/
2)*b*x)*A*b+1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C-A/a/x+B/a*ln(x)
```

maxima [A] time = 3.00, size = 67, normalized size = 0.88

$$\frac{B\log(x)}{a} + \frac{(Ca - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} + \frac{(Da - Bb)\log\left(bx^2 + a\right)}{2\,ab} - \frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*log(x)/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D*a - B
*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

mupad [B] time = 1.21, size = 78, normalized size = 1.03

$$\frac{\ln\left(b\,x^2 + a\right)\,D}{2\,b} - \frac{A}{a\,x} - \frac{B\,\left(\ln\left(b\,x^2 + a\right) - 2\,\ln(x)\right)}{2\,a} - \frac{A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}} + \frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)),x)
```

```
[Out] (log(a + b*x^2)*D)/(2*b) - A/(a*x) - (B*(log(a + b*x^2) - 2*log(x)))/(2*a)
- (A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2) + (C*atan((b^(1/2)*x)/a^(1/
2)))/(a^(1/2)*b^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.93 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)} \, dx$

Optimal. Leaf size=92

$$-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B}{ax}$$

```
[Out] -1/2*A/a/x^2-B/a/x-(A*b-C*a)*ln(x)/a^2+1/2*(A*b-C*a)*ln(b*x^2+a)/a^2-(B*b-D
*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{2ax^2}-\frac{B}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] -A/(2*a*x^2) - B/(a*x) - ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)
*Sqrt[b]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2])/(2*a^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}\,dx = \int\left(\frac{A}{ax^3}+\frac{B}{ax^2}+\frac{-Ab+aC}{a^2x}+\frac{-a(bB-aD)+b(Ab-aC)x}{a^2\left(a+bx^2\right)}\right)dx$$

$$= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{\int\frac{-a(bB-aD)+b(Ab-aC)x}{a+bx^2}\,dx}{a^2}$$

$$= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(b(Ab-aC))\int\frac{x}{a+bx^2}\,dx}{a^2}-\frac{(bB-aD)\int\frac{1}{a+bx^2}\,dx}{a}$$

$$= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}$$

Mathematica [A] time = 0.08, size = 84, normalized size = 0.91

$$\frac{(Ab-aC)\log\left(a+bx^2\right)+2\log(x)(aC-Ab)-\frac{aA}{x^2}+\frac{2\sqrt{a}\,(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{2aB}{x}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (2*Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/Sqrt[b] + 2*(-(A*b) + a*C)*Log[x] + (A*b - a*C)*Log[a + b*x^2])/(2*
a^2)
```

fricas [A] time = 0.56, size = 205, normalized size = 2.23

$$\left[-\frac{(Da-Bb)\sqrt{-ab}\,x^2\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,Babx+\left(Cab-Ab^2\right)x^2\log\left(bx^2+a\right)-2\left(Cab-Ab^2\right)x^2\log(x)+Aa\ldots}{2\,a^2bx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*((D*a - B*b)*sqrt(-a*b)*x^2*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)) + 2*B*a*b*x + (C*a*b - A*b^2)*x^2*log(b*x^2 + a) - 2*(C*a*b - A*b^2)*x
^2*log(x) + A*a*b)/(a^2*b*x^2), 1/2*(2*(D*a - B*b)*sqrt(a*b)*x^2*arctan(sqr
t(a*b)*x/a) - 2*B*a*b*x - (C*a*b - A*b^2)*x^2*log(b*x^2 + a) + 2*(C*a*b - A
*b^2)*x^2*log(x) - A*a*b)/(a^2*b*x^2)]
```

giac [A] time = 0.39, size = 80, normalized size = 0.87

$$\frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{(Ca-Ab)\log\left(bx^2+a\right)}{2\,a^2}+\frac{(Ca-Ab)\log\left(|x|\right)}{a^2}-\frac{2\,Bax+Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^2
 + a)/a^2 + (C*a - A*b)*log(abs(x))/a^2 - 1/2*(2*B*a*x + A*a)/(a^2*x^2)
```

maple [A] time = 0.01, size = 102, normalized size = 1.11

$$-\frac{Bb\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{D\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{Ab\ln(x)}{a^2}+\frac{Ab\ln\left(b\,x^2+a\right)}{2a^2}+\frac{C\ln(x)}{a}-\frac{C\ln\left(b\,x^2+a\right)}{2a}-\frac{B}{ax}-\frac{A}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x)
```

```
[Out] 1/2*A/a^2*b*ln(b*x^2+a)-1/2/a*ln(b*x^2+a)*C-1/(a*b)^(1/2)*B/a*b*arctan(1/(a
*b)^(1/2)*b*x)+1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D-1/2*A/a/x^2-B/a/x-
A/a^2*b*ln(x)+1/a*ln(x)*C
```

maxima [A] time = 3.05, size = 76, normalized size = 0.83

$$\frac{(Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} - \frac{(Ca - Ab)\log\left(bx^2 + a\right)}{2\,a^2} + \frac{(Ca - Ab)\log(x)}{a^2} - \frac{2\,Bx + A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^2
 + a)/a^2 + (C*a - A*b)*log(x)/a^2 - 1/2*(2*B*x + A)/(a*x^2)
```

mupad [B] time = 1.30, size = 97, normalized size = 1.05

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)D}{\sqrt{a}\,\sqrt{b}} - \frac{B}{a\,x} - \frac{C\,\left(\ln\left(b\,x^2 + a\right) - 2\,\ln(x)\right)}{2\,a} - \frac{A}{2\,a\,x^2} + \frac{A\,b\,\ln\left(b\,x^2 + a\right)}{2\,a^2} - \frac{A\,b\,\ln(x)}{a^2} - \frac{B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*D)/(a^(1/2)*b^(1/2)) - B/(a*x) - (C*(log(a + b*x
^2) - 2*log(x)))/(2*a) - A/(2*a*x^2) + (A*b*log(a + b*x^2))/(2*a^2) - (A*b*
log(x))/a^2 - (B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.94 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=176

$$-\frac{\sqrt{a}\,(3Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(3Ab-5aC)}{2b^3}-\frac{x^3(3Ab-5aC)}{6ab^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}-\frac{a(2bB-3aD)\,\mathrm{lo}}{2b^4}$$

```
[Out] 1/2*(3*A*b-5*C*a)*x/b^3+1/2*(2*B*b-3*D*a)*x^2/b^3-1/6*(3*A*b-5*C*a)*x^3/a/b
^2+1/4*D*x^4/b^2-1/2*x^4*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)-1/2*a*(2*B
*b-3*D*a)*ln(b*x^2+a)/b^4-1/2*(3*A*b-5*C*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/
2)/b^(7/2)
```

Rubi [A] time = 0.27, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 1802, 635, 205, 260}

$$-\frac{x^3(3Ab-5aC)}{6ab^2}+\frac{x(3Ab-5aC)}{2b^3}-\frac{\sqrt{a}\,(3Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{x^2(2bB-3aD)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((3*A*b - 5*a*C)*x)/(2*b^3) + ((2*b*B - 3*a*D)*x^2)/(2*b^3) - ((3*A*b - 5*a
*C)*x^3)/(6*a*b^2) + (D*x^4)/(4*b^2) - (x^4*(a*(B - (a*D)/b) - (A*b - a*C)*
x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]*(3*A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(7/2)) - (a*(2*b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
```

```
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}dx = -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int \frac{x^3\left(-4a\left(B-\frac{aD}{b}\right)+(3Ab-5aC)x-2aDx^2\right)}{a+bx^2}dx}{2ab}$$

$$= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int\left(-\frac{a(3Ab-5aC)}{b^2}-\frac{2a(2bB-3aD)x}{b^2}+\frac{(3Ab-5aC)}{b}\right.}{2}$$

$$= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right.\right.}{2ab(}$$

$$= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right.\right.}{2ab(}$$

$$= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right.\right.}{2ab(}$$

Mathematica [A] time = 0.13, size = 139, normalized size = 0.79

$$\frac{\frac{6a\left(a^2D-ab(B+Cx)+Ab^2x\right)}{a+bx^2}+12bx(Ab-2aC)+6\sqrt{a}\sqrt{b}(5aC-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+6bx^2(bB-2aD)+6a(3aD-2b}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (12*b*(A*b - 2*a*C)*x + 6*b*(b*B - 2*a*D)*x^2 + 4*b^2*C*x^3 + 3*b^2*D*x^4 +
 (6*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2) + 6*Sqrt[a]*Sqrt[b]*(-
3*A*b + 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-2*b*B + 3*a*D)*Log[a + b
*x^2])/(12*b^4)
```

fricas [A] time = 0.58, size = 468, normalized size = 2.66

$$\left[\frac{3\,Db^3x^6+4\,Cb^3x^5-3\left(3\,Dab^2-2\,Bb^3\right)x^4+6\,Da^3-6\,Ba^2b-4\left(5\,Cab^2-3\,Ab^3\right)x^3-6\left(2\,Da^2b-Bab^2\right)x^2-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 + 6*D*a^3 -
6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^2)*x^2 - 3*(
5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(-a/b)*log((b*x^2 -
2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*a
^3 - 2*B*a^2*b + (3*D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*
b^4), 1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 + 6*D*a
^3 - 6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^2)*x^2
```

```
+ 6*(5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(a/b)*arctan(b*
x*sqrt(a/b)/a) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*a^3 - 2*B*a^2*b + (3*
D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)]
```

giac [A] time = 0.43, size = 159, normalized size = 0.90

$$\frac{\left(5\,Ca^2-3\,Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{\left(3\,Da^2-2\,Bab\right)\log\left(bx^2+a\right)}{2\,b^4}+\frac{Da^3-Ba^2b-\left(Ca^2b-Aab^2\right)x}{2\left(bx^2+a\right)b^4}+\frac{3\,Db^6x^4+4\,Cb^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*C*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(3*D*a^
2 - 2*B*a*b)*log(b*x^2 + a)/b^4 + 1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b^2
)*x)/((b*x^2 + a)*b^4) + 1/12*(3*D*b^6*x^4 + 4*C*b^6*x^3 - 12*D*a*b^5*x^2 +
 6*B*b^6*x^2 - 24*C*a*b^5*x + 12*A*b^6*x)/b^8
```

maple [A] time = 0.01, size = 201, normalized size = 1.14

$$\frac{Dx^4}{4b^2}+\frac{C\,x^3}{3b^2}+\frac{Aax}{2\left(b\,x^2+a\right)b^2}-\frac{3Aa\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2}+\frac{B\,x^2}{2b^2}-\frac{C\,a^2x}{2\left(b\,x^2+a\right)b^3}+\frac{5C\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^3}-\frac{Da\,x^2}{b^3}+\frac{Ax}{b^2}-\frac{B}{2\left(b\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/4*D*x^4/b^2+1/3/b^2*C*x^3+1/2*B/b^2*x^2-1/b^3*D*x^2*a+1/b^2*A*x-2/b^3*a*C
*x+1/2*a/b^2/(b*x^2+a)*A*x-1/2*a^2/b^3/(b*x^2+a)*C*x-1/2/(b*x^2+a)*B*a^2/b^
3+1/2*a^3/b^4/(b*x^2+a)*D-B*a/b^3*ln(b*x^2+a)+3/2*a^2/b^4*ln(b*x^2+a)*D-3/2
*a/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+5/2*a^2/b^3/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)*C
```

maxima [A] time = 2.83, size = 150, normalized size = 0.85

$$\frac{Da^3-Ba^2b-\left(Ca^2b-Aab^2\right)x}{2\left(b^5x^2+ab^4\right)}+\frac{\left(5\,Ca^2-3\,Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{3\,Dbx^4+4\,Cbx^3-6\,(2\,Da-Bb)x^2-12\,(2\,Ca-}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b^2)*x)/(b^5*x^2 + a*b^4) + 1/2*(5*C*
a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/12*(3*D*b*x^4 + 4*
C*b*x^3 - 6*(2*D*a - B*b)*x^2 - 12*(2*C*a - A*b)*x)/b^3 + 1/2*(3*D*a^2 - 2*
B*a*b)*log(b*x^2 + a)/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

sympy [B] time = 4.77, size = 335, normalized size = 1.90

$$\frac{Cx^3}{3b^2}+\frac{Dx^4}{4b^2}+x^2\left(\frac{B}{2b^2}-\frac{Da}{b^3}\right)+x\left(\frac{A}{b^2}-\frac{2Ca}{b^3}\right)+\left(\frac{a\left(-2Bb+3Da\right)}{2b^4}-\frac{\sqrt{-ab^9}\left(-3Ab+5Ca\right)}{4b^8}\right)\log\left(x+\frac{4Bab-6Da}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**3/(3*b**2) + D*x**4/(4*b**2) + x**2*(B/(2*b**2) - D*a/b**3) + x*(A/b**
2 - 2*C*a/b**3) + (a*(-2*B*b + 3*D*a)/(2*b**4) - sqrt(-a*b**9)*(-3*A*b + 5*
C*a)/(4*b**8))*log(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*
b**4) - sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) +
(a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8))*log
(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a*b*
*9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) + (-B*a**2*b + D*a**
3 + x*(A*a*b**2 - C*a**2*b))/(2*a*b**4 + 2*b**5*x**2)
```

3.95 $\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=154

$$\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}-\frac{x^2(Ab-2aC)}{2ab^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}-\frac{\sqrt{a}\,(3bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(3bB-}{2b}$$

```
[Out] 1/2*(3*B*b-5*D*a)*x/b^3-1/2*(A*b-2*C*a)*x^2/a/b^2+1/3*D*x^3/b^2-1/2*x^3*(a*
(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*(A*b-2*C*a)*ln(b*x^2+a)/b^3-1/2*(3
*B*b-5*D*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(7/2)
```

Rubi [A] time = 0.24, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 1802, 635, 205, 260}

$$-\frac{x^2(Ab-2aC)}{2ab^2}+\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{x(3bB-5aD)}{2b^3}-\frac{\sqrt{a}\,(3bB-5aD)\tan^{}}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((3*b*B - 5*a*D)*x)/(2*b^3) - ((A*b - 2*a*C)*x^2)/(2*a*b^2) + (D*x^3)/(3*b^
2) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]
*(3*b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*
Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
```

```
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}dx = -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int \frac{x^2\left(-3a\left(B-\frac{aD}{b}\right)+2(Ab-2aC)x-2aDx^2\right)}{a+bx^2}dx}{2ab}$$

$$= -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int\left(-\frac{a(3bB-5aD)}{b^2}+\frac{2(Ab-2aC)x}{b}-\frac{2aDx^2}{b}+\frac{a^2(}{}\right.}{2ab}$$

$$= \frac{(3bB-5aD)x}{2b^3} - \frac{(Ab-2aC)x^2}{2ab^2} + \frac{Dx^3}{3b^2} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} -$$

$$= \frac{(3bB-5aD)x}{2b^3} - \frac{(Ab-2aC)x^2}{2ab^2} + \frac{Dx^3}{3b^2} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} +$$

$$= \frac{(3bB-5aD)x}{2b^3} - \frac{(Ab-2aC)x^2}{2ab^2} + \frac{Dx^3}{3b^2} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} -$$

Mathematica [A] time = 0.08, size = 128, normalized size = 0.83

$$\frac{a(-a(C+Dx)+Ab+bBx)}{2b^3\left(a+bx^2\right)}+\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}+\frac{\sqrt{a}\,(5aD-3bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(bB-2aD)}{b^3}+\frac{Cx^2}{2b^2}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((b*B - 2*a*D)*x)/b^3 + (C*x^2)/(2*b^2) + (D*x^3)/(3*b^2) + (a*(A*b + b*B*x
 - a*(C + D*x)))/(2*b^3*(a + b*x^2)) + (Sqrt[a]*(-3*b*B + 5*a*D)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*Log[a + b*x^2])/(2*b^3)
```

fricas [A] time = 0.69, size = 372, normalized size = 2.42

$$\left[\frac{4\,Db^2x^5+6\,Cb^2x^4+6\,Cabx^2-4\left(5\,Dab-3\,Bb^2\right)x^3-6\,Ca^2+6\,Aab+3\left(5\,Da^2-3\,Bab+\left(5\,Dab-3\,Bb^2\right)x^2\right.}{12\left(b^4x^2+a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*D*b^2*x^5 + 6*C*b^2*x^4 + 6*C*a*b*x^2 - 4*(5*D*a*b - 3*B*b^2)*x^3
- 6*C*a^2 + 6*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*b^2)*x^2)*sqrt(
-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*D*a^2 - 3*B*a*
b)*x - 6*(2*C*a^2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2
 + a*b^3), 1/6*(2*D*b^2*x^5 + 3*C*b^2*x^4 + 3*C*a*b*x^2 - 2*(5*D*a*b - 3*B*
b^2)*x^3 - 3*C*a^2 + 3*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*b^2)*x
^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*(5*D*a^2 - 3*B*a*b)*x - 3*(2*C*a^
2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 0.38, size = 131, normalized size = 0.85

$$-\frac{(2\,Ca-Ab)\log\left(bx^{2}+a\right)}{2\,b^{3}}+\frac{\left(5\,Da^{2}-3\,Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{3}}-\frac{Ca^{2}-Aab+\left(Da^{2}-Bab\right)x}{2\left(bx^{2}+a\right)b^{3}}+\frac{2\,Db^{4}x^{3}+3\,Cb^{4}x^{2}-1}{6\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*a - A*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*x^
2 + a)*b^3) + 1/6*(2*D*b^4*x^3 + 3*C*b^4*x^2 - 12*D*a*b^3*x + 6*B*b^4*x)/b^
6
```

maple [A] time = 0.01, size = 177, normalized size = 1.15

$$\frac{Dx^{3}}{3b^{2}}+\frac{Bax}{2\left(b\,x^{2}+a\right)b^{2}}-\frac{3Ba\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{2}}+\frac{C\,x^{2}}{2b^{2}}-\frac{Da^{2}x}{2\left(b\,x^{2}+a\right)b^{3}}+\frac{5Da^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{3}}+\frac{Aa}{2\left(b\,x^{2}+a\right)b^{2}}+\frac{A\ln\left(b\,x^{2}\right.}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*D*x^3/b^2+1/2/b^2*C*x^2+1/b^2*B*x-2/b^3*a*D*x+1/2/b^2/(b*x^2+a)*B*x*a-1
/2/b^3/(b*x^2+a)*a^2*D*x+1/2/(b*x^2+a)*A*a/b^2-1/2/b^3/(b*x^2+a)*a^2*C+1/2*
A/b^2*ln(b*x^2+a)-1/b^3*ln(b*x^2+a)*a*C-3/2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)*a*B+5/2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^2*D
```

maxima [A] time = 3.00, size = 127, normalized size = 0.82

$$-\frac{Ca^{2}-Aab+\left(Da^{2}-Bab\right)x}{2\left(b^{4}x^{2}+ab^{3}\right)}-\frac{(2\,Ca-Ab)\log\left(bx^{2}+a\right)}{2\,b^{3}}+\frac{\left(5\,Da^{2}-3\,Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{3}}+\frac{2\,Dbx^{3}+3\,Cbx^{2}-6\,(}{6\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/(b^4*x^2 + a*b^3) - 1/2*(2*C*a - A
*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*b^3) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(2*D*a - B*b)*x)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3}\,\left(A+B\,x+C\,x^{2}+x^{3}\,D\right)}{{\left(b\,x^{2}+a\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

sympy [B] time = 3.87, size = 289, normalized size = 1.88

$$\frac{Cx^{2}}{2b^{2}}+\frac{Dx^{3}}{3b^{2}}+x\left(\frac{B}{b^{2}}-\frac{2Da}{b^{3}}\right)+\left(-\frac{-Ab+2Ca}{2b^{3}}-\frac{\sqrt{-ab^{7}}\,(-3Bb+5Da)}{4b^{7}}\right)\log\left(x+\frac{-2Ab+4Ca+4b^{3}\left(-\frac{-Ab+2Ca}{2b^{3}}-\right.}{-3Bb+5Da}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**2/(2*b**2) + D*x**3/(3*b**2) + x*(B/b**2 - 2*D*a/b**3) + (-(-A*b + 2*C
*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4
*C*a + 4*b**3*(-(-A*b + 2*C*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4
*b**7)))/(-3*B*b + 5*D*a)) + (-(-A*b + 2*C*a)/(2*b**3) + sqrt(-a*b**7)*(-3*
B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4*C*a + 4*b**3*(-(-A*b + 2*C*a)/(2
*b**3) + sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7)))/(-3*B*b + 5*D*a)) + (A*a
*b - C*a**2 + x*(B*a*b - D*a**2))/(2*a*b**3 + 2*b**4*x**2)
```

3.96 $\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=134

$$\frac{(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}-\frac{x(Ab-3aC)}{2ab^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-2aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Dx^2}{2b^2}$$

```
[Out] -1/2*(A*b-3*C*a)*x/a/b^2+1/2*D*x^2/b^2-1/2*x^2*(a*(B-a*D/b)-(A*b-C*a)*x)/a/
b/(b*x^2+a)+1/2*(B*b-2*D*a)*ln(b*x^2+a)/b^3+1/2*(A*b-3*C*a)*arctan(x*b^(1/2
)/a^(1/2))/b^(5/2)/a^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 1802, 635, 205, 260}

$$-\frac{x(Ab-3aC)}{2ab^2}+\frac{(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-2aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Dx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] -((A*b - 3*a*C)*x)/(2*a*b^2) + (D*x^2)/(2*b^2) - (x^2*(a*(B - (a*D)/b) - (A
*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*Sqrt[a]*b^(5/2)) + ((b*B - 2*a*D)*Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
```

```
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}dx = -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int \frac{x\left(-2a\left(B-\frac{aD}{b}\right)+(Ab-3aC)x-2aDx^2\right)}{a+bx^2}dx}{2ab}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int\left(A-\frac{3aC}{b}-\frac{2aDx}{b}-\frac{a(Ab-3aC)+2a(bB-2aD)}{b\left(a+bx^2\right)}\right.}{2ab}$$

$$= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{\int \frac{a(Ab-3aC)+2a(bB-}{a+bx^2}}{2ab^2}$$

$$= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(Ab-3aC)\int\frac{1}{a+b}}{2b^2}$$

$$= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(Ab-3aC)\tan^{-}}{2\sqrt{a}\,b^{5/2}}$$

Mathematica [A] time = 0.08, size = 100, normalized size = 0.75

$$\frac{\frac{a^2(-D)+ab(B+Cx)-Ab^2x}{a+bx^2}+\frac{\sqrt{b}\,(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-2aD)\log\left(a+bx^2\right)+2bCx+bDx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (2*b*C*x + b*D*x^2 + (-(a^2*D) - A*b^2*x + a*b*(B + C*x))/(a + b*x^2) + (Sq
rt[b]*(A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (b*B - 2*a*D)*Lo
g[a + b*x^2])/(2*b^3)
```

fricas [A] time = 0.62, size = 357, normalized size = 2.66

$$\left[\frac{2\,Dab^2x^4+4\,Cab^2x^3+2\,Da^2bx^2-2\,Da^3+2\,Ba^2b+\left(3\,Ca^2-Aab+\left(3\,Cab-Ab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}}{bx^2+a}\right.}{4\left(ab^4x^2+a^2b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*D*a*b^2*x^4 + 4*C*a*b^2*x^3 + 2*D*a^2*b*x^2 - 2*D*a^3 + 2*B*a^2*b +
 (3*C*a^2 - A*a*b + (3*C*a*b - A*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 2*(3*C*a^2*b - A*a*b^2)*x - 2*(2*D*a^3 - B*a^2*b
 + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), 1/2*(D
*a*b^2*x^4 + 2*C*a*b^2*x^3 + D*a^2*b*x^2 - D*a^3 + B*a^2*b - (3*C*a^2 - A*a
*b + (3*C*a*b - A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*C*a^2*b -
A*a*b^2)*x - (2*D*a^3 - B*a^2*b + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x^2 + a)
)/(a*b^4*x^2 + a^2*b^3)]
```

giac [A] time = 0.39, size = 111, normalized size = 0.83

$$-\frac{(3\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}-\frac{(2\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^3}+\frac{Db^2x^2+2\,Cb^2x}{2\,b^4}-\frac{Da^2-Bab-\left(Cab-Ab^2\right)x}{2\,\left(bx^2+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(2*D*a - B*b
)*log(b*x^2 + a)/b^3 + 1/2*(D*b^2*x^2 + 2*C*b^2*x)/b^4 - 1/2*(D*a^2 - B*a*b
 - (C*a*b - A*b^2)*x)/((b*x^2 + a)*b^3)
```

maple [A] time = 0.01, size = 154, normalized size = 1.15

$$-\frac{Ax}{2\left(b\,x^2+a\right)b}+\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\,b}+\frac{Cax}{2\left(b\,x^2+a\right)b^2}-\frac{3Ca\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\,b^2}+\frac{Dx^2}{2b^2}+\frac{Ba}{2\left(b\,x^2+a\right)b^2}+\frac{B\ln\left(b\,x^2+a\right)}{2b^2}+\frac{Cx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*D*x^2/b^2+1/b^2*C*x-1/2/b/(b*x^2+a)*A*x+1/2/b^2/(b*x^2+a)*a*C*x+1/2/b^2
/(b*x^2+a)*B*a-1/2/b^3/(b*x^2+a)*a^2*D+1/2/b^2*ln(b*x^2+a)*B-1/b^3*ln(b*x^2
+a)*a*D+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A-3/2/b^2/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x)*a*C
```

maxima [A] time = 2.96, size = 108, normalized size = 0.81

$$-\frac{Da^2-Bab-\left(Cab-Ab^2\right)x}{2\left(b^4x^2+ab^3\right)}-\frac{(3\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{Dx^2+2\,Cx}{2\,b^2}-\frac{(2\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(b^4*x^2 + a*b^3) - 1/2*(3*C*a - A
*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*x^2 + 2*C*x)/b^2 - 1/2*(
2*D*a - B*b)*log(b*x^2 + a)/b^3
```

mupad [B] time = 1.29, size = 152, normalized size = 1.13

$$\frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{x^2\,D}{2\,b^2}+\frac{C\,x}{b^2}-\frac{a^2\,D}{2\,b^3\,\left(b\,x^2+a\right)}+\frac{B\,a}{2\,b^2\,\left(b\,x^2+a\right)}-\frac{A\,x}{2\,b\,\left(b\,x^2+a\right)}+\frac{C\,a\,x}{2\,\left(b^3\,x^2+a\,b^2\right)}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] (B*log(a + b*x^2))/(2*b^2) + (x^2*D)/(2*b^2) + (C*x)/b^2 - (a^2*D)/(2*b^3*(
a + b*x^2)) + (B*a)/(2*b^2*(a + b*x^2)) - (A*x)/(2*b*(a + b*x^2)) + (C*a*x)
/(2*(a*b^2 + b^3*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
- (3*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(5/2)) - (a*log(a + b*x^2)*D
)/b^3
```

sympy [B] time = 4.61, size = 284, normalized size = 2.12

$$\frac{Cx}{b^2}+\frac{Dx^2}{2b^2}+\left(-\frac{-Bb+2Da}{2b^3}-\frac{\sqrt{-ab^7}\,\left(-Ab+3Ca\right)}{4ab^6}\right)\log\left(x+\frac{2Bab-4Da^2-4ab^3\left(-\frac{-Bb+2Da}{2b^3}-\frac{\sqrt{-ab^7}\,(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2+3Cab}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x/b**2 + D*x**2/(2*b**2) + (-(-B*b + 2*D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*
b + 3*C*a)/(4*a*b**6))*log(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D
*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b
)) + (-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6))*l
og(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b
**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b)) + (B*a*b - D*a**2 + x
*(-A*b**2 + C*a*b))/(2*a*b**3 + 2*b**4*x**2)
```

3.97 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{Dx}{b^2}$$

```
[Out] D*x/b^2-1/2*x*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*C*ln(b*x^2+a)/b^2
+1/2*(B*b-3*D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(5/2)/a^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1804, 1810, 635, 205, 260}

$$-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{Dx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 - (x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((b
*B - 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b
*x^2])/(2*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int \frac{-a\left(B-\frac{aD}{b}\right)-2aCx-2aDx^2}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int\left(-\frac{2aD}{b}-\frac{a(bB-3aD)+2abCx}{b\left(a+bx^2\right)}\right)dx}{2ab}$$

$$= \frac{Dx}{b^2} - \frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} + \frac{\int \frac{a(bB-3aD)+2abCx}{a+bx^2}\,dx}{2ab^2}$$

$$= \frac{Dx}{b^2} - \frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} + \frac{C\int \frac{x}{a+bx^2}\,dx}{b} + \frac{(bB-3aD)\int \frac{1}{a+bx^2}\,dx}{2b^2}$$

$$= \frac{Dx}{b^2} - \frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} + \frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}} + \frac{C\log\left(a+\right.}{2b^2}$$

Mathematica [A] time = 0.05, size = 92, normalized size = 0.91

$$\frac{aC+aDx-Ab-bBx}{2b^2\left(a+bx^2\right)} - \frac{(3aD-bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}} + \frac{C\log\left(a+bx^2\right)}{2b^2} + \frac{Dx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 + (-(A*b) + a*C - b*B*x + a*D*x)/(2*b^2*(a + b*x^2)) - ((-(b*B) +
 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b*x^2
])/(2*b^2)
```

fricas [A] time = 0.60, size = 287, normalized size = 2.84

$$\left[\frac{4\,Dab^2x^3+2\,Ca^2b-2\,Aab^2-\left(3\,Da^2-Bab+\left(3\,Dab-Bb^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(3\,Da^2b-Bab\right.}{4\left(ab^4x^2+a^2b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*D*a*b^2*x^3 + 2*C*a^2*b - 2*A*a*b^2 - (3*D*a^2 - B*a*b + (3*D*a*b -
 B*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(
3*D*a^2*b - B*a*b^2)*x + 2*(C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^4*x
^2 + a^2*b^3), 1/2*(2*D*a*b^2*x^3 + C*a^2*b - A*a*b^2 - (3*D*a^2 - B*a*b +
(3*D*a*b - B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^2*b - B*a*b
^2)*x + (C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

giac [A] time = 0.43, size = 81, normalized size = 0.80

$$\frac{Dx}{b^2} + \frac{C\log\left(bx^2+a\right)}{2\,b^2} - \frac{(3\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2} + \frac{Ca-Ab+(Da-Bb)x}{2\left(bx^2+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] D*x/b^2 + 1/2*C*log(b*x^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^2) + 1/2*(C*a - A*b + (D*a - B*b)*x)/((b*x^2 + a)*b^2)
```

maple [A] time = 0.01, size = 127, normalized size = 1.26

$$-\frac{Bx}{2\left(b\,x^{2}+a\right)b}+\frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}+\frac{Dax}{2\left(b\,x^{2}+a\right)b^{2}}-\frac{3Da\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{2}}-\frac{A}{2\left(b\,x^{2}+a\right)b}+\frac{Ca}{2\left(b\,x^{2}+a\right)b^{2}}+\frac{C\ln\left(b\,x^{2}\right.}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] D*x/b^2-1/2/b/(b*x^2+a)*B*x+1/2/b^2/(b*x^2+a)*a*D*x-1/2/b/(b*x^2+a)*A+1/2/b
^2/(b*x^2+a)*a*C+1/2*C*ln(b*x^2+a)/b^2+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/
2)*b*x)*B-3/2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*D
```

maxima [A] time = 3.02, size = 84, normalized size = 0.83

$$\frac{Ca-Ab+(Da-Bb)x}{2\left(b^{3}x^{2}+ab^{2}\right)}+\frac{Dx}{b^{2}}+\frac{C\log\left(bx^{2}+a\right)}{2\,b^{2}}-\frac{(3\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(C*a - A*b + (D*a - B*b)*x)/(b^3*x^2 + a*b^2) + D*x/b^2 + 1/2*C*log(b*x
^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(A+B\,x+C\,x^{2}+x^{3}\,D\right)}{\left(b\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

sympy [B] time = 5.76, size = 212, normalized size = 2.10

$$\frac{Dx}{b^{2}}+\left(\frac{C}{2b^{2}}-\frac{\sqrt{-ab^{5}}\,(-Bb+3Da)}{4ab^{5}}\right)\log\left(x+\frac{2Ca-4ab^{2}\left(\frac{C}{2b^{2}}-\frac{\sqrt{-ab^{5}}\,(-Bb+3Da)}{4ab^{5}}\right)}{-Bb+3Da}\right)+\left(\frac{C}{2b^{2}}+\frac{\sqrt{-ab^{5}}\,(-Bb+3Da)}{4ab^{5}}\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] D*x/b**2 + (C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(x + (
2*C*a - 4*a*b**2*(C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)))/(-
B*b + 3*D*a)) + (C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(
x + (2*C*a - 4*a*b**2*(C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)
))/(-B*b + 3*D*a)) + (-A*b + C*a + x*(-B*b + D*a))/(2*a*b**2 + 2*b**3*x**2)
```

3.98 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=93

$$\frac{(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(Ab-aC)-a\left(B-\frac{aD}{b}\right)}{2ab\left(a+bx^2\right)}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] 1/2*(-a*(B-a*D/b)+(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*(A*b+C*a)*arctan(x*b^(1/2)
/a^(1/2))/a^(3/2)/b^(3/2)+1/2*D*ln(b*x^2+a)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1814, 635, 205, 260}

$$\frac{(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*C)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (D*Log[a + b*x^2])/(2*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx = -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\left(a+bx^2\right)} - \frac{\int \frac{-\frac{Ab+aC}{b}-\frac{2aDx}{b}}{a+bx^2}\,dx}{2a}$$

$$= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\left(a+bx^2\right)} + \frac{(Ab+aC)\int \frac{1}{a+bx^2}\,dx}{2ab} + \frac{D\int \frac{x}{a+bx^2}\,dx}{b}$$

$$= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\left(a+bx^2\right)} + \frac{(Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} + \frac{D\log\left(a+bx^2\right)}{2b^2}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 0.89

$$\frac{\frac{\sqrt{b}\,(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}} + \frac{a^2D-ab(B+Cx)+Ab^2x}{a\left(a+bx^2\right)} + D\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] ((a^2*D + A*b^2*x - a*b*(B + C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + a*C)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + D*Log[a + b*x^2])/(2*b^2)
```

fricas [A] time = 0.69, size = 257, normalized size = 2.76

$$\left[\frac{2\,Da^3 - 2\,Ba^2b - \left(Ca^2 + Aab + \left(Cab + Ab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right) - 2\left(Ca^2b - Aab^2\right)x + 2\left(Da^2bx^2 + D\right.}{4\left(a^2b^3x^2 + a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*D*a^3 - 2*B*a^2*b - (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(C*a^2*b - A*a*b^2)*x +
 2*(D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), 1/2*(D*a^
3 - B*a^2*b + (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a
*b)*x/a) - (C*a^2*b - A*a*b^2)*x + (D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a
^2*b^3*x^2 + a^3*b^2)]
```

giac [A] time = 0.38, size = 88, normalized size = 0.95

$$\frac{D\log\left(bx^2+a\right)}{2\,b^2} + \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab} - \frac{(Ca-Ab)x - \frac{Da^2-Bab}{b}}{2\left(bx^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*D*log(b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a*b) - 1/2*((C*a - A*b)*x - (D*a^2 - B*a*b)/b)/((b*x^2 + a)*a*b)
```

maple [A] time = 0.01, size = 97, normalized size = 1.04

$$\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a} + \frac{C\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b} + \frac{D\ln\left(b\,x^2+a\right)}{2b^2} + \frac{\frac{(Ab-aC)x}{2ab} - \frac{bB-aD}{2b^2}}{b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] (1/2*(A*b-C*a)/a/b*x-1/2*(B*b-D*a)/b^2)/(b*x^2+a)+1/2*D*ln(b*x^2+a)/b^2+1/2
/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)
^(1/2)*b*x)*C
```

maxima [A] time = 2.94, size = 89, normalized size = 0.96

$$\frac{Da^2 - Bab - \left(Cab - Ab^2\right)x}{2\left(ab^3x^2 + a^2b^2\right)} + \frac{D\log\left(bx^2 + a\right)}{2\,b^2} + \frac{(Ca + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(a*b^3*x^2 + a^2*b^2) + 1/2*D*log(b
*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

mupad [B] time = 1.32, size = 110, normalized size = 1.18

$$\frac{\left(\ln\left(b\,x^2+a\right)+\frac{a}{b\,x^2+a}\right)D}{2\,b^2}-\frac{B}{2\,b\,\left(b\,x^2+a\right)}+\frac{A\,x}{2\,a\,\left(b\,x^2+a\right)}-\frac{C\,x}{2\,b\,\left(b\,x^2+a\right)}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^2,x)
```

```
[Out] ((log(a + b*x^2) + a/(a + b*x^2))*D)/(2*b^2) - B/(2*b*(a + b*x^2)) + (A*x)/
(2*a*(a + b*x^2)) - (C*x)/(2*b*(a + b*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)))
/(2*a^(3/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
```

sympy [B] time = 3.09, size = 233, normalized size = 2.51

$$\left(\frac{D}{2b^2} - \frac{\sqrt{-a^3b^5}\,(Ab + Ca)}{4a^3b^4}\right)\log\left(x + \frac{-2Da^2 + 4a^2b^2\left(\frac{D}{2b^2} - \frac{\sqrt{-a^3b^5}\,(Ab+Ca)}{4a^3b^4}\right)}{Ab^2 + Cab}\right) + \left(\frac{D}{2b^2} + \frac{\sqrt{-a^3b^5}\,(Ab + Ca)}{4a^3b^4}\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] (D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))*log(x + (-2*D*a**
2 + 4*a**2*b**2*(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4)))/
(A*b**2 + C*a*b)) + (D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4
))*log(x + (-2*D*a**2 + 4*a**2*b**2*(D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C
*a)/(4*a**3*b**4)))/(A*b**2 + C*a*b)) + (-B*a*b + D*a**2 + x*(A*b**2 - C*a*
b))/(2*a**2*b**2 + 2*a*b**3*x**2)
```

3.99 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2} dx$

Optimal. Leaf size=95

$$\frac{(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

```
[Out] 1/2*(A*b-a*C+(B*b-D*a)*x)/a/b/(b*x^2+a)+1/2*(B*b+D*a)*arctan(x*b^(1/2)/a^(1
/2))/a^(3/2)/b^(3/2)+A*ln(x)/a^2-1/2*A*ln(b*x^2+a)/a^2
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1805, 801, 635, 205, 260}

$$-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(2*a*b*(a + b*x^2)) + ((b*B + a*D)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2])/
(2*a^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx=\frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-2A-\frac{(bB+aD)x}{b}}{x\left(a+bx^2\right)}\,dx}{2a}$$

$$=\frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}-\frac{\int \left(-\frac{2A}{ax}+\frac{-abB-a^2D+2Ab^2x}{ab\left(a+bx^2\right)}\right)dx}{2a}$$

$$=\frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{\int \frac{-abB-a^2D+2Ab^2x}{a+bx^2}\,dx}{2a^2b}$$

$$=\frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{(Ab)\int \frac{x}{a+bx^2}\,dx}{a^2}+\frac{(bB+aD)\int \frac{1}{a+bx^2}\,dx}{2ab}$$

$$=\frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{(bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^2\right)}{2a^2}$$

Mathematica [A] time = 0.08, size = 85, normalized size = 0.89

$$\frac{\frac{a(-a(C+Dx)+Ab+bBx)}{b\left(a+bx^2\right)}-A\log\left(a+bx^2\right)+\frac{\sqrt{a}\,(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}+2A\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(A*b + b*B*x - a*(C + D*x)))/(b*(a + b*x^2)) + (Sqrt[a]*(b*B + a*D)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + 2*A*Log[x] - A*Log[a + b*x^2])/(2*a^2)
```

fricas [A] time = 0.76, size = 296, normalized size = 3.12

$$\left[-\frac{2\,Ca^2b-2\,Aab^2+\left(Da^2+Bab+\left(Dab+Bb^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(Da^2b-Bab^2\right)x+2\left(Ab^3x^2\right.}{4\left(a^2b^3x^2+a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*C*a^2*b - 2*A*a*b^2 + (D*a^2 + B*a*b + (D*a*b + B*b^2)*x^2)*sqrt(-
a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(D*a^2*b - B*a*b^2)*
x + 2*(A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 4*(A*b^3*x^2 + A*a*b^2)*log(x)
)/(a^2*b^3*x^2 + a^3*b^2), -1/2*(C*a^2*b - A*a*b^2 - (D*a^2 + B*a*b + (D*a*
b + B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^2*b - B*a*b^2)*x + (
A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 2*(A*b^3*x^2 + A*a*b^2)*log(x))/(a^2*
b^3*x^2 + a^3*b^2)]
```

giac [A] time = 0.35, size = 93, normalized size = 0.98

$$-\frac{A\log\left(bx^2+a\right)}{2\,a^2}+\frac{A\log\left(|x|\right)}{a^2}+\frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab}-\frac{Ca^2-Aab+\left(Da^2-Bab\right)x}{2\left(bx^2+a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*A*log(b*x^2 + a)/a^2 + A*log(abs(x))/a^2 + 1/2*(D*a + B*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*x^
2 + a)*a^2*b)
```

maple [A] time = 0.01, size = 125, normalized size = 1.32

$$\frac{Bx}{2\left(b\,x^2+a\right)a}+\frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{Dx}{2\left(b\,x^2+a\right)b}+\frac{D\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}+\frac{A}{2\left(b\,x^2+a\right)a}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^2+a\right)}{2a^2}-\frac{\cdots}{2\left(b\,x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a/(b*x^2+a)*B*x-1/2/(b*x^2+a)/b*x*D+1/2/a/(b*x^2+a)*A-1/2/(b*x^2+a)/b*C
-1/2*A*ln(b*x^2+a)/a^2+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B+1/2/b/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D+A*ln(x)/a^2
```

maxima [A] time = 2.91, size = 87, normalized size = 0.92

$$-\frac{Ca-Ab+(Da-Bb)x}{2\left(ab^2x^2+a^2b\right)}-\frac{A\log\left(bx^2+a\right)}{2\,a^2}+\frac{A\log(x)}{a^2}+\frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*a - A*b + (D*a - B*b)*x)/(a*b^2*x^2 + a^2*b) - 1/2*A*log(b*x^2 + a)
/a^2 + A*log(x)/a^2 + 1/2*(D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{A+B\,x+C\,x^2+x^3\,D}{x\,{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.100 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}-\frac{B\log\left(a+bx^2\right)}{2a^2}+\frac{B\log(x)}{a^2}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

```
[Out] -A/a^2/x+1/2*(b*B-a*D-b*(A*b/a-C)*x)/a/b/(b*x^2+a)+B*ln(x)/a^2-1/2*B*ln(b*x
^2+a)/a^2-1/2*(3*A*b-C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$-\frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}-\frac{B\log\left(a+bx^2\right)}{2a^2}+\frac{B\log(x)}{a^2}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(2*a*b*(a + b*x^2)) - ((3*A*
b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log[x])/a^2
- (B*Log[a + b*x^2])/(2*a^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx = \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)} - \frac{\int \frac{-2A-2Bx+\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{2a}$$

$$= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)} - \frac{\int \left(-\frac{2A}{ax^2}-\frac{2B}{ax}+\frac{3Ab-aC+2bBx}{a\left(a+bx^2\right)}\right)dx}{2a}$$

$$= -\frac{A}{a^2x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)} + \frac{B\log(x)}{a^2} - \frac{\int \frac{3Ab-aC+2bBx}{a+bx^2}\,dx}{2a^2}$$

$$= -\frac{A}{a^2x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)} + \frac{B\log(x)}{a^2} - \frac{(bB)\int \frac{x}{a+bx^2}\,dx}{a^2} - \frac{(3Ab-aC)\int \frac{1}{a+b}$$

$$= -\frac{A}{a^2x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)} - \frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} + \frac{B\log(x)}{a^2} - \frac{B\log\left(a\right.}{2a}$$

Mathematica [A] time = 0.07, size = 110, normalized size = 1.00

$$\frac{(aC-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} + \frac{a^2(-D)+abB+abCx-Ab^2x}{2a^2b\left(a+bx^2\right)} - \frac{A}{a^2x} - \frac{B\log\left(a+bx^2\right)}{2a^2} + \frac{B\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(2*a^2*b*(a + b*x^2)) +
((-3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log[x
])/a^2 - (B*Log[a + b*x^2])/(2*a^2)
```

fricas [A] time = 0.68, size = 336, normalized size = 3.05

$$\left[-\frac{4\,Aa^2b-2\left(Ca^2b-3\,Aab^2\right)x^2-\left(\left(Cab-3\,Ab^2\right)x^3+\left(Ca^2-3\,Aab\right)x\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(Da^3-Ba\right.}{4\left(a^3b^2x^3+a^4bx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*A*a^2*b - 2*(C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 + (
C*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) + 2*(D*a^3 - B*a^2*b)*x + 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(b*x^2 + a) - 4*
(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*A*a^2*b
- (C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 + (C*a^2 - 3*A*a*b)*x)
*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^3 - B*a^2*b)*x + (B*a*b^2*x^3 + B*a
^2*b*x)*log(b*x^2 + a) - 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^2*x^3 +
 a^4*b*x)]
```

giac [A] time = 0.34, size = 103, normalized size = 0.94

$$-\frac{B\log\left(bx^2+a\right)}{2\,a^2}+\frac{B\log\left(|x|\right)}{a^2}+\frac{(Ca-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}+\frac{Cabx^2-3\,Ab^2x^2-Da^2x+Babx-2\,Aab}{2\left(bx^3+ax\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*B*log(b*x^2 + a)/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a - 3*A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(C*a*b*x^2 - 3*A*b^2*x^2 - D*a^2*x + B*a
*b*x - 2*A*a*b)/((b*x^3 + a*x)*a^2*b)
```

maple [A] time = 0.01, size = 136, normalized size = 1.24

$$-\frac{Abx}{2\left(b\,x^2+a\right)a^2}-\frac{3Ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^2}+\frac{Cx}{2\left(b\,x^2+a\right)a}+\frac{C\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}+\frac{B}{2\left(b\,x^2+a\right)a}+\frac{B\ln(x)}{a^2}-\frac{B\ln\left(b\,x^2+a\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2/(b*x^2+a)*A*b*x+1/2/a/(b*x^2+a)*C*x+1/2/a/(b*x^2+a)*B-1/2/(b*x^2+a
)/b*D-1/2*B/a^2*ln(b*x^2+a)-3/2/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A
*b+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C-A/a^2/x+B/a^2*ln(x)
```

maxima [A] time = 2.98, size = 105, normalized size = 0.95

$$-\frac{2\,Aab-\left(Cab-3\,Ab^2\right)x^2+\left(Da^2-Bab\right)x}{2\left(a^2b^2x^3+a^3bx\right)}-\frac{B\log\left(bx^2+a\right)}{2\,a^2}+\frac{B\log(x)}{a^2}+\frac{(Ca-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*A*a*b - (C*a*b - 3*A*b^2)*x^2 + (D*a^2 - B*a*b)*x)/(a^2*b^2*x^3 + a
^3*b*x) - 1/2*B*log(b*x^2 + a)/a^2 + B*log(x)/a^2 + 1/2*(C*a - 3*A*b)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

mupad [B] time = 1.41, size = 133, normalized size = 1.21

$$\frac{B}{2\,a\,\left(b\,x^2+a\right)}-\frac{\frac{A}{a}+\frac{3\,A\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{B\,\ln\left(b\,x^2+a\right)}{2\,a^2}+\frac{B\,\ln(x)}{a^2}-\frac{D}{2\,b\,\left(b\,x^2+a\right)}+\frac{C\,x}{2\,a\,\left(b\,x^2+a\right)}-\frac{3\,A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}+\frac{C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] B/(2*a*(a + b*x^2)) - (A/a + (3*A*b*x^2)/(2*a^2))/(a*x + b*x^3) - (B*log(a
+ b*x^2))/(2*a^2) + (B*log(x))/a^2 - D/(2*b*(a + b*x^2)) + (C*x)/(2*a*(a +
b*x^2)) - (3*A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(5/2)) + (C*atan((b^
(1/2)*x)/a^(1/2)))/(2*a^(3/2)*b^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.101 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=135

$$-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

```
[Out] -1/2*A/a^2/x^2-B/a^2/x+1/2*(-A*b/a+C-(b*B/a-D)*x)/a/(b*x^2+a)-(2*A*b-C*a)*l
n(x)/a^3+1/2*(2*A*b-C*a)*ln(b*x^2+a)/a^3-1/2*(3*B*b-D*a)*arctan(x*b^(1/2)/a
^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{A}{2a^2x^2}-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -A/(2*a^2*x^2) - B/(a^2*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(2*a*(a + b*x^
2)) - ((3*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) - ((2
*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}dx = -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)} - \frac{\int \frac{-2A-2Bx+2\left(\frac{Ab}{a}-C\right)x^2+\left(\frac{bB}{a}-D\right)x^3}{x^3\left(a+bx^2\right)}dx}{2a}$$

$$= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)} - \frac{\int \left(-\frac{2A}{ax^3}-\frac{2B}{ax^2}-\frac{2(-2Ab+aC)}{a^2x}+\frac{a(3bB-aD)-2b(2Ab-aC)x}{a^2\left(a+bx^2\right)}\right) dx}{2a}$$

$$= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(2Ab-aC)\log(x)}{a^3}-\frac{\int \frac{a(3bB-aD)-2b(2Ab}{a+bx^2}}{2a^3}$$

$$= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(2Ab-aC)\log(x)}{a^3}+\frac{(b(2Ab-aC))\int}{a^3}$$

$$= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{(2Ab-aC)}{a^3}$$

Mathematica [A] time = 0.10, size = 112, normalized size = 0.83

$$\frac{\frac{a(a(C+Dx)-Ab-bBx)}{a+bx^2}+(2Ab-aC)\log\left(a+bx^2\right)+2\log(x)(aC-2Ab)-\frac{aA}{x^2}+\frac{\sqrt{a}(aD-3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{2aB}{x}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (a*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)
+ (Sqrt[a]*(-3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 2*(-2*A*b
+ a*C)*Log[x] + (2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

fricas [A] time = 0.82, size = 441, normalized size = 3.27

$$\left[-\frac{4Ba^2bx+2Aa^2b-2\left(Da^2b-3Bab^2\right)x^3-2\left(Ca^2b-2Aab^2\right)x^2+\left(\left(Dab-3Bb^2\right)x^4+\left(Da^2-3Bab\right)x^2\right)\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*B*a^2*b*x + 2*A*a^2*b - 2*(D*a^2*b - 3*B*a*b^2)*x^3 - 2*(C*a^2*b -
 2*A*a*b^2)*x^2 + ((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*((C*a*b^2 - 2*A*b^3)*x^
4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*x^2 + a) - 4*((C*a*b^2 - 2*A*b^3)*x^4
+ (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))/(a^3*b^2*x^4 + a^4*b*x^2), -1/2*(2*B*a
^2*b*x + A*a^2*b - (D*a^2*b - 3*B*a*b^2)*x^3 - (C*a^2*b - 2*A*a*b^2)*x^2 -
((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a) + ((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*x^2 + a)
 - 2*((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))/(a^3*b^2
*x^4 + a^4*b*x^2)]
```

giac [A] time = 0.42, size = 126, normalized size = 0.93

$$\frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{(Ca-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3}+\frac{(Ca-2\,Ab)\log\left(|x|\right)}{a^3}-\frac{2\,Ba^2x-\left(Da^2-3\,Bab\right)x^3+Aa^2-}{2\left(bx^2+a\right)a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*a - 2*A*b)
*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(abs(x))/a^3 - 1/2*(2*B*a^2*x - (D*a
^2 - 3*B*a*b)*x^3 + A*a^2 - (C*a^2 - 2*A*a*b)*x^2)/((b*x^2 + a)*a^3*x^2)
```

maple [A] time = 0.02, size = 169, normalized size = 1.25

$$-\frac{Bbx}{2\left(b\,x^2+a\right)a^2}-\frac{3Bb\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^2}+\frac{Dx}{2\left(b\,x^2+a\right)a}+\frac{D\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{Ab}{2\left(b\,x^2+a\right)a^2}-\frac{2Ab\ln(x)}{a^3}+\frac{Ab\ln\left(b\,x^2+\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2/(b*x^2+a)*B/a^2*b*x+1/2/a/(b*x^2+a)*D*x-1/2/a^2/(b*x^2+a)*A*b+1/2/a/(b
*x^2+a)*C+1/a^3*b*ln(b*x^2+a)*A-1/2/a^2*ln(b*x^2+a)*C-3/2/(a*b)^(1/2)*B/a^2
*b*arctan(1/(a*b)^(1/2)*b*x)+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D-
1/2*A/a^2/x^2-B/a^2/x-2/a^3*ln(x)*A*b+1/a^2*ln(x)*C
```

maxima [A] time = 2.94, size = 117, normalized size = 0.87

$$\frac{(Da-3\,Bb)x^3-2\,Bax+(Ca-2\,Ab)x^2-Aa}{2\left(a^2bx^4+a^3x^2\right)}+\frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{(Ca-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3}+\frac{(Ca-2\,A}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((D*a - 3*B*b)*x^3 - 2*B*a*x + (C*a - 2*A*b)*x^2 - A*a)/(a^2*b*x^4 + a^
3*x^2) + 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*a
 - 2*A*b)*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(x)/a^3
```

mupad [B] time = 1.35, size = 158, normalized size = 1.17

$$\frac{C}{2\,a\,\left(b\,x^2+a\right)}-\frac{\frac{A}{2\,a}+\frac{A\,b\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{\frac{B}{a}+\frac{3\,B\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{C\,\ln\left(b\,x^2+a\right)}{2\,a^2}+\frac{C\,\ln(x)}{a^2}+\frac{A\,b\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{2\,A\,b\,\ln(x)}{a^3}+\frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] C/(2*a*(a + b*x^2)) - (A/(2*a) + (A*b*x^2)/a^2)/(a*x^2 + b*x^4) - (B/a + (3
*B*b*x^2)/(2*a^2))/(a*x + b*x^3) - (C*log(a + b*x^2))/(2*a^2) + (C*log(x))/
a^2 + (A*b*log(a + b*x^2))/a^3 - (2*A*b*log(x))/a^3 + (x*D*hypergeom([1/2,
2], 3/2, -(b*x^2)/a))/a^2 - (3*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(5
/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.102 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$

Optimal. Leaf size=185

$$\frac{3(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}-\frac{3x(Ab-5aC)}{8ab^3}+\frac{x^3(4x(bB-2aD)-5aC+Ab)}{8ab^2\left(a+bx^2\right)}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{(bB-\cdots}{\cdots}$$

```
[Out] -3/8*(A*b-5*C*a)*x/a/b^3-1/2*(B*b-3*D*a)*x^2/a/b^3-1/4*x^4*(a*(B-a*D/b)-(A*
b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*x^3*(A*b-5*a*C+4*(B*b-2*D*a)*x)/a/b^2/(b*x^2+
a)+1/2*(B*b-3*D*a)*ln(b*x^2+a)/b^4+3/8*(A*b-5*C*a)*arctan(x*b^(1/2)/a^(1/2)
)/b^(7/2)/a^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 801, 635, 205, 260}

$$\frac{x^3(4x(bB-2aD)-5aC+Ab)}{8ab^2\left(a+bx^2\right)}-\frac{3x(Ab-5aC)}{8ab^3}+\frac{3(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(bB\cdots}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(A*b - 5*a*C)*x)/(8*a*b^3) - ((b*B - 3*a*D)*x^2)/(2*a*b^3) - (x^4*(a*(B
 - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) + (x^3*(A*b - 5*a*C + 4
*(b*B - 2*a*D)*x))/(8*a*b^2*(a + b*x^2)) + (3*(A*b - 5*a*C)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + ((b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
```

```
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{x^3\left(-4a\left(B-\frac{aD}{b}\right)+(Ab-5aC)x-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int \frac{x^2(-3}{}}{}$$

$$= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int\left(-\frac{3a}{}\right.}{}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab}{}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab}{}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab}{}$$

Mathematica [A] time = 0.12, size = 139, normalized size = 0.75

$$\frac{\frac{2a\left(a^2D-ab(B+Cx)+Ab^2x\right)}{\left(a+bx^2\right)^2}+\frac{-12a^2D+8abB+9abCx-5Ab^2x}{a+bx^2}+\frac{3\sqrt{b}(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+4(bB-3aD)\log\left(a+bx^2\right)+8bCx+}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (8*b*C*x + 4*b*D*x^2 + (8*a*b*B - 12*a^2*D - 5*A*b^2*x + 9*a*b*C*x)/(a + b*
x^2) + (2*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2)^2 + (3*Sqrt[b]*(
A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + 4*(b*B - 3*a*D)*Log[a +
 b*x^2])/(8*b^4)
```

fricas [A] time = 0.76, size = 574, normalized size = 3.10

$$\left[\frac{8\,Dab^3x^6+16\,Cab^3x^5+16\,Da^2b^2x^4-20\,Da^4+12\,Ba^3b+10\left(5\,Ca^2b^2-Aab^3\right)x^3-16\left(Da^3b-Ba^2b^2\right)x^2+3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*D*a*b^3*x^6 + 16*C*a*b^3*x^5 + 16*D*a^2*b^2*x^4 - 20*D*a^4 + 12*B*
a^3*b + 10*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 16*(D*a^3*b - B*a^2*b^2)*x^2 + 3*(
```

```
(5*C*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*x^2)*
sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*C*a^3*b - A
*a^2*b^2)*x - 8*(3*D*a^4 - B*a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(3*D*a
^3*b - B*a^2*b^2)*x^2)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4
), 1/8*(4*D*a*b^3*x^6 + 8*C*a*b^3*x^5 + 8*D*a^2*b^2*x^4 - 10*D*a^4 + 6*B*a^
3*b + 5*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 8*(D*a^3*b - B*a^2*b^2)*x^2 - 3*((5*C
*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*x^2)*sqrt
(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*C*a^3*b - A*a^2*b^2)*x - 4*(3*D*a^4 - B*
a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(3*D*a^3*b - B*a^2*b^2)*x^2)*log(b*
x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

giac [A] time = 0.41, size = 157, normalized size = 0.85

$$-\frac{3\,(5\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}-\frac{(3\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^4}+\frac{Db^3x^2+2\,Cb^3x}{2\,b^6}-\frac{10\,Da^3-6\,Ba^2b-\left(9\,Cab^2-5\,Ab^3\right)\cdots}{8\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(3*D*a - B*b
)*log(b*x^2 + a)/b^4 + 1/2*(D*b^3*x^2 + 2*C*b^3*x)/b^6 - 1/8*(10*D*a^3 - 6*
B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*B*a*b^2)*x^2 - (7*C*
a^2*b - 3*A*a*b^2)*x)/((b*x^2 + a)^2*b^4)
```

maple [A] time = 0.01, size = 235, normalized size = 1.27

$$-\frac{5A\,x^3}{8\left(b\,x^2+a\right)^2 b}+\frac{9Ca\,x^3}{8\left(b\,x^2+a\right)^2 b^2}+\frac{Ba\,x^2}{\left(b\,x^2+a\right)^2 b^2}-\frac{3Da^2x^2}{2\left(b\,x^2+a\right)^2 b^3}-\frac{3Aax}{8\left(b\,x^2+a\right)^2 b^2}+\frac{7C\,a^2x}{8\left(b\,x^2+a\right)^2 b^3}+\frac{3A\arctan\cdots}{8\sqrt{ab}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/2/b^3*D*x^2+1/b^3*C*x-5/8/(b*x^2+a)^2*A/b*x^3+9/8/b^2/(b*x^2+a)^2*C*x^3*a
+1/b^2/(b*x^2+a)^2*B*x^2*a-3/2/b^3/(b*x^2+a)^2*D*x^2*a^2-3/8/(b*x^2+a)^2*A*
a/b^2*x+7/8/b^3/(b*x^2+a)^2*a^2*C*x+3/4/(b*x^2+a)^2*B*a^2/b^3-5/4/b^4/(b*x^
2+a)^2*a^3*D+1/2*B/b^3*ln(b*x^2+a)-3/2/b^4*ln(b*x^2+a)*a*D+3/8/(a*b)^(1/2)*
A/b^2*arctan(1/(a*b)^(1/2)*b*x)-15/8/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)*a*C
```

maxima [A] time = 3.07, size = 165, normalized size = 0.89

$$-\frac{10\,Da^3-6\,Ba^2b-\left(9\,Cab^2-5\,Ab^3\right)x^3+4\left(3\,Da^2b-2\,Bab^2\right)x^2-\left(7\,Ca^2b-3\,Aab^2\right)x}{8\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}-\frac{3\,(5\,Ca-Ab)\arctan\left(\frac{b\cdots}{\sqrt{\cdots}}\right)}{8\,\sqrt{ab}\,b^3}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(10*D*a^3 - 6*B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*B
*a*b^2)*x^2 - (7*C*a^2*b - 3*A*a*b^2)*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
- 3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(D*x^2 + 2*
C*x)/b^3 - 1/2*(3*D*a - B*b)*log(b*x^2 + a)/b^4
```

mupad [B] time = 1.56, size = 232, normalized size = 1.25

$$\frac{\frac{7\,C\,a^2\,x}{8}+\frac{9\,C\,b\,a\,x^3}{8}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}-\frac{\frac{5\,A\,x^3}{8\,b}+\frac{3\,A\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\frac{3\,B\,a^2}{4\,b^3}+\frac{B\,a\,x^2}{b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{D\left(3\,a\,\ln\left(b\,x^2+a\right)-b\,x^2+\frac{3\,a^2}{b\,x^2+a}-\frac{a^3}{2\left(b\,x^2+\cdots\right.}\cdots\right.}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] ((7*C*a^2*x)/8 + (9*C*a*b*x^3)/8)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) - ((5*A
*x^3)/(8*b) + (3*A*a*x)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((3*B*a^2)/(
4*b^3) + (B*a*x^2)/b^2)/(a^2 + b^2*x^4 + 2*a*b*x^2) - (D*(3*a*log(a + b*x^2
) - b*x^2 + (3*a^2)/(a + b*x^2) - a^3/(2*(a + b*x^2)^2)))/(2*b^4) + (B*log(
a + b*x^2))/(2*b^3) + (C*x)/b^3 + (3*A*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2
)*b^(5/2)) - (15*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(7/2))
```

sympy [B] time = 29.59, size = 357, normalized size = 1.93

$$\frac{Cx}{b^3}+\frac{Dx^2}{2b^3}+\left(-\frac{-Bb+3Da}{2b^4}-\frac{3\sqrt{-ab^9}\,(-Ab+5Ca)}{16ab^8}\right)\log\left(x+\frac{8Bab-24Da^2-16ab^4\left(-\frac{-Bb+3Da}{2b^4}-\frac{3\sqrt{-ab^9}\,(-Ab+}{16ab^8}\right.}{-3Ab^2+15Cab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] C*x/b**3 + D*x**2/(2*b**3) + (-(-B*b + 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-
A*b + 5*C*a)/(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b
+ 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2
 + 15*C*a*b)) + (-(-B*b + 3*D*a)/(2*b**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/
(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b + 3*D*a)/(2*b
**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2 + 15*C*a*b))
 + (6*B*a**2*b - 10*D*a**3 + x**3*(-5*A*b**3 + 9*C*a*b**2) + x**2*(8*B*a*b*
*2 - 12*D*a**2*b) + x*(-3*A*a*b**2 + 7*C*a**2*b))/(8*a**2*b**4 + 16*a*b**5*
x**2 + 8*b**6*x**4)
```

3.103 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}-\frac{3x(bB-5aD)}{8ab^3}+\frac{C\log\left(a+bx^2\right)}{2b^3}-\frac{x^2(4aC-x(3bB-}{8ab^2\left(a+bx^2\right.}$$

```
[Out] -3/8*(B*b-5*D*a)*x/a/b^3-1/4*x^3*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2-
1/8*x^2*(4*a*C-(3*B*b-7*D*a)*x)/a/b^2/(b*x^2+a)+1/2*C*ln(b*x^2+a)/b^3+3/8*(
B*b-5*D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1804, 774, 635, 205, 260}

$$-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-x(3bB-7aD))}{8ab^2\left(a+bx^2\right)}-\frac{3x(bB-5aD)}{8ab^3}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}+\frac{C\log\left(a+\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(b*B - 5*a*D)*x)/(8*a*b^3) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4
*a*b*(a + b*x^2)^2) - (x^2*(4*a*C - (3*b*B - 7*a*D)*x))/(8*a*b^2*(a + b*x^2
)) + (3*(b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C
*Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
```

```
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{x^2\left(-3a\left(B-\frac{aD}{b}\right)-4aCx-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)} + \frac{\int \frac{x\left(8a^2C-3a(b\right.}{a+bx}}{8a^2}$$

$$= -\frac{3(bB-5aD)x}{8ab^3} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{x^2(4aC-(3bB-7aD)x}{8ab^2\left(a+bx^2\right)}$$

$$= -\frac{3(bB-5aD)x}{8ab^3} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{x^2(4aC-(3bB-7aD)x}{8ab^2\left(a+bx^2\right)}$$

$$= -\frac{3(bB-5aD)x}{8ab^3} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{x^2(4aC-(3bB-7aD)x}{8ab^2\left(a+bx^2\right)}$$

Mathematica [A] time = 0.08, size = 126, normalized size = 0.81

$$\frac{a(-a(C+Dx)+Ab+bBx)}{4b^3\left(a+bx^2\right)^2}+\frac{8aC+9aDx-4Ab-5bBx}{8b^3\left(a+bx^2\right)}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}+\frac{C\log\left(a+bx^2\right)}{2b^3}+\frac{Dx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (D*x)/b^3 + (-4*A*b + 8*a*C - 5*b*B*x + 9*a*D*x)/(8*b^3*(a + b*x^2)) + (a*(
A*b + b*B*x - a*(C + D*x)))/(4*b^3*(a + b*x^2)^2) + (3*(b*B - 5*a*D)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C*Log[a + b*x^2])/(2*b^3)
```

fricas [A] time = 0.74, size = 480, normalized size = 3.10

$$\left[\frac{16\,Dab^3x^5+12\,Ca^3b-4\,Aa^2b^2+10\left(5\,Da^2b^2-Bab^3\right)x^3+8\left(2\,Ca^2b^2-Aab^3\right)x^2-3\left(\left(5\,Dab^2-Bb^3\right)x^4+5\right.}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*D*a*b^3*x^5 + 12*C*a^3*b - 4*A*a^2*b^2 + 10*(5*D*a^2*b^2 - B*a*b^
3)*x^3 + 8*(2*C*a^2*b^2 - A*a*b^3)*x^2 - 3*((5*D*a*b^2 - B*b^3)*x^4 + 5*D*a
^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(
-a*b)*x - a)/(b*x^2 + a)) + 6*(5*D*a^3*b - B*a^2*b^2)*x + 8*(C*a*b^3*x^4 +
2*C*a^2*b^2*x^2 + C*a^3*b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3
*b^4), 1/8*(8*D*a*b^3*x^5 + 6*C*a^3*b - 2*A*a^2*b^2 + 5*(5*D*a^2*b^2 - B*a*
b^3)*x^3 + 4*(2*C*a^2*b^2 - A*a*b^3)*x^2 - 3*((5*D*a*b^2 - B*b^3)*x^4 + 5*D
*a^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
```

a) + 3*(5*D*a^3*b - B*a^2*b^2)*x + 4*(C*a*b^3*x^4 + 2*C*a^2*b^2*x^2 + C*a^3 *b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]

giac [A] time = 0.38, size = 122, normalized size = 0.79

$$\frac{Dx}{b^3}+\frac{C\log\left(bx^2+a\right)}{2\,b^3}-\frac{3\,(5\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{\left(9\,Dab-5\,Bb^2\right)x^3+6\,Ca^2-2\,Aab+4\left(2\,Cab-Ab^2\right)x^2+(7}{8\left(bx^2+a\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")

[Out] D*x/b^3 + 1/2*C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*(2*C*a*b - A*b^2)*x^2 + (7*D*a^2 - 3*B*a*b)*x)/((b*x^2 + a)^2*b^3)

maple [A] time = 0.01, size = 206, normalized size = 1.33

$$-\frac{5B\,x^3}{8\left(b\,x^2+a\right)^2b}+\frac{9Da\,x^3}{8\left(b\,x^2+a\right)^2b^2}-\frac{A\,x^2}{2\left(b\,x^2+a\right)^2b}+\frac{Ca\,x^2}{\left(b\,x^2+a\right)^2b^2}-\frac{3Bax}{8\left(b\,x^2+a\right)^2b^2}+\frac{7Da^2x}{8\left(b\,x^2+a\right)^2b^3}-\frac{Aa}{4\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)

[Out] D/b^3*x-5/8/b/(b*x^2+a)^2*B*x^3+9/8/b^2/(b*x^2+a)^2*D*x^3*a-1/2/b/(b*x^2+a)^2*A*x^2+1/b^2/(b*x^2+a)^2*C*x^2*a-3/8/b^2/(b*x^2+a)^2*B*x*a+7/8/b^3/(b*x^2+a)^2*a^2*D*x-1/4/(b*x^2+a)^2*A*a/b^2+3/4/b^3/(b*x^2+a)^2*a^2*C+1/2*C*ln(b*x^2+a)/b^3+3/8/(a*b)^(1/2)*B/b^2*arctan(1/(a*b)^(1/2)*b*x)-15/8/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*D

maxima [A] time = 2.98, size = 136, normalized size = 0.88

$$\frac{\left(9\,Dab-5\,Bb^2\right)x^3+6\,Ca^2-2\,Aab+4\left(2\,Cab-Ab^2\right)x^2+\left(7\,Da^2-3\,Bab\right)x}{8\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}+\frac{Dx}{b^3}+\frac{C\log\left(bx^2+a\right)}{2\,b^3}-\frac{3\,(5\,Da-B}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")

[Out] 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*(2*C*a*b - A*b^2)*x^2 + (7*D*a^2 - 3*B*a*b)*x)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + D*x/b^3 + 1/2*C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\left(b\,x^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)

[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)

sympy [B] time = 29.73, size = 282, normalized size = 1.82

$$\frac{Dx}{b^3}+\left(\frac{C}{2b^3}-\frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)\log\left(x+\frac{8Ca-16ab^3\left(\frac{C}{2b^3}-\frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right)+\left(\frac{C}{2b^3}+\frac{3\sqrt{-ab^7}\,(-Bb+}{16ab^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] D*x/b**3 + (C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**7))*log(x
+ (8*C*a - 16*a*b**3*(C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**
7)))/(-3*B*b + 15*D*a)) + (C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*
a*b**7))*log(x + (8*C*a - 16*a*b**3*(C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5
*D*a)/(16*a*b**7)))/(-3*B*b + 15*D*a)) + (-2*A*a*b + 6*C*a**2 + x**3*(-5*B*
b**2 + 9*D*a*b) + x**2*(-4*A*b**2 + 8*C*a*b) + x*(-3*B*a*b + 7*D*a**2))/(8*
a**2*b**3 + 16*a*b**4*x**2 + 8*b**5*x**4)
```

3.104 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$

Optimal. Leaf size=136

$$\frac{(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{8ab^2(a+bx^2)}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2}+\frac{D\log(a+bx^2)}{2b^3}$$

```
[Out] -1/4*x^2*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2-1/8*x*(A*b+3*a*C-2*(B*b-
3*D*a)*x)/a/b^2/(b*x^2+a)+1/8*(A*b+3*C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)
/b^(5/2)+1/2*D*ln(b*x^2+a)/b^3
```

Rubi [A] time = 0.16, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1804, 635, 205, 260}

$$\frac{(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{8ab^2(a+bx^2)}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2}+\frac{D\log(a+bx^2)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) - (x*(A*b +
3*a*C - 2*(b*B - 3*a*D)*x))/(8*a*b^2*(a + b*x^2)) + ((A*b + 3*a*C)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)) + (D*Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{x\left(-2a\left(B-\frac{aD}{b}\right)-(Ab+3aC)x-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)} + \frac{\int \frac{a(Ab+}{}}{}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)} + \frac{(Ab+3}{}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)} + \frac{(Ab+3}{}$$

Mathematica [A] time = 0.10, size = 122, normalized size = 0.90

$$\frac{\frac{\sqrt{b}(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}} + \frac{-2a^2D+2ab(B+Cx)-2Ab^2x}{\left(a+bx^2\right)^2} + \frac{8a^2D-ab(4B+5Cx)+Ab^2x}{a\left(a+bx^2\right)} + 4D\log\left(a+bx^2\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] ((-2*a^2*D - 2*A*b^2*x + 2*a*b*(B + C*x))/(a + b*x^2)^2 + (8*a^2*D + A*b^2*
x - a*b*(4*B + 5*C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + 3*a*C)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/a^(3/2) + 4*D*Log[a + b*x^2])/(8*b^3)
```

fricas [A] time = 0.76, size = 447, normalized size = 3.29

$$\left[\frac{12\,Da^4 - 4\,Ba^3b - 2\left(5\,Ca^2b^2 - Aab^3\right)x^3 + 8\left(2\,Da^3b - Ba^2b^2\right)x^2 - \left(\left(3\,Cab^2 + Ab^3\right)x^4 + 3\,Ca^3 + Aa^2b + 2\left(3\right.\right.}{16\left(a^2b^5x^4 +\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*D*a^4 - 4*B*a^3*b - 2*(5*C*a^2*b^2 - A*a*b^3)*x^3 + 8*(2*D*a^3*b
- B*a^2*b^2)*x^2 - ((3*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*a^
2*b + A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*(3*C*a^3*b + A*a^2*b^2)*x + 8*(D*a^2*b^2*x^4 + 2*D*a^3*b*x^2 + D*a^4)
*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), 1/8*(6*D*a^4 - 2*
B*a^3*b - (5*C*a^2*b^2 - A*a*b^3)*x^3 + 4*(2*D*a^3*b - B*a^2*b^2)*x^2 + ((3
*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*a^2*b + A*a*b^2)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) - (3*C*a^3*b + A*a^2*b^2)*x + 4*(D*a^2*b^2*x^
4 + 2*D*a^3*b*x^2 + D*a^4)*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a
^4*b^3)]
```

giac [A] time = 0.42, size = 128, normalized size = 0.94

$$\frac{D\log\left(bx^2+a\right)}{2\,b^3} + \frac{(3\,Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2} - \frac{\left(5\,Cab-Ab^2\right)x^3 - 4\left(2\,Da^2-Bab\right)x^2 + \left(3\,Ca^2+Aab\right)x - \frac{2(3\,Da}{}}{8\left(bx^2+a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a*b^2) - 1/8*((5*C*a*b - A*b^2)*x^3 - 4*(2*D*a^2 - B*a*b)*x^2 + (3*C*a^2
 + A*a*b)*x - 2*(3*D*a^3 - B*a^2*b)/b)/((b*x^2 + a)^2*a*b^2)
```

maple [A] time = 0.01, size = 133, normalized size = 0.98

$$\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab}+\frac{3C\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^{2}}+\frac{D\ln\left(bx^{2}+a\right)}{2b^{3}}+\frac{\frac{(Ab-5aC)x^{3}}{8ab}-\frac{(bB-2aD)x^{2}}{2b^{2}}-\frac{(Ab+3aC)x}{8b^{2}}-\frac{(bB-3aD)a}{4b^{3}}}{\left(bx^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(A*b-5*C*a)/a/b*x^3-1/2*(B*b-2*D*a)/b^2*x^2-1/8*(A*b+3*C*a)/b^2*x-1/4*
a*(B*b-3*D*a)/b^3)/(b*x^2+a)^2+1/2*D*ln(b*x^2+a)/b^3+1/8/(a*b)^(1/2)*A/a/b*
arctan(1/(a*b)^(1/2)*b*x)+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C
```

maxima [A] time = 2.98, size = 146, normalized size = 1.07

$$\frac{6\,Da^{3}-2\,Ba^{2}b-\left(5\,Cab^{2}-Ab^{3}\right)x^{3}+4\left(2\,Da^{2}b-Bab^{2}\right)x^{2}-\left(3\,Ca^{2}b+Aab^{2}\right)x}{8\left(ab^{5}x^{4}+2\,a^{2}b^{4}x^{2}+a^{3}b^{3}\right)}+\frac{D\log\left(bx^{2}+a\right)}{2\,b^{3}}+\frac{(3\,Ca+Ab)\,\mathrm{a}}{8\,\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(6*D*a^3 - 2*B*a^2*b - (5*C*a*b^2 - A*b^3)*x^3 + 4*(2*D*a^2*b - B*a*b^2
)*x^2 - (3*C*a^2*b + A*a*b^2)*x)/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3) + 1/
2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a*b^2)
```

mupad [B] time = 1.39, size = 195, normalized size = 1.43

$$\frac{\frac{Ax^{3}}{8a}-\frac{Ax}{8b}}{a^{2}+2abx^{2}+b^{2}x^{4}}-\frac{\frac{Bx^{2}}{2b}+\frac{Ba}{4b^{2}}}{a^{2}+2abx^{2}+b^{2}x^{4}}-\frac{\frac{5Cx^{3}}{8b}+\frac{3Cax}{8b^{2}}}{a^{2}+2abx^{2}+b^{2}x^{4}}+\frac{D\left(\ln\left(bx^{2}+a\right)+\frac{2a}{bx^{2}+a}-\frac{a^{2}}{2\left(bx^{2}+a\right)^{2}}\right)}{2b^{3}}+\frac{A\,\mathrm{atan}\Big(}{8a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] ((A*x^3)/(8*a) - (A*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((B*x^2)/(2*b)
+ (B*a)/(4*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((5*C*x^3)/(8*b) + (3*C*a*x)
/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (D*(log(a + b*x^2) + (2*a)/(a + b*x
^2) - a^2/(2*(a + b*x^2)^2)))/(2*b^3) + (A*atan((b^(1/2)*x)/a^(1/2)))/(8*a^
(3/2)*b^(3/2)) + (3*C*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*b^(5/2))
```

sympy [B] time = 20.01, size = 304, normalized size = 2.24

$$\left(\frac{D}{2b^{3}}-\frac{\sqrt{-a^{3}b^{7}}\,(Ab+3Ca)}{16a^{3}b^{6}}\right)\log\left(x+\frac{-8Da^{2}+16a^{2}b^{3}\left(\frac{D}{2b^{3}}-\frac{\sqrt{-a^{3}b^{7}}\,(Ab+3Ca)}{16a^{3}b^{6}}\right)}{Ab^{2}+3Cab}\right)+\left(\frac{D}{2b^{3}}+\frac{\sqrt{-a^{3}b^{7}}\,(Ab+3Ca)}{16a^{3}b^{6}}\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)

[Out] (D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b**6))*log(x + (-8*D*
a**2 + 16*a**2*b**3*(D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b
**6)))/(A*b**2 + 3*C*a*b)) + (D/(2*b**3) + sqrt(-a**3*b**7)*(A*b + 3*C*a)/(
16*a**3*b**6))*log(x + (-8*D*a**2 + 16*a**2*b**3*(D/(2*b**3) + sqrt(-a**3*b
**7)*(A*b + 3*C*a)/(16*a**3*b**6)))/(A*b**2 + 3*C*a*b)) + (-2*B*a**2*b + 6*
D*a**3 + x**3*(A*b**3 - 5*C*a*b**2) + x**2*(-4*B*a*b**2 + 8*D*a**2*b) + x*(
-A*a*b**2 - 3*C*a**2*b))/(8*a**3*b**3 + 16*a**2*b**4*x**2 + 8*a*b**5*x**4)
```

3.105 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=119

$$\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{2(aC+Ab)-x(bB-5aD)}{8ab^2\left(a+bx^2\right)}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

```
[Out] -1/4*x*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*(-2*A*b-2*a*C+(B*b-5*D
*a)*x)/a/b^2/(b*x^2+a)+1/8*(B*b+3*D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^
(5/2)
```

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1804, 1814, 12, 205}

$$\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{2(aC+Ab)-x(bB-5aD)}{8ab^2\left(a+bx^2\right)}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) - (2*(A*b + a*
C) - (b*B - 5*a*D)*x)/(8*a*b^2*(a + b*x^2)) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}dx = -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{-a\left(B-\frac{aD}{b}\right)-2(Ab+aC)x-4aDx^2}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)} + \frac{\int \frac{a\left(B+\frac{3aD}{b}\right)}{a+bx^2}d}{8a^2b}$$

$$= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)} + \frac{(bB+3aD)}{8ab}$$

$$= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)} + \frac{(bB+3aD)}{8a^{3/2}}$$

Mathematica [**A**] time = 0.13, size = 99, normalized size = 0.83

$$\frac{\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{b}\left(-a^2(2C+3Dx)-ab\left(2A+x\left(B+4Cx+5Dx^2\right)\right)+b^2Bx^3\right)}{a\left(a+bx^2\right)^2}}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[b]*(b^2*B*x^3 - a^2*(2*C + 3*D*x) - a*b*(2*A + x*(B + 4*C*x + 5*D*x^
2))))/(a*(a + b*x^2)^2) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/
2))/(8*b^(5/2))
```

fricas [**A**] time = 0.73, size = 357, normalized size = 3.00

$$\left[-\frac{8\,Ca^2b^2x^2+4\,Ca^3b+4\,Aa^2b^2+2\left(5\,Da^2b^2-Bab^3\right)x^3+\left(\left(3\,Dab^2+Bb^3\right)x^4+3\,Da^3+Ba^2b+2\left(3\,Da^2b+\right.\right.}{16\left(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*C*a^2*b^2*x^2 + 4*C*a^3*b + 4*A*a^2*b^2 + 2*(5*D*a^2*b^2 - B*a*b^
3)*x^3 + ((3*D*a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*a*
b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(3*D
*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), -1/8*(4*C*a
^2*b^2*x^2 + 2*C*a^3*b + 2*A*a^2*b^2 + (5*D*a^2*b^2 - B*a*b^3)*x^3 - ((3*D*
a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*a*b^2)*x^2)*sqrt(
a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^
3*b^4*x^2 + a^4*b^3)]
```

giac [**A**] time = 0.39, size = 97, normalized size = 0.82

$$\frac{(3\,Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2}-\frac{5\,Dabx^3-Bb^2x^3+4\,Cabx^2+3\,Da^2x+Babx+2\,Ca^2+2\,Aab}{8\left(bx^2+a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(5*D*a*b*x^
3 - B*b^2*x^3 + 4*C*a*b*x^2 + 3*D*a^2*x + B*a*b*x + 2*C*a^2 + 2*A*a*b)/((b*
x^2 + a)^2*a*b^2)
```

maple [A] time = 0.01, size = 110, normalized size = 0.92

$$\frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;ab}+\frac{3D\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^2}+\frac{-\frac{Cx^2}{2b}+\frac{(bB-5aD)x^3}{8ab}-\frac{(bB+3aD)x}{8b^2}-\frac{Ab+aC}{4b^2}}{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(B*b-5*D*a)/a/b*x^3-1/2*C/b*x^2-1/8*(B*b+3*D*a)/b^2*x-1/4*(A*b+C*a)/b^
2)/(b*x^2+a)^2+1/8/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B+3/8/b^2/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D
```

maxima [A] time = 2.99, size = 111, normalized size = 0.93

$$-\frac{4\,Cabx^2+\left(5\,Dab-Bb^2\right)x^3+2\,Ca^2+2\,Aab+\left(3\,Da^2+Bab\right)x}{8\left(ab^4x^4+2\,a^2b^3x^2+a^3b^2\right)}+\frac{(3\,Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*C*a*b*x^2 + (5*D*a*b - B*b^2)*x^3 + 2*C*a^2 + 2*A*a*b + (3*D*a^2 +
B*a*b)*x)/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2) + 1/8*(3*D*a + B*b)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(A+B\,x+C\,x^2+x^3\,D\right)}{\left(b\,x^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

sympy [A] time = 16.37, size = 178, normalized size = 1.50

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Bb+3Da)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Bb+3Da)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{-2Aab-2Ca^2-4C}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x
)/16 + sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(a**2*b**2*sqrt(-1/(a**3*b**5)
) + x)/16 + (-2*A*a*b - 2*C*a**2 - 4*C*a*b*x**2 + x**3*(B*b**2 - 5*D*a*b) +
 x*(-B*a*b - 3*D*a**2))/(8*a**3*b**2 + 16*a**2*b**3*x**2 + 8*a*b**4*x**4)
```

3.106 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=116

$$\frac{(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{4a^2D-bx(aC+3Ab)}{8a^2b^2\left(a+bx^2\right)}+\frac{x(Ab-aC)-a\left(B-\frac{aD}{b}\right)}{4ab\left(a+bx^2\right)^2}$$

```
[Out] 1/4*(-a*(B-a*D/b)+(A*b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*(-4*a^2*D+b*(3*A*b+C*a)*
x)/a^2/b^2/(b*x^2+a)+1/8*(3*A*b+C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3
/2)
```

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1814, 639, 205}

$$-\frac{4a^2D-bx(aC+3Ab)}{8a^2b^2\left(a+bx^2\right)}+\frac{(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(4*a*b*(a + b*x^2)^2) - (4*a^2*D - b*(3*
A*b + a*C)*x)/(8*a^2*b^2*(a + b*x^2)) + ((3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3}\,dx &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{-3A-\frac{aC}{b}-\frac{4aDx}{b}}{\left(a+bx^2\right)^2}\,dx}{4a} \\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{4a^2D-b(3Ab+aC)x}{8a^2b^2\left(a+bx^2\right)}+\frac{(3Ab+aC)\int \frac{1}{a+bx^2}\,dx}{8a^2b} \\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{4a^2D-b(3Ab+aC)x}{8a^2b^2\left(a+bx^2\right)}+\frac{(3Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}
\end{aligned}
$$

<u>Mathematica</u> [A] time = 0.11, size = 104, normalized size = 0.90

$$
\frac{\frac{\sqrt{a}\left(-2a^3D-a^2b(2B+x(C+4Dx))+ab^2x\left(5A+Cx^2\right)+3Ab^3x^3\right)}{\left(a+bx^2\right)^2}+\sqrt{b}\,(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-2*a^3*D + 3*A*b^3*x^3 + a*b^2*x*(5*A + C*x^2) - a^2*b*(2*B + x*
(C + 4*D*x))))/(a + b*x^2)^2 + Sqrt[b]*(3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(5/2)*b^2)
```

<u>fricas</u> [A] time = 0.73, size = 346, normalized size = 2.98

$$
\left[-\frac{8\,Da^3bx^2+4\,Da^4+4\,Ba^3b-2\left(Ca^2b^2+3\,Aab^3\right)x^3+\left(\left(Cab^2+3\,Ab^3\right)x^4+Ca^3+3\,Aa^2b+2\left(Ca^2b+3\,Aab^2\right)\right.}{16\left(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*D*a^3*b*x^2 + 4*D*a^4 + 4*B*a^3*b - 2*(C*a^2*b^2 + 3*A*a*b^3)*x^3
 + ((C*a*b^2 + 3*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x
^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^3*b -
 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2), -1/8*(4*D*a^3*b*x
^2 + 2*D*a^4 + 2*B*a^3*b - (C*a^2*b^2 + 3*A*a*b^3)*x^3 - ((C*a*b^2 + 3*A*b^
3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a) + (C*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 +
a^5*b^2)]
```

<u>giac</u> [A] time = 0.39, size = 106, normalized size = 0.91

$$
\frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}+\frac{Cab^2x^3+3\,Ab^3x^3-4\,Da^2bx^2-Ca^2bx+5\,Aab^2x-2\,Da^3-2\,Ba^2b}{8\left(bx^2+a\right)^2a^2b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(C*a*b^2*x^
3 + 3*A*b^3*x^3 - 4*D*a^2*b*x^2 - C*a^2*b*x + 5*A*a*b^2*x - 2*D*a^3 - 2*B*a
^2*b)/((b*x^2 + a)^2*a^2*b^2)
```

maple [A] time = 0.01, size = 111, normalized size = 0.96

$$\frac{3A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2}+\frac{C\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab}+\frac{-\frac{Dx^2}{2b}+\frac{(3Ab+aC)x^3}{8a^2}+\frac{(5Ab-aC)x}{8ab}-\frac{bB+aD}{4b^2}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*b+C*a)/a^2*x^3-1/2*D/b*x^2+1/8*(5*A*b-C*a)/a/b*x-1/4*(B*b+D*a)/b^
2)/(b*x^2+a)^2+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+1/8/a/b/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C
```

maxima [A] time = 3.00, size = 122, normalized size = 1.05

$$-\frac{4\,Da^2bx^2+2\,Da^3+2\,Ba^2b-\left(Cab^2+3\,Ab^3\right)x^3+\left(Ca^2b-5\,Aab^2\right)x}{8\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)}+\frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*D*a^2*b*x^2 + 2*D*a^3 + 2*B*a^2*b - (C*a*b^2 + 3*A*b^3)*x^3 + (C*a^
2*b - 5*A*a*b^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(C*a + 3*
A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

mupad [B] time = 1.33, size = 163, normalized size = 1.41

$$\frac{\frac{C\,x^3}{8\,a}-\frac{C\,x}{8\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\frac{5\,A\,x}{8\,a}+\frac{3\,A\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{B}{4\,b\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}-\frac{\left(2\,b\,x^2+a\right)\,D}{4\,b^2\,\left(b\,x^2+a\right)^2}+\frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}+\frac{C\,a}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^3,x)
```

```
[Out] ((C*x^3)/(8*a) - (C*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*A*x)/(8*a)
+ (3*A*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - B/(4*b*(a^2 + b^2*x^4
+ 2*a*b*x^2)) - ((a + 2*b*x^2)*D)/(4*b^2*(a + b*x^2)^2) + (3*A*atan((b^(1/2
)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(3/
2)*b^(3/2))
```

sympy [A] time = 11.27, size = 184, normalized size = 1.59

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(3Ab+Ca)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,(3Ab+Ca)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{-2Ba^2b-2Da^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (-2*B*a**2*b - 2*D*a**3 - 4*D*a**2*b*x**2 + x**3*(3*A*b**3 + C*a*b**2
) + x*(5*A*a*b**2 - C*a**2*b))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b*
*4*x**4)
```

3.107 $\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=130

$$\frac{(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{x(aD+3bB)+4Ab}{8a^2b\left(a+bx^2\right)}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

```
[Out] 1/4*(A*b-a*C+(B*b-D*a)*x)/a/b/(b*x^2+a)^2+1/8*(4*A*b+(3*B*b+D*a)*x)/a^2/b/(
b*x^2+a)+1/8*(3*B*b+D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)+A*ln(x)/
a^3-1/2*A*ln(b*x^2+a)/a^3
```

Rubi [A] time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1805, 823, 801, 635, 205, 260}

$$\frac{x(aD+3bB)+4Ab}{8a^2b\left(a+bx^2\right)}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(4*a*b*(a + b*x^2)^2) + (4*A*b + (3*b*B + a*D)*
x)/(8*a^2*b*(a + b*x^2)) + ((3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(3/2)) + (A*Log[x])/a^3 - (A*Log[a + b*x^2])/(2*a^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx = \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{-4A-\frac{(3bB+aD)x}{b}}{x\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{\int \frac{8aAb+a(3bB+aD)x}{x\left(a+bx^2\right)}\,dx}{8a^3b}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{\int \left(\frac{8Ab}{x}+\frac{3abB+a^2D-8Ab^2x}{a+bx^2}\right)dx}{8a^3b}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{A\log(x)}{a^3} + \frac{\int \frac{3abB+a^2D-8Ab^2x}{a+bx^2}\,d}{8a^3b}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{A\log(x)}{a^3} - \frac{(Ab)\int \frac{x}{a+bx^2}\,dx}{a^3} +$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{(3bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}} + \frac{A}{}$$

Mathematica [A] time = 0.11, size = 117, normalized size = 0.90

$$\frac{\frac{2a^2(-a(C+Dx)+Ab+bBx)}{b\left(a+bx^2\right)^2}+\frac{a(aDx+4Ab+3bBx)}{b\left(a+bx^2\right)}-4A\log\left(a+bx^2\right)+\frac{\sqrt{a}\,(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}+8A\log(x)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(4*A*b + 3*b*B*x + a*D*x))/(b*(a + b*x^2)) + (2*a^2*(A*b + b*B*x - a*(C
 + D*x)))/(b*(a + b*x^2)^2) + (Sqrt[a]*(3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(3/2) + 8*A*Log[x] - 4*A*Log[a + b*x^2])/(8*a^3)
```

fricas [B] time = 0.81, size = 488, normalized size = 3.75

$$\left[\frac{8\,Aab^3x^2-4\,Ca^3b+12\,Aa^2b^2+2\left(Da^2b^2+3\,Bab^3\right)x^3-\left(\left(Dab^2+3\,Bb^3\right)x^4+Da^3+3\,Ba^2b+2\left(Da^2b+3\,B\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*A*a*b^3*x^2 - 4*C*a^3*b + 12*A*a^2*b^2 + 2*(D*a^2*b^2 + 3*B*a*b^3)
*x^3 - ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*b + 2*(D*a^2*b + 3*B*a*b^
2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(D*a^3
*b - 5*B*a^2*b^2)*x - 8*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x^2 +
 a) + 16*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4 + 2*a
^4*b^3*x^2 + a^5*b^2), 1/8*(4*A*a*b^3*x^2 - 2*C*a^3*b + 6*A*a^2*b^2 + (D*a^
2*b^2 + 3*B*a*b^3)*x^3 + ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*b + 2*(
D*a^2*b + 3*B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*a^3*b - 5*B*
a^2*b^2)*x - 4*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x^2 + a) + 8*(
A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4 + 2*a^4*b^3*x^2
 + a^5*b^2)]
```

giac [A] time = 0.48, size = 128, normalized size = 0.98

$$-\frac{A\log\left(bx^2+a\right)}{2\,a^3}+\frac{A\log\left(|x|\right)}{a^3}+\frac{(Da+3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}+\frac{4\,Aab^2x^2-2\,Ca^3+6\,Aa^2b+\left(Da^2b+3\,Bab^2\right)x^3-\left(D\right.}{8\left(bx^2+a\right)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*A*log(b*x^2 + a)/a^3 + A*log(abs(x))/a^3 + 1/8*(D*a + 3*B*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(4*A*a*b^2*x^2 - 2*C*a^3 + 6*A*a^2*b +
 (D*a^2*b + 3*B*a*b^2)*x^3 - (D*a^3 - 5*B*a^2*b)*x)/((b*x^2 + a)^2*a^3*b)
```

maple [A] time = 0.02, size = 184, normalized size = 1.42

$$\frac{3Bb\,x^3}{8\left(b\,x^2+a\right)^2a^2}+\frac{Dx^3}{8\left(b\,x^2+a\right)^2a}+\frac{Ab\,x^2}{2\left(b\,x^2+a\right)^2a^2}+\frac{5Bx}{8\left(b\,x^2+a\right)^2a}-\frac{Dx}{8\left(b\,x^2+a\right)^2b}+\frac{3A}{4\left(b\,x^2+a\right)^2a}+\frac{3B\arctan\left(\frac{b}{\sqrt{}}\right.}{8\sqrt{ab}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x)
```

```
[Out] 3/8/(b*x^2+a)^2*B/a^2*b*x^3+1/8/a/(b*x^2+a)^2*D*x^3+1/2/a^2/(b*x^2+a)^2*A*x
^2*b+5/8/(b*x^2+a)^2*B/a*x-1/8/(b*x^2+a)^2/b*x*D+3/4/(b*x^2+a)^2*A/a-1/4/(b
*x^2+a)^2/b*C-1/2*A/a^3*ln(b*x^2+a)+3/8/(a*b)^(1/2)*B/a^2*arctan(1/(a*b)^(1
/2)*b*x)+1/8/a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D+A/a^3*ln(x)
```

maxima [A] time = 2.90, size = 133, normalized size = 1.02

$$\frac{4\,Ab^2x^2+\left(Dab+3\,Bb^2\right)x^3-2\,Ca^2+6\,Aab-\left(Da^2-5\,Bab\right)x}{8\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)}-\frac{A\log\left(bx^2+a\right)}{2\,a^3}+\frac{A\log(x)}{a^3}+\frac{(Da+3\,Bb)\arctan}{8\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*A*b^2*x^2 + (D*a*b + 3*B*b^2)*x^3 - 2*C*a^2 + 6*A*a*b - (D*a^2 - 5*B
*a*b)*x)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b) - 1/2*A*log(b*x^2 + a)/a^3 +
 A*log(x)/a^3 + 1/8*(D*a + 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x+C\,x^2+x^3\,D}{x\left(b\,x^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.108 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=144

$$-\frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3}+\frac{B\log(x)}{a^3}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{8a^2\left(a+bx^2\right)}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

```
[Out] -A/a^3/x+1/4*(b*B-a*D-b*(A*b/a-C)*x)/a/b/(b*x^2+a)^2+1/8*(4*B-(7*A*b/a-3*C)
*x)/a^2/(b*x^2+a)+B*ln(x)/a^3-1/2*B*ln(b*x^2+a)/a^3-3/8*(5*A*b-C*a)*arctan(
x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{8a^2\left(a+bx^2\right)}-\frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3}+\frac{B\log(x)}{a^3}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(4*a*b*(a + b*x^2)^2) + (4*B
 - ((7*A*b)/a - 3*C)*x)/(8*a^2*(a + b*x^2)) - (3*(5*A*b - a*C)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a + b*x^2])/
(2*a^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{-4A-4Bx+3\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{\int \frac{8A+8Bx-\left(\frac{7Ab}{a}-3C\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{8a^2}$$

$$= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{\int \left(\frac{8A}{ax^2}+\frac{8B}{ax}+\frac{-15Ab+3aC-8bBx}{a\left(a+bx^2\right)}\right)}{8a^2}$$

$$= -\frac{A}{a^3x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{B\log(x)}{a^3} + \frac{\int \frac{-15Ab+3a}{a+bx}}{8a}$$

$$= -\frac{A}{a^3x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{B\log(x)}{a^3} - \frac{(bB)\int \frac{x}{a+bx}}{a^3}$$

$$= -\frac{A}{a^3x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} - \frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{8a^{7/2}\sqrt{b}}$$

Mathematica [A] time = 0.10, size = 141, normalized size = 0.98

$$\frac{3(aC-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} + \frac{4aB+3aCx-7Abx}{8a^3\left(a+bx^2\right)} - \frac{A}{a^3x} - \frac{B\log\left(a+bx^2\right)}{2a^3} + \frac{B\log(x)}{a^3} + \frac{a^2(-D)+abB+abCx-A}{4a^2b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(4*a^2*b*(a + b*x^2)^2)
+ (4*a*B - 7*A*b*x + 3*a*C*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*C)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a + b
*x^2])/(2*a^3)
```

fricas [B] time = 0.70, size = 524, normalized size = 3.64

$$\left[\frac{8\,Ba^2b^2x^3 - 16\,Aa^3b + 6\left(Ca^2b^2 - 5\,Aab^3\right)x^4 + 10\left(Ca^3b - 5\,Aa^2b^2\right)x^2 + 3\left(\left(Cab^2 - 5\,Ab^3\right)x^5 + 2\left(Ca^2b - 5\,\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*B*a^2*b^2*x^3 - 16*A*a^3*b + 6*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 10*(C
*a^3*b - 5*A*a^2*b^2)*x^2 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A*a
```

```
*b^2)*x^3 + (C*a^3 - 5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x -
 a)/(b*x^2 + a)) - 4*(D*a^4 - 3*B*a^3*b)*x - 8*(B*a*b^3*x^5 + 2*B*a^2*b^2*x
^3 + B*a^3*b*x)*log(b*x^2 + a) + 16*(B*a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*
b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), 1/8*(4*B*a^2*b^2*x^3
- 8*A*a^3*b + 3*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 5*(C*a^3*b - 5*A*a^2*b^2)*x^2
 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A*a*b^2)*x^3 + (C*a^3 - 5*A*
a^2*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 2*(D*a^4 - 3*B*a^3*b)*x - 4*(B*
a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(b*x^2 + a) + 8*(B*a*b^3*x^5 +
2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x
)]
```

giac [A] time = 0.49, size = 141, normalized size = 0.98

$$-\frac{B\log\left(bx^2+a\right)}{2\,a^3}+\frac{B\log\left(|x|\right)}{a^3}+\frac{3\,(Ca-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}+\frac{4\,Bab^2x^3+3\left(Cab^2-5\,Ab^3\right)x^4-8\,Aa^2b+5\left(Ca^2b-\right.}{8\left(bx^2+a\right)^2a^3bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*B*log(b*x^2 + a)/a^3 + B*log(abs(x))/a^3 + 3/8*(C*a - 5*A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x
^4 - 8*A*a^2*b + 5*(C*a^2*b - 5*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/((b
*x^2 + a)^2*a^3*b*x)
```

maple [A] time = 0.02, size = 195, normalized size = 1.35

$$-\frac{7A\,b^2x^3}{8\left(b\,x^2+a\right)^2a^3}+\frac{3Cb\,x^3}{8\left(b\,x^2+a\right)^2a^2}+\frac{Bb\,x^2}{2\left(b\,x^2+a\right)^2a^2}-\frac{9Abx}{8\left(b\,x^2+a\right)^2a^2}+\frac{5Cx}{8\left(b\,x^2+a\right)^2a}-\frac{15Ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^3}+\frac{}{4\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -7/8/(b*x^2+a)^2*A/a^3*b^2*x^3+3/8/a^2/(b*x^2+a)^2*C*x^3*b+1/2/a^2/(b*x^2+a
)^2*B*x^2*b-9/8/(b*x^2+a)^2*A/a^2*b*x+5/8/a/(b*x^2+a)^2*C*x+3/4/(b*x^2+a)^2
*B/a-1/4/(b*x^2+a)^2/b*D-1/2*B/a^3*ln(b*x^2+a)-15/8/(a*b)^(1/2)*A/a^3*b*arc
tan(1/(a*b)^(1/2)*b*x)+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C-A/a^
3/x+B/a^3*ln(x)
```

maxima [A] time = 2.99, size = 152, normalized size = 1.06

$$\frac{4\,Bab^2x^3+3\left(Cab^2-5\,Ab^3\right)x^4-8\,Aa^2b+5\left(Ca^2b-5\,Aab^2\right)x^2-2\left(Da^3-3\,Ba^2b\right)x}{8\left(a^3b^3x^5+2\,a^4b^2x^3+a^5bx\right)}-\frac{B\log\left(bx^2+a\right)}{2\,a^3}+\frac{B\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x^4 - 8*A*a^2*b + 5*(C*a^2*b - 5
*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/(a^3*b^3*x^5 + 2*a^4*b^2*x^3 + a^5
*b*x) - 1/2*B*log(b*x^2 + a)/a^3 + B*log(x)/a^3 + 3/8*(C*a - 5*A*b)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 1.40, size = 202, normalized size = 1.40

$$\frac{\frac{3\,B}{4\,a}+\frac{B\,b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\frac{5\,C\,x}{8\,a}+\frac{3\,C\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{A}{a}+\frac{25\,A\,b\,x^2}{8\,a^2}+\frac{15\,A\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}-\frac{D}{4\,b\left(b\,x^2+a\right)^2}-\frac{B\,\ln\left(b\,x^2+a\right)}{2\,a^3}+\frac{B\,\ln(x)}{a^3}-\frac{15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] ((3*B)/(4*a) + (B*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*C*x)/(8
*a) + (3*C*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/a + (25*A*b*x^2
)/(8*a^2) + (15*A*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 + 2*a*b*x^3) - D/(4*b*
(a + b*x^2)^2) - (B*log(a + b*x^2))/(2*a^3) + (B*log(x))/a^3 - (15*A*b^(1/2
)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2)) + (3*C*atan((b^(1/2)*x)/a^(1/2)))/
(8*a^(5/2)*b^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.109 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=174

$$-\frac{3(5bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{(3Ab-aC)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aC)}{a^4}-\frac{4(2Ab-aC)+x(7bB-3aD)}{8a^3\left(a+bx^2\right)}-\frac{A}{2a^3x^2}$$

```
[Out] -1/2*A/a^3/x^2-B/a^3/x+1/4*(-A*b/a+C-(b*B/a-D)*x)/a/(b*x^2+a)^2+1/8*(-8*A*b
+4*a*C-(7*B*b-3*D*a)*x)/a^3/(b*x^2+a)-(3*A*b-C*a)*ln(x)/a^4+1/2*(3*A*b-C*a)
*ln(b*x^2+a)/a^4-3/8*(5*B*b-D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$-\frac{4(2Ab-aC)+x(7bB-3aD)}{8a^3\left(a+bx^2\right)}+\frac{(3Ab-aC)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aC)}{a^4}-\frac{A}{2a^3x^2}-\frac{3(5bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -A/(2*a^3*x^2) - B/(a^3*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(4*a*(a + b*x^
2)^2) - (4*(2*A*b - a*C) + (7*b*B - 3*a*D)*x)/(8*a^3*(a + b*x^2)) - (3*(5*b
*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) - ((3*A*b - a*C)
*Log[x])/a^4 + ((3*A*b - a*C)*Log[a + b*x^2])/(2*a^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx = -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{-4A-4Bx+4\left(\frac{Ab}{a}-C\right)x^2+3\left(\frac{bB}{a}-D\right)x^3}{x^3\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}+\frac{\int \frac{8A+8Bx-8\left(\frac{2Ab}{a}-C\right)x^2-(}{x^3\left(a+bx^2\right)}}{8a^2}$$

$$= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}+\frac{\int\left(\frac{8A}{ax^3}+\frac{8B}{ax^2}+\frac{8(-3Ab}{a^2x}\right.}{}$$

$$= -\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}-\frac{(3Ab}{}$$

$$= -\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}-\frac{(3Ab}{}$$

$$= -\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}-\frac{3(5bB}{}$$

Mathematica [A] time = 0.16, size = 147, normalized size = 0.84

$$\frac{\frac{2a^2(a(C+Dx)-Ab-bBx)}{\left(a+bx^2\right)^2}+\frac{a(4aC+3aDx-8Ab-7bBx)}{a+bx^2}+4(3Ab-aC)\log\left(a+bx^2\right)+8\log(x)(aC-3Ab)-\frac{4aA}{x^2}+\frac{3\sqrt{a}\,(aD-}{}}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-4*a*A)/x^2 - (8*a*B)/x + (a*(-8*A*b + 4*a*C - 7*b*B*x + 3*a*D*x))/(a + b
*x^2) + (2*a^2*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)^2 + (3*Sqrt[a]*(
-5*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 8*(-3*A*b + a*C)*Log[x
] + 4*(3*A*b - a*C)*Log[a + b*x^2])/(8*a^4)
```

fricas [B] time = 1.02, size = 696, normalized size = 4.00

$$\left[-\frac{16\,Ba^3bx-6\left(Da^2b^2-5\,Bab^3\right)x^5+8\,Aa^3b-8\left(Ca^2b^2-3\,Aab^3\right)x^4-10\left(Da^3b-5\,Ba^2b^2\right)x^3-12\left(Ca^3b-3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*B*a^3*b*x - 6*(D*a^2*b^2 - 5*B*a*b^3)*x^5 + 8*A*a^3*b - 8*(C*a^2
*b^2 - 3*A*a*b^3)*x^4 - 10*(D*a^3*b - 5*B*a^2*b^2)*x^3 - 12*(C*a^3*b - 3*A*
```

```
a^2*b^2)*x^2 + 3*((D*a*b^2 - 5*B*b^3)*x^6 + 2*(D*a^2*b - 5*B*a*b^2)*x^4 + (
D*a^3 - 5*B*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
+ a)) + 8*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3
*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 16*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*
a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(x))/(a^4*b^3*x^
6 + 2*a^5*b^2*x^4 + a^6*b*x^2), -1/8*(8*B*a^3*b*x - 3*(D*a^2*b^2 - 5*B*a*b^
3)*x^5 + 4*A*a^3*b - 4*(C*a^2*b^2 - 3*A*a*b^3)*x^4 - 5*(D*a^3*b - 5*B*a^2*b
^2)*x^3 - 6*(C*a^3*b - 3*A*a^2*b^2)*x^2 - 3*((D*a*b^2 - 5*B*b^3)*x^6 + 2*(D
*a^2*b - 5*B*a*b^2)*x^4 + (D*a^3 - 5*B*a^2*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*
b)*x/a) + 4*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a
^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 8*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C
*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(x))/(a^4*b^3*x
^6 + 2*a^5*b^2*x^4 + a^6*b*x^2)]
```

giac [A] time = 0.38, size = 162, normalized size = 0.93

$$\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}-\frac{(Ca-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\log\left(|x|\right)}{a^4}+\frac{3\,Dabx^5-15\,Bb^2x^5+4\,Cabx^4-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(C*a - 3*A*b)
*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(abs(x))/a^4 + 1/8*(3*D*a*b*x^5 - 15
*B*b^2*x^5 + 4*C*a*b*x^4 - 12*A*b^2*x^4 + 5*D*a^2*x^3 - 25*B*a*b*x^3 + 6*C*
a^2*x^2 - 18*A*a*b*x^2 - 8*B*a^2*x - 4*A*a^2)/((b*x^3 + a*x)^2*a^3)
```

maple [A] time = 0.02, size = 250, normalized size = 1.44

$$-\frac{7B\,b^2x^3}{8\left(b\,x^2+a\right)^2a^3}+\frac{3Db\,x^3}{8\left(b\,x^2+a\right)^2a^2}-\frac{A\,b^2x^2}{\left(b\,x^2+a\right)^2a^3}+\frac{Cb\,x^2}{2\left(b\,x^2+a\right)^2a^2}-\frac{9Bbx}{8\left(b\,x^2+a\right)^2a^2}+\frac{5Dx}{8\left(b\,x^2+a\right)^2a}-\frac{5Ab}{4\left(b\,x^2+a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x)
```

```
[Out] -7/8/a^3/(b*x^2+a)^2*B*x^3*b^2+3/8/a^2/(b*x^2+a)^2*D*x^3*b-1/a^3/(b*x^2+a)^
2*A*x^2*b^2+1/2/a^2/(b*x^2+a)^2*C*x^2*b-9/8/a^2/(b*x^2+a)^2*B*x*b+5/8/a/(b*
x^2+a)^2*D*x-5/4/(b*x^2+a)^2*A/a^2*b+3/4/a/(b*x^2+a)^2*C+3/2*A/a^4*b*ln(b*x
^2+a)-1/2/a^3*ln(b*x^2+a)*C-15/8/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*
b*B+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D-1/2*A/a^3/x^2-B/a^3/x-3
*A/a^4*b*ln(x)+1/a^3*ln(x)*C
```

maxima [A] time = 2.99, size = 172, normalized size = 0.99

$$\frac{3\left(Dab-5\,Bb^2\right)x^5+4\left(Cab-3\,Ab^2\right)x^4-8\,Ba^2x+5\left(Da^2-5\,Bab\right)x^3-4\,Aa^2+6\left(Ca^2-3\,Aab\right)x^2}{8\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)}+\frac{3\,(Da-5\,B\cdots}{8\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(D*a*b - 5*B*b^2)*x^5 + 4*(C*a*b - 3*A*b^2)*x^4 - 8*B*a^2*x + 5*(D*a
^2 - 5*B*a*b)*x^3 - 4*A*a^2 + 6*(C*a^2 - 3*A*a*b)*x^2)/(a^3*b^2*x^6 + 2*a^4
*b*x^4 + a^5*x^2) + 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
 - 1/2*(C*a - 3*A*b)*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(x)/a^4
```

mupad [B] time = 1.46, size = 229, normalized size = 1.32

$$\frac{\frac{3C}{4a}+\frac{Cbx^2}{2a^2}}{a^2+2abx^2+b^2x^4}-\frac{\frac{A}{2a}+\frac{9Abx^2}{4a^2}+\frac{3Ab^2x^4}{2a^3}}{a^2x^2+2abx^4+b^2x^6}-\frac{\frac{B}{a}+\frac{25Bbx^2}{8a^2}+\frac{15Bb^2x^4}{8a^3}}{a^2x+2abx^3+b^2x^5}-\frac{C\ln\left(bx^2+a\right)}{2a^3}+\frac{C\ln(x)}{a^3}+\frac{3Ab\ln\left(bx^{2}\cdots\right.}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^3),x)
```

```
[Out] ((3*C)/(4*a) + (C*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/(2*a) +
(9*A*b*x^2)/(4*a^2) + (3*A*b^2*x^4)/(2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x^4
) - (B/a + (25*B*b*x^2)/(8*a^2) + (15*B*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5
+ 2*a*b*x^3) - (C*log(a + b*x^2))/(2*a^3) + (C*log(x))/a^3 + (3*A*b*log(a +
 b*x^2))/(2*a^4) - (3*A*b*log(x))/a^4 + (x*D*hypergeom([1/2, 3], 3/2, -(b*x
^2)/a))/a^3 - (15*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.110 $\int \frac{-x+4x^3}{\left(5+x^2\right)^2} dx$

Optimal. Leaf size=20

$$\frac{21}{2\left(x^2+5\right)}+2\log\left(x^2+5\right)$$

```
[Out] 21/2/(x^2+5)+2*ln(x^2+5)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1593, 444, 43}

$$\frac{21}{2\left(x^2+5\right)}+2\log\left(x^2+5\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 4*x^3)/(5 + x^2)^2,x]
```

```
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-x+4x^3}{\left(5+x^2\right)^2}\,dx &= \int \frac{x\left(-1+4x^2\right)}{\left(5+x^2\right)^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+4x}{(5+x)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{21}{(5+x)^2}+\frac{4}{5+x}\right)dx, x, x^2\right)\\
&= \frac{21}{2\left(5+x^2\right)}+2\log\left(5+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{21}{2\left(x^2+5\right)}+2\log\left(x^2+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 4*x^3)/(5 + x^2)^2,x]
```

```
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

fricas [A] time = 0.64, size = 24, normalized size = 1.20

$$\frac{4\left(x^2+5\right)\log\left(x^2+5\right)+21}{2\left(x^2+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(x^2 + 5)*log(x^2 + 5) + 21)/(x^2 + 5)
```

giac [A] time = 0.32, size = 25, normalized size = 1.25

$$-\frac{4\,x^2-1}{2\left(x^2+5\right)}+2\,\log\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*x^2 - 1)/(x^2 + 5) + 2*log(x^2 + 5)
```

maple [A] time = 0.01, size = 19, normalized size = 0.95

$$2\ln\left(x^2+5\right)+\frac{21}{2\left(x^2+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-x)/(x^2+5)^2,x)
```

```
[Out] 21/2/(x^2+5)+2*ln(x^2+5)
```

maxima [A] time = 1.34, size = 18, normalized size = 0.90

$$\frac{21}{2\left(x^2+5\right)}+2\,\log\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="maxima")
```

```
[Out] 21/2/(x^2 + 5) + 2*log(x^2 + 5)
```

mupad [B] time = 0.91, size = 20, normalized size = 1.00

$$2\,\ln\left(x^2+5\right)+\frac{21}{2\,\left(x^2+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 4*x^3)/(x^2 + 5)^2,x)
```

```
[Out] 2*log(x^2 + 5) + 21/(2*(x^2 + 5))
```

sympy [A] time = 0.17, size = 15, normalized size = 0.75

$$2\log\left(x^2+5\right)+\frac{21}{2x^2+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-x)/(x**2+5)**2,x)
```

```
[Out] 2*log(x**2 + 5) + 21/(2*x**2 + 10)
```

3.111 $\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}\left(x^2-2\right)^{3/2}+\sqrt{x^2-2}$$

```
[Out] 1/3*(x^2-2)^(3/2)+(x^2-2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1593, 444, 43}

$$\frac{1}{3}\left(x^2-2\right)^{3/2}+\sqrt{x^2-2}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/Sqrt[-2 + x^2],x]
```

```
[Out] Sqrt[-2 + x^2] + (-2 + x^2)^(3/2)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx &= \int \frac{x\left(-1+x^2\right)}{\sqrt{-2+x^2}}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+x}{\sqrt{-2+x}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt{-2+x}}+\sqrt{-2+x}\right)dx,x,x^2\right)\\
&= \sqrt{-2+x^2}+\frac{1}{3}\left(-2+x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{1}{3}\sqrt{x^2-2}\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/Sqrt[-2 + x^2],x]
```

```
[Out] (Sqrt[-2 + x^2]*(1 + x^2))/3
```

fricas [A] time = 0.69, size = 14, normalized size = 0.61

$$\frac{1}{3}\left(x^2+1\right)\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 + 1)*sqrt(x^2 - 2)
```

giac [A] time = 0.40, size = 17, normalized size = 0.74

$$\frac{1}{3}\left(x^2-2\right)^{\frac{3}{2}}+\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 2)^(3/2) + sqrt(x^2 - 2)
```

maple [A] time = 0.01, size = 15, normalized size = 0.65

$$\frac{\left(x^2+1\right)\sqrt{x^2-2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(x^2-2)^(1/2),x)
```

```
[Out] 1/3*(x^2+1)*(x^2-2)^(1/2)
```

maxima [A] time = 1.34, size = 22, normalized size = 0.96

$$\frac{1}{3}\sqrt{x^2-2}\,x^2+\frac{1}{3}\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^2 - 2)*x^2 + 1/3*sqrt(x^2 - 2)
```

mupad [B] time = 0.10, size = 14, normalized size = 0.61

$$\frac{\left(x^2+1\right)\sqrt{x^2-2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^3)/(x^2 - 2)^(1/2),x)
```

```
[Out] ((x^2 + 1)*(x^2 - 2)^(1/2))/3
```

sympy [A] time = 0.69, size = 22, normalized size = 0.96

$$\frac{x^2\sqrt{x^2-2}}{3}+\frac{\sqrt{x^2-2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(x**2-2)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 - 2)/3 + sqrt(x**2 - 2)/3
```

3.112 $\int \frac{-x^2+2x^4}{1+2x^2} \, dx$

Optimal. Leaf size=25

$$\frac{x^3}{3} - x + \frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] -x+1/3*x^3+1/2*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1593, 459, 321, 203}

$$\frac{x^3}{3} - x + \frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-x^2+2x^4}{1+2x^2}\,dx &= \int \frac{x^2\left(-1+2x^2\right)}{1+2x^2}\,dx\\ &= \frac{x^3}{3}-2\int \frac{x^2}{1+2x^2}\,dx\\ &= -x+\frac{x^3}{3}+\int \frac{1}{1+2x^2}\,dx\\ &= -x+\frac{x^3}{3}+\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^3}{3}-x+\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

fricas [A] time = 0.63, size = 20, normalized size = 0.80

$$\frac{1}{3}\,x^3+\frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

giac [A] time = 0.35, size = 20, normalized size = 0.80

$$\frac{1}{3}\,x^3+\frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{x^3}{3}-x+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-x^2)/(2*x^2+1),x)
[Out] -x+1/3*x^3+1/2*arctan(x*2^(1/2))*2^(1/2)
```

maxima [A] time = 2.92, size = 20, normalized size = 0.80

$$\frac{1}{3}\,x^3+\frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

mupad [B] time = 0.04, size = 20, normalized size = 0.80

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}-x+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 2*x^4)/(2*x^2 + 1),x)
[Out] (2^(1/2)*atan(2^(1/2)*x))/2 - x + x^3/3
```

sympy [A] time = 0.14, size = 20, normalized size = 0.80

$$\frac{x^3}{3}-x+\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-x**2)/(2*x**2+1),x)
[Out] x**3/3 - x + sqrt(2)*atan(sqrt(2)*x)/2
```

3.113 $\int \frac{x^3+x^4}{1+x^2} \, dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

[Out] -x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 801, 635, 203, 260}

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^3 + x^4)/(1 + x^2),x]

[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3+x^4}{1+x^2}\,dx &= \int \frac{x^3(1+x)}{1+x^2}\,dx \\
&= \int \left(-1+x+x^2+\frac{1-x}{1+x^2}\right)dx \\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\int \frac{1-x}{1+x^2}\,dx \\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx \\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3 + x^4)/(1 + x^2),x]
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

fricas [A] time = 0.56, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3+\frac{1}{2}x^2-x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

giac [A] time = 0.39, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3+\frac{1}{2}x^2-x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{x^3}{3}+\frac{x^2}{2}-x+\arctan(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3)/(x^2+1),x)
[Out] -x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

maxima [A] time = 2.90, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3+\frac{1}{2}x^2-x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\operatorname{atan}(x) - \frac{\ln\left(x^2+1\right)}{2} - x + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + x^4)/(x^2 + 1),x)
[Out] atan(x) - log(x^2 + 1)/2 - x + x^2/2 + x^3/3
```

sympy [A] time = 0.11, size = 22, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} - x - \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3)/(x**2+1),x)
[Out] x**3/3 + x**2/2 - x - log(x**2 + 1)/2 + atan(x)
```

3.114 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2} dx$

Optimal. Leaf size=210

$$\frac{x^7(a^2f-abe+b^2d)}{7b^3}+\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{b^6}-\frac{ax^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5}+\frac{x^5(a^3(-f)+a^2be-}{5b^4}$$

```
[Out] a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^6-1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)*x^3/b^5+1/5*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^5/b^4+1/7*(a^2*f-a*b*e+b^2*
d)*x^7/b^3+1/9*(-a*f+b*e)*x^9/b^2+1/11*f*x^11/b-a^(5/2)*(-a^3*f+a^2*b*e-a*b
^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(13/2)
```

Rubi [A] time = 0.16, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{x^5(a^2be+a^3(-f)-ab^2d+b^3c)}{5b^4}-\frac{ax^3(a^2be+a^3(-f)-ab^2d+b^3c)}{3b^5}+\frac{a^2x(a^2be+a^3(-f)-ab^2d+b^3c)}{b^6}-\frac{a^{5/2}\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b
^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (
f*x^11)/(11*b) - (a^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(13/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2} dx = \int \left(\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{b^6}-\frac{a(b^3c-ab^2d+a^2be-a^3f)x^2}{b^5}+\frac{(b^3c-ab^2}{}\right.$$

$$=\frac{a^2(b^3c-ab^2d+a^2be-a^3f)x}{b^6}-\frac{a(b^3c-ab^2d+a^2be-a^3f)x^3}{3b^5}+\frac{(b^3c-ab^2d}{}$$

$$=\frac{a^2(b^3c-ab^2d+a^2be-a^3f)x}{b^6}-\frac{a(b^3c-ab^2d+a^2be-a^3f)x^3}{3b^5}+\frac{(b^3c-ab^2d}{}$$

Mathematica [A] time = 0.16, size = 210, normalized size = 1.00

$$\frac{x^7(a^2f-abe+b^2d)}{7b^3}-\frac{a^2x(a^3f-a^2be+ab^2d-b^3c)}{b^6}+\frac{ax^3(a^3f-a^2be+ab^2d-b^3c)}{3b^5}+\frac{x^5(a^3(-f)+a^2be-ab^2d+}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b^
2*d - a^2*b*e + a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9
*b^2) + (f*x^11)/(11*b) + (a^(5/2)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(13/2)
```

fricas [A] time = 0.57, size = 452, normalized size = 2.15

$$\left[\frac{630\,b^5 f x^{11} + 770\left(b^5 e - a b^4 f\right)x^9 + 990\left(b^5 d - a b^4 e + a^2 b^3 f\right)x^7 + 1386\left(b^5 c - a b^4 d + a^2 b^3 e - a^3 b^2 f\right)x^5 - 2310\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6930*(630*b^5*f*x^11 + 770*(b^5*e - a*b^4*f)*x^9 + 990*(b^5*d - a*b^4*e
+ a^2*b^3*f)*x^7 + 1386*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5 - 231
0*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^3*b
^2*d + a^4*b*e - a^5*f)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) + 6930*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6, 1/3465*(31
5*b^5*f*x^11 + 385*(b^5*e - a*b^4*f)*x^9 + 495*(b^5*d - a*b^4*e + a^2*b^3*f
)*x^7 + 693*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5 - 1155*(a*b^4*c -
 a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^3*b^2*d + a^4*b
*e - a^5*f)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 3465*(a^2*b^3*c - a^3*b^2*d
 + a^4*b*e - a^5*f)*x)/b^6]
```

giac [A] time = 0.46, size = 250, normalized size = 1.19

$$-\frac{\left(a^3 b^3 c - a^4 b^2 d - a^6 f + a^5 b e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^6} + \frac{315\,b^{10} f x^{11} - 385\,a b^9 f x^9 + 385\,b^{10} x^9 e + 495\,b^{10} d x^7 + 495\,a^2 b^8\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + 1/3465*(315*b^10*f*x^11 - 385*a*b^9*f*x^9 + 385*b^10*x^9*e + 495*b^
10*d*x^7 + 495*a^2*b^8*f*x^7 - 495*a*b^9*x^7*e + 693*b^10*c*x^5 - 693*a*b^9
*d*x^5 - 693*a^3*b^7*f*x^5 + 693*a^2*b^8*x^5*e - 1155*a*b^9*c*x^3 + 1155*a^
2*b^8*d*x^3 + 1155*a^4*b^6*f*x^3 - 1155*a^3*b^7*x^3*e + 3465*a^2*b^8*c*x -
3465*a^3*b^7*d*x - 3465*a^5*b^5*f*x + 3465*a^4*b^6*x*e)/b^11
```

maple [A] time = 0.00, size = 278, normalized size = 1.32

$$\frac{f x^{11}}{11b} - \frac{a f x^9}{9b^2} + \frac{e x^9}{9b} + \frac{a^2 f x^7}{7b^3} - \frac{a e x^7}{7b^2} + \frac{d x^7}{7b} - \frac{a^3 f x^5}{5b^4} + \frac{a^2 e x^5}{5b^3} - \frac{a d x^5}{5b^2} + \frac{c x^5}{5b} + \frac{a^4 f x^3}{3b^5} - \frac{a^3 e x^3}{3b^4} + \frac{a^2 d x^3}{3b^3} - \frac{a c x^3}{3b^2} + \frac{a^6 f \operatorname{arc}\cdots}{\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/11*f*x^11/b-1/9/b^2*x^9*a*f+1/9/b*x^9*e+1/7/b^3*x^7*a^2*f-1/7/b^2*x^7*a*e
+1/7/b*x^7*d-1/5/b^4*x^5*a^3*f+1/5/b^3*x^5*a^2*e-1/5/b^2*x^5*a*d+1/5/b*x^5*
c+1/3/b^5*x^3*a^4*f-1/3/b^4*x^3*a^3*e+1/3/b^3*x^3*a^2*d-1/3/b^2*x^3*a*c-1/b
```

```
^6*a^5*f*x+1/b^5*a^4*e*x-1/b^4*a^3*d*x+1/b^3*a^2*c*x+a^6/b^6/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x)*f-a^5/b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+a
^4/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-a^3/b^3/(a*b)^(1/2)*arctan(1
/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.90, size = 213, normalized size = 1.01

$$-\frac{\left(a^3b^3c-a^4b^2d+a^5be-a^6f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^6}+\frac{315\,b^5fx^{11}+385\left(b^5e-ab^4f\right)x^9+495\left(b^5d-ab^4e+a^2b^3f\right)x^7+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + 1/3465*(315*b^5*f*x^11 + 385*(b^5*e - a*b^4*f)*x^9 + 495*(b^5*d - a
*b^4*e + a^2*b^3*f)*x^7 + 693*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5
 - 1155*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 + 3465*(a^2*b^3*c -
 a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6
```

mupad [B] time = 0.93, size = 289, normalized size = 1.38

$$x^9\left(\frac{e}{9\,b}-\frac{a\,f}{9\,b^2}\right)+x^7\left(\frac{d}{7\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{7\,b}\right)+x^5\left(\frac{c}{5\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{5\,b}\right)+\frac{f\,x^{11}}{11\,b}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c}{f\,a^6-e\,a^5\,b+d\,a^4\,b^2-c\,a^3\,b^3}\right.}{b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^9*(e/(9*b) - (a*f)/(9*b^2)) + x^7*(d/(7*b) - (a*(e/b - (a*f)/b^2))/(7*b))
 + x^5*(c/(5*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(5*b)) + (f*x^11)/(11
*b) + (a^(5/2)*atan((a^(5/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))
/(a^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*
b*e))/b^(13/2) - (a*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b
) + (a^2*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b^2
```

sympy [A] time = 1.65, size = 384, normalized size = 1.83

$$x^9\left(-\frac{af}{9b^2}+\frac{e}{9b}\right)+x^7\left(\frac{a^2f}{7b^3}-\frac{ae}{7b^2}+\frac{d}{7b}\right)+x^5\left(-\frac{a^3f}{5b^4}+\frac{a^2e}{5b^3}-\frac{ad}{5b^2}+\frac{c}{5b}\right)+x^3\left(\frac{a^4f}{3b^5}-\frac{a^3e}{3b^4}+\frac{a^2d}{3b^3}-\frac{ac}{3b^2}\right)+x\left(-\frac{a^5f}{b^6}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**9*(-a*f/(9*b**2) + e/(9*b)) + x**7*(a**2*f/(7*b**3) - a*e/(7*b**2) + d/(
7*b)) + x**5*(-a**3*f/(5*b**4) + a**2*e/(5*b**3) - a*d/(5*b**2) + c/(5*b))
+ x**3*(a**4*f/(3*b**5) - a**3*e/(3*b**4) + a**2*d/(3*b**3) - a*c/(3*b**2))
 + x*(-a**5*f/b**6 + a**4*e/b**5 - a**3*d/b**4 + a**2*c/b**3) - sqrt(-a**5/
b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**6*sqrt(-a**5/b**13)*
(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**5*f - a**4*b*e + a**3*b**2*d -
a**2*b**3*c) + x)/2 + sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**
3*c)*log(b**6*sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a*
*5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x)/2 + f*x**11/(11*b)
```

3.115 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2} dx$

Optimal. Leaf size=172

$$\frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3}-\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5}+\frac{x^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^{\cdot}\right.}{}$$

```
[Out] -a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^5+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
x^3/b^4+1/5*(a^2*f-a*b*e+b^2*d)*x^5/b^3+1/7*(-a*f+b*e)*x^7/b^2+1/9*f*x^9/b+
a^(3/2)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A] time = 0.12, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{x^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4}-\frac{ax\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^5}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+\right.}{b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^5)/(5*b^3) + ((b*e -
 a*f)*x^7)/(7*b^2) + (f*x^9)/(9*b) + (a^(3/2)*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx &= \int \left(-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{b^4}+\frac{\left(b^2d-a\right.}{}\right.\\
&= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^4}+\frac{\left(b^2d-abe\right.}{5l}\\
&= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^4}+\frac{\left(b^2d-abe\right.}{5l}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 162, normalized size = 0.94

$$\frac{x\left(315a^4f-105a^3b\left(3e+fx^2\right)+21a^2b^2\left(15d+5ex^2+3fx^4\right)-3ab^3\left(105c+35dx^2+21ex^4+15fx^6\right)+b^4x^2\left(\right.\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(315*a^4*f - 105*a^3*b*(3*e + f*x^2) + 21*a^2*b^2*(15*d + 5*e*x^2 + 3*f*
x^4) - 3*a*b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6) + b^4*x^2*(105*c +
63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5) - (a^(3/2)*(-(b^3*c) + a*b^2*d
- a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

fricas [A] time = 0.59, size = 368, normalized size = 2.14

$$\left[\frac{70\,b^4fx^9+90\left(b^4e-ab^3f\right)x^7+126\left(b^4d-ab^3e+a^2b^2f\right)x^5+210\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^3-315\left(ab^3c-a\right.}{630\,b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*f*x^9 + 90*(b^4*e - a*b^3*f)*x^7 + 126*(b^4*d - a*b^3*e + a^
2*b^2*f)*x^5 + 210*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 315*(a*b^3
*c - a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b)
- a)/(b*x^2 + a)) - 630*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5, 1/3
15*(35*b^4*f*x^9 + 45*(b^4*e - a*b^3*f)*x^7 + 63*(b^4*d - a*b^3*e + a^2*b^2
*f)*x^5 + 105*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 + 315*(a*b^3*c -
a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(a*b^3
*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5]
```

giac [A] time = 0.37, size = 200, normalized size = 1.16

$$\frac{\left(a^2b^3c-a^3b^2d-a^5f+a^4be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^5}+\frac{35\,b^8fx^9-45\,ab^7fx^7+45\,b^8x^7e+63\,b^8dx^5+63\,a^2b^6fx^5-63\,ab^7x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^5) + 1/315*(35*b^8*f*x^9 - 45*a*b^7*f*x^7 + 45*b^8*x^7*e + 63*b^8*d*x^5 +
 63*a^2*b^6*f*x^5 - 63*a*b^7*x^5*e + 105*b^8*c*x^3 - 105*a*b^7*d*x^3 - 105*
a^3*b^5*f*x^3 + 105*a^2*b^6*x^3*e - 315*a*b^7*c*x + 315*a^2*b^6*d*x + 315*a
^4*b^4*f*x - 315*a^3*b^5*x*e)/b^9
```

maple [A] time = 0.01, size = 230, normalized size = 1.34

$$\frac{fx^9}{9b}-\frac{afx^7}{7b^2}+\frac{ex^7}{7b}+\frac{a^2fx^5}{5b^3}-\frac{aex^5}{5b^2}+\frac{dx^5}{5b}-\frac{a^3fx^3}{3b^4}+\frac{a^2ex^3}{3b^3}-\frac{adx^3}{3b^2}+\frac{cx^3}{3b}-\frac{a^5f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^5}+\frac{a^4e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^4}-\frac{a^3d\,}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/9*f*x^9/b-1/7/b^2*x^7*a*f+1/7/b*x^7*e+1/5/b^3*x^5*a^2*f-1/5/b^2*x^5*a*e+1
/5/b*x^5*d-1/3/b^4*x^3*a^3*f+1/3/b^3*x^3*a^2*e-1/3/b^2*x^3*a*d+1/3/b*x^3*c+
1/b^5*a^4*f*x-1/b^4*a^3*e*x+1/b^3*a^2*d*x-1/b^2*a*c*x-a^5/b^5/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x)*f+a^4/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-
a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+a^2/b^2/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.93, size = 172, normalized size = 1.00

$$\frac{\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^5}+\frac{35\,b^4fx^9+45\left(b^4e-ab^3f\right)x^7+63\left(b^4d-ab^3e+a^2b^2f\right)x^5+105\left(b^4\right.}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^5) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e - a*b^3*f)*x^7 + 63*(b^4*d - a*b^3*e
 + a^2*b^2*f)*x^5 + 105*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 315*(
a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5
```

mupad [B] time = 0.94, size = 243, normalized size = 1.41

$$x^7\left(\frac{e}{7\,b}-\frac{a\,f}{7\,b^2}\right)+x^5\left(\frac{d}{5\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{5\,b}\right)+x^3\left(\frac{c}{3\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^9}{9\,b}-\frac{a\,x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b}-\frac{a^{3/2}\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^7*(e/(7*b) - (a*f)/(7*b^2)) + x^5*(d/(5*b) - (a*(e/b - (a*f)/b^2))/(5*b))
 + x^3*(c/(3*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(3*b)) + (f*x^9)/(9*b
) - (a*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b - (a^(3/2)*atan((
a^(3/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^5*f - a^2*b^3*c +
 a^3*b^2*d - a^4*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/b^(11/2)
```

sympy [B] time = 1.32, size = 337, normalized size = 1.96

$$x^7\left(-\frac{af}{7b^2}+\frac{e}{7b}\right)+x^5\left(\frac{a^2f}{5b^3}-\frac{ae}{5b^2}+\frac{d}{5b}\right)+x^3\left(-\frac{a^3f}{3b^4}+\frac{a^2e}{3b^3}-\frac{ad}{3b^2}+\frac{c}{3b}\right)+x\left(\frac{a^4f}{b^5}-\frac{a^3e}{b^4}+\frac{a^2d}{b^3}-\frac{ac}{b^2}\right)+\frac{\sqrt{-\frac{a^3}{b^{11}}}\,(a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**7*(-a*f/(7*b**2) + e/(7*b)) + x**5*(a**2*f/(5*b**3) - a*e/(5*b**2) + d/(
5*b)) + x**3*(-a**3*f/(3*b**4) + a**2*e/(3*b**3) - a*d/(3*b**2) + c/(3*b))
+ x*(a**4*f/b**5 - a**3*e/b**4 + a**2*d/b**3 - a*c/b**2) + sqrt(-a**3/b**11
)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**5*sqrt(-a**3/b**11)*(a**3
*f - a**2*b*e + a*b**2*d - b**3*c)/(a**4*f - a**3*b*e + a**2*b**2*d - a*b**
3*c) + x)/2 - sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log
(b**5*sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**4*f - a
**3*b*e + a**2*b**2*d - a*b**3*c) + x)/2 + f*x**9/(9*b)
```

3.116 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

Optimal. Leaf size=136

$$\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^{9/2}}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{x^5(be-a}{5b^2}$$

```
[Out] (-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4+1/3*(a^2*f-a*b*e+b^2*d)*x^3/b^3+1/5*(-
a*f+b*e)*x^5/b^2+1/7*f*x^7/b-(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2
)/a^(1/2))*a^(1/2)/b^(9/2)
```

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^{9/2}}+\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{x^5(be-a}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^3)
/(3*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^7)/(7*b) - (Sqrt[a]*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx &= \int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^2}{b^3}+\frac{(be-af)x^4}{b^2}+\frac{fx^6}{b}+\frac{-a\right.\\ &= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^3}{3b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^7}{7b}-\frac{(a\\ &= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^3}{3b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^7}{7b}-\frac{\sqrt{a}\end{aligned}$$

Mathematica [A] time = 0.10, size = 128, normalized size = 0.94

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^{9/2}}+\frac{x\left(-105a^3f+35a^2b\left(3e+fx^2\right)-7ab^2\left(15d+5ex^2+3fx^4\right)+b^3(10}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(-105*a^3*f + 35*a^2*b*(3*e + f*x^2) - 7*a*b^2*(15*d + 5*e*x^2 + 3*f*x^4
) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4) + (Sqrt[a]*(-(
b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

fricas [A] time = 0.69, size = 286, normalized size = 2.10

$$\left[\frac{30\,b^3 f x^7 + 42\left(b^3 e - a b^2 f\right)x^5 + 70\left(b^3 d - a b^2 e + a^2 b f\right)x^3 - 105\left(b^3 c - a b^2 d + a^2 b e - a^3 f\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{b x^2 + 2\,b x\sqrt{\cdots}}{b x^2 + \cdots}\right.}{210\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*f*x^7 + 42*(b^3*e - a*b^2*f)*x^5 + 70*(b^3*d - a*b^2*e + a^2
*b*f)*x^3 - 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a/b)*log((b*x^2 +
 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*x)/b^4, 1/105*(15*b^3*f*x^7 + 21*(b^3*e - a*b^2*f)*x^5 + 35*(b^3*d - a*b
^2*e + a^2*b*f)*x^3 - 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(a/b)*arc
tan(b*x*sqrt(a/b)/a) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4]
```

giac [A] time = 0.42, size = 152, normalized size = 1.12

$$-\frac{\left(a b^3 c - a^2 b^2 d - a^4 f + a^3 b e\right)\arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^4} + \frac{15\, b^6 f x^7 - 21\, a b^5 f x^5 + 21\, b^6 x^5 e + 35\, b^6 d x^3 + 35\, a^2 b^4 f x^3 - 35\, a\cdots}{105\, b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^4) + 1/105*(15*b^6*f*x^7 - 21*a*b^5*f*x^5 + 21*b^6*x^5*e + 35*b^6*d*x^3 +
35*a^2*b^4*f*x^3 - 35*a*b^5*x^3*e + 105*b^6*c*x - 105*a*b^5*d*x - 105*a^3*b
^3*f*x + 105*a^2*b^4*x*e)/b^7
```

maple [A] time = 0.00, size = 182, normalized size = 1.34

$$\frac{f x^7}{7b} - \frac{a f x^5}{5b^2} + \frac{e x^5}{5b} + \frac{a^2 f x^3}{3b^3} - \frac{a e x^3}{3b^2} + \frac{d x^3}{3b} + \frac{a^4 f \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\; b^4} - \frac{a^3 e \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\; b^3} + \frac{a^2 d \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\; b^2} - \frac{a c \arctan\left(\cdots\right.}{\sqrt{a b}\; b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/7*f*x^7/b-1/5/b^2*x^5*a*f+1/5/b*x^5*e+1/3/b^3*x^3*a^2*f-1/3/b^2*x^3*a*e+1
/3/b*x^3*d-1/b^4*a^3*f*x+1/b^3*a^2*e*x-1/b^2*a*d*x+1/b*c*x+a^4/b^4/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)*f-a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*
x)*e+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-a/b/(a*b)^(1/2)*arctan
(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.96, size = 133, normalized size = 0.98

$$-\frac{\left(a b^3 c - a^2 b^2 d + a^3 b e - a^4 f\right)\arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^4} + \frac{15\, b^3 f x^7 + 21\left(b^3 e - a b^2 f\right)x^5 + 35\left(b^3 d - a b^2 e + a^2 b f\right)x^3 + 105\,(\cdots}{105\, b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^4) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - a*b^2*f)*x^5 + 35*(b^3*d - a*b^2*e
+ a^2*b*f)*x^3 + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4
```

mupad [B] time = 0.91, size = 193, normalized size = 1.42

$$x^5\left(\frac{e}{5\,b}-\frac{a\,f}{5\,b^2}\right)+x^3\left(\frac{d}{3\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{3\,b}\right)+x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)+\frac{f\,x^7}{7\,b}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3}\right)}{b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^5*(e/(5*b) - (a*f)/(5*b^2)) + x^3*(d/(3*b) - (a*(e/b - (a*f)/b^2))/(3*b))
 + x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b) + (f*x^7)/(7*b) + (a^(1/
2)*atan((a^(1/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^4*f + a^
2*b^2*d - a*b^3*c - a^3*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/b^(9/2)
```

sympy [A] time = 1.13, size = 185, normalized size = 1.36

$$x^5\left(-\frac{af}{5b^2}+\frac{e}{5b}\right)+x^3\left(\frac{a^2f}{3b^3}-\frac{ae}{3b^2}+\frac{d}{3b}\right)+x\left(-\frac{a^3f}{b^4}+\frac{a^2e}{b^3}-\frac{ad}{b^2}+\frac{c}{b}\right)-\frac{\sqrt{-\frac{a}{b^9}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-b^4\sqrt{-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**5*(-a*f/(5*b**2) + e/(5*b)) + x**3*(a**2*f/(3*b**3) - a*e/(3*b**2) + d/(
3*b)) + x*(-a**3*f/b**4 + a**2*e/b**3 - a*d/b**2 + c/b) - sqrt(-a/b**9)*(a*
*3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**4*sqrt(-a/b**9) + x)/2 + sqrt(
-a/b**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(b**4*sqrt(-a/b**9) + x
)/2 + f*x**7/(7*b)
```

3.117 $\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2} \, dx$

Optimal. Leaf size=100

$$\frac{x\left(a^2f-abe+b^2d\right)}{b^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{a}\,b^{7/2}}+\frac{x^3(be-af)}{3b^2}+\frac{fx^5}{5b}$$

```
[Out] (a^2*f-a*b*e+b^2*d)*x/b^3+1/3*(-a*f+b*e)*x^3/b^2+1/5*f*x^5/b+(-a^3*f+a^2*b*
e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1810, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{a}\,b^{7/2}}+\frac{x\left(a^2f-abe+b^2d\right)}{b^3}+\frac{x^3(be-af)}{3b^2}+\frac{fx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^3)/(3*b^2) + (f*x^5)/(5*b)
 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx &= \int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x^2}{b^2}+\frac{fx^4}{b}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\left(a+bx^2\right)}\right)dx\\ &= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^3}{3b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{a+bx^2}\,dx}{b^3}\\ &= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^3}{3b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 98, normalized size = 0.98

$$\frac{x\left(15a^2f-5ab\left(3e+fx^2\right)+b^2\left(15d+5ex^2+3fx^4\right)\right)}{15b^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{a}\,b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

```
[Out] (x*(15*a^2*f - 5*a*b*(3*e + f*x^2) + b^2*(15*d + 5*e*x^2 + 3*f*x^4)))/(15*b
^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sq
rt[a]*b^(7/2))
```

fricas [A] time = 0.72, size = 236, normalized size = 2.36

$$\left[\frac{6\,ab^3 f x^5 + 10\left(ab^3e - a^2b^2f\right)x^3 + 15\left(b^3c - ab^2d + a^2be - a^3f\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right) + 30\left(ab^3d - a^2b^2e + a\right.}{30\,ab^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*f*x^5 + 10*(a*b^3*e - a^2*b^2*f)*x^3 + 15*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a))
+ 30*(a*b^3*d - a^2*b^2*e + a^3*b*f)*x)/(a*b^4), 1/15*(3*a*b^3*f*x^5 + 5*(a
*b^3*e - a^2*b^2*f)*x^3 + 15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a) + 15*(a*b^3*d - a^2*b^2*e + a^3*b*f)*x)/(a*b^4)]
```

giac [A] time = 0.45, size = 106, normalized size = 1.06

$$\frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3\,b^4fx^5 - 5\,ab^3fx^3 + 5\,b^4x^3e + 15\,b^4dx + 15\,a^2b^2fx - 15\,ab^3xe}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) +
 1/15*(3*b^4*f*x^5 - 5*a*b^3*f*x^3 + 5*b^4*x^3*e + 15*b^4*d*x + 15*a^2*b^2*
f*x - 15*a*b^3*x*e)/b^5
```

maple [A] time = 0.00, size = 135, normalized size = 1.35

$$\frac{f\,x^5}{5b} - \frac{af\,x^3}{3b^2} + \frac{e\,x^3}{3b} - \frac{a^3f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3} + \frac{a^2e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2} - \frac{ad\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} + \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{a^2fx}{b^3} - \frac{aex}{b^2} + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/5*f*x^5/b-1/3/b^2*x^3*a*f+1/3/b*x^3*e+1/b^3*a^2*f*x-1/b^2*a*e*x+1/b*d*x-1
/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^3*f+1/b^2/(a*b)^(1/2)*arctan(1
/(a*b)^(1/2)*b*x)*a^2*e-1/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*d+1/(a*
b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.97, size = 94, normalized size = 0.94

$$\frac{\left(b^3c - ab^2d + a^2be - a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3\,b^2fx^5 + 5\left(b^2e - abf\right)x^3 + 15\left(b^2d - abe + a^2f\right)x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) +
 1/15*(3*b^2*f*x^5 + 5*(b^2*e - a*b*f)*x^3 + 15*(b^2*d - a*b*e + a^2*f)*x)/
b^3
```

mupad [B] time = 0.94, size = 96, normalized size = 0.96

$$x^3\left(\frac{e}{3\,b}-\frac{a\,f}{3\,b^2}\right)+x\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)+\frac{f\,x^5}{5\,b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x)
```

```
[Out] x^3*(e/(3*b) - (a*f)/(3*b^2)) + x*(d/b - (a*(e/b - (a*f)/b^2))/b) + (f*x^5)
/(5*b) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a
^(1/2)*b^(7/2))
```

sympy [A] time = 1.15, size = 160, normalized size = 1.60

$$x^3\left(-\frac{af}{3b^2}+\frac{e}{3b}\right)+x\left(\frac{a^2f}{b^3}-\frac{ae}{b^2}+\frac{d}{b}\right)+\frac{\sqrt{-\frac{1}{ab^7}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^7}}\left(a^3f-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**3*(-a*f/(3*b**2) + e/(3*b)) + x*(a**2*f/b**3 - a*e/b**2 + d/b) + sqrt(-1
/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a*b**3*sqrt(-1/(a*b
**7)) + x)/2 - sqrt(-1/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*lo
g(a*b**3*sqrt(-1/(a*b**7)) + x)/2 + f*x**5/(5*b)
```

3.118 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=84

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

```
[Out] -c/a/x+(-a*f+b*e)*x/b^2+1/3*f*x^3/b-(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x
*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)}\,dx &= \int \left(\frac{be-af}{b^2}+\frac{c}{ax^2}+\frac{fx^2}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2\left(a+bx^2\right)}\right)dx\\ &= -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^2}\,dx}{ab^2}\\ &= -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) + ((-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

fricas [A] time = 0.52, size = 211, normalized size = 2.51

$$\left[\frac{2\,a^2b^2fx^4 - 6\,ab^3c + 3\left(b^3c - ab^2d + a^2be - a^3f\right)\sqrt{-ab}\,x\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 6\left(a^2b^2e - a^3bf\right)x^2}{6\,a^2b^3x}, \frac{a^2b^2fx^4 -}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b^2*f*x^4 - 6*a*b^3*c + 3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*s
qrt(-a*b)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a^2*b^2*e -
a^3*b*f)*x^2)/(a^2*b^3*x), 1/3*(a^2*b^2*f*x^4 - 3*a*b^3*c - 3*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a) + 3*(a^2*b^2*e - a
^3*b*f)*x^2)/(a^2*b^3*x)]
```

giac [A] time = 0.34, size = 86, normalized size = 1.02

$$-\frac{c}{ax} - \frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2} + \frac{b^2fx^3 - 3\,abfx + 3\,b^2xe}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -c/(a*x) - (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a*b^2) + 1/3*(b^2*f*x^3 - 3*a*b*f*x + 3*b^2*x*e)/b^3
```

maple [A] time = 0.01, size = 114, normalized size = 1.36

$$\frac{fx^3}{3b} + \frac{a^2f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2} - \frac{ae\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} - \frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a} + \frac{d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{afx}{b^2} + \frac{ex}{b} - \frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x)
```

```
[Out] 1/3*f*x^3/b-1/b^2*a*f*x+1/b*e*x+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*
x)*f-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+1/(a*b)^(1/2)*arctan(1/(a*
b)^(1/2)*b*x)*d-1/(a*b)^(1/2)/a*b*c*arctan(1/(a*b)^(1/2)*b*x)-1/a*c/x
```

maxima [A] time = 2.93, size = 80, normalized size = 0.95

$$\frac{bfx^3 + 3\left(be - af\right)x}{3\,b^2} - \frac{c}{ax} - \frac{\left(b^3c - ab^2d + a^2be - a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b*f*x^3 + 3*(b*e - a*f)*x)/b^2 - c/(a*x) - (b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

mupad [B] time = 1.07, size = 76, normalized size = 0.90

$$x\left(\frac{e}{b} - \frac{a\,f}{b^2}\right) - \frac{c}{a\,x} + \frac{f\,x^3}{3\,b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{a^{3/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x)
```

```
[Out] x*(e/b - (a*f)/b^2) - c/(a*x) + (f*x^3)/(3*b) - (atan((b^(1/2)*x)/a^(1/2))*
(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(3/2)*b^(5/2))
```

sympy [B] time = 1.64, size = 150, normalized size = 1.79

$$x\left(-\frac{af}{b^2}+\frac{e}{b}\right)-\frac{\sqrt{-\frac{1}{a^3b^5}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b^5}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] x*(-a*f/b**2 + e/b) - sqrt(-1/(a**3*b**5))*(a**3*f - a**2*b*e + a*b**2*d -
b**3*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/2 + sqrt(-1/(a**3*b**5))*(
a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**2*b**2*sqrt(-1/(a**3*b**5)) +
 x)/2 + f*x**3/(3*b) - c/(a*x)
```

3.119 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=82

$$\frac{bc-ad}{a^2x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{5/2}b^{3/2}}-\frac{c}{3ax^3}+\frac{fx}{b}$$

```
[Out] -1/3*c/a/x^3+(-a*d+b*c)/a^2/x+f*x/b+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x
*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{5/2}b^{3/2}}+\frac{bc-ad}{a^2x}-\frac{c}{3ax^3}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b + ((b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)}\,dx &= \int \left(\frac{f}{b}+\frac{c}{ax^4}+\frac{-bc+ad}{a^2x^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b\left(a+bx^2\right)}\right)dx\\
&= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{a+bx^2}\,dx}{a^2b}\\
&= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 1.01

$$\frac{bc-ad}{a^2x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{5/2}b^{3/2}}-\frac{c}{3ax^3}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b - ((-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

fricas [A] time = 0.68, size = 216, normalized size = 2.63

$$\left[\frac{6\,a^3bfx^4+3\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-ab}\,x^3\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,a^2b^2c+6\left(ab^3c-a^2b^2d\right)x^2}{6\,a^3b^2x^3},\ \frac{3\,a^3bfx^4+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^3*b*f*x^4 + 3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a*b)*x^3*
log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*a^2*b^2*c + 6*(a*b^3*c -
a^2*b^2*d)*x^2)/(a^3*b^2*x^3), 1/3*(3*a^3*b*f*x^4 + 3*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*sqrt(a*b)*x^3*arctan(sqrt(a*b)*x/a) - a^2*b^2*c + 3*(a*b^3*c
 - a^2*b^2*d)*x^2)/(a^3*b^2*x^3)]
```

giac [A] time = 0.36, size = 81, normalized size = 0.99

$$\frac{fx}{b}+\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2b}+\frac{3\,bcx^2-3\,adx^2-ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] f*x/b + (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^2*b) + 1/3*(3*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^2*x^3)
```

maple [A] time = 0.01, size = 115, normalized size = 1.40

$$-\frac{af\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}-\frac{bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{b^2c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}+\frac{e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{fx}{b}-\frac{d}{ax}+\frac{bc}{a^2x}-\frac{c}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x)
```

```
[Out] f*x/b-1/b*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+1/(a*b)^(1/2)*arctan(1/
(a*b)^(1/2)*b*x)*e-b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+b^2/a^2/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/3*c/a/x^3-1/a/x*d+1/a^2/x*b*c
```

maxima [A] time = 2.94, size = 79, normalized size = 0.96

$$\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2b}+\frac{3\,(bc-ad)x^2-ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] f*x/b + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^2*b) + 1/3*(3*(b*c - a*d)*x^2 - a*c)/(a^2*x^3)
```

mupad [B] time = 0.11, size = 80, normalized size = 0.98

$$\frac{f\,x}{b}-\frac{\frac{b\,c}{3\,a}+\frac{b\,x^2\,(a\,d-b\,c)}{a^2}}{b\,x^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x)
```

```
[Out] (f*x)/b - ((b*c)/(3*a) + (b*x^2*(a*d - b*c))/a^2)/(b*x^3) + (atan((b^(1/2)*
x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(5/2)*b^(3/2))
```

sympy [B] time = 2.27, size = 151, normalized size = 1.84

$$\frac{\sqrt{-\frac{1}{a^5b^3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{a^5b^3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**3*b*sq
rt(-1/(a**5*b**3)) + x)/2 - sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**
2*d - b**3*c)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/2 + f*x/b + (-a*c + x**2
*(-3*a*d + 3*b*c))/(3*a**2*x**3)
```

3.120 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)} \, dx$

Optimal. Leaf size=104

$$\frac{bc-ad}{3a^2x^3}-\frac{a^2e-abd+b^2c}{a^3x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{c}{5ax^5}$$

```
[Out] -1/5*c/a/x^5+1/3*(-a*d+b*c)/a^2/x^3+(-a^2*e+a*b*d-b^2*c)/a^3/x-(-a^3*f+a^2*
b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1802, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{a^2e-abd+b^2c}{a^3x}+\frac{bc-ad}{3a^2x^3}-\frac{c}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(3*a^2*x^3) - (b^2*c - a*b*d + a^2*e)/(a^3*x) -
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*
Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)} \, dx &= \int \left(\frac{c}{ax^6}+\frac{-bc+ad}{a^2x^4}+\frac{b^2c-abd+a^2e}{a^3x^2}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3\left(a+bx^2\right)}\right) dx\\
&= -\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^2} \, dx}{a^3}\\
&= -\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}-\frac{b^2c-abd+a^2e}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 103, normalized size = 0.99

$$\frac{bc-ad}{3a^2x^3}+\frac{a^2(-e)+abd-b^2c}{a^3x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{c}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/5*c/(a*x^5) + (b*c - a*d)/(3*a^2*x^3) + (-(b^2*c) + a*b*d - a^2*e)/(a^3*
x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
a^(7/2)*Sqrt[b])
```

fricas [A] time = 0.78, size = 246, normalized size = 2.37

$$\left[\frac{15\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\sqrt{-ab}\,x^{5}\log\left(\frac{bx^{2}-2\sqrt{-ab}\,x-a}{bx^{2}+a}\right)-6\,a^{3}bc-30\left(ab^{3}c-a^{2}b^{2}d+a^{3}be\right)x^{4}+10\left(a^{2}b^{2}c-\right.}{30\,a^{4}bx^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a*b)*x^5*log((b*x^2 - 2
*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*a^3*b*c - 30*(a*b^3*c - a^2*b^2*d + a^3
*b*e)*x^4 + 10*(a^2*b^2*c - a^3*b*d)*x^2)/(a^4*b*x^5), -1/15*(15*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*sqrt(a*b)*x^5*arctan(sqrt(a*b)*x/a) + 3*a^3*b*c +
 15*(a*b^3*c - a^2*b^2*d + a^3*b*e)*x^4 - 5*(a^2*b^2*c - a^3*b*d)*x^2)/(a^4
*b*x^5)]
```

giac [A] time = 0.37, size = 105, normalized size = 1.01

$$-\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{3}}-\frac{15\,b^{2}cx^{4}-15\,abdx^{4}+15\,a^{2}x^{4}e-5\,abcx^{2}+5\,a^{2}dx^{2}+3\,a^{2}c}{15\,a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
- 1/15*(15*b^2*c*x^4 - 15*a*b*d*x^4 + 15*a^2*x^4*e - 5*a*b*c*x^2 + 5*a^2*d*
x^2 + 3*a^2*c)/(a^3*x^5)
```

maple [A] time = 0.01, size = 142, normalized size = 1.37

$$-\frac{be\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{b^{2}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^{2}}-\frac{b^{3}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^{3}}+\frac{f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{e}{ax}+\frac{bd}{a^{2}x}-\frac{b^{2}c}{a^{3}x}-\frac{d}{3a\,x^{3}}+\frac{bc}{3a^{2}x^{3}}-\frac{c}{5a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-1/a/(a*b)^(1/2)*arctan(1/(a*b)^(1
/2)*b*x)*b*e+1/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^2*d-1/a^3/(a*b)^
(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^3*c-1/5*c/a/x^5-1/3/a/x^3*d+1/3/a^2/x^3*b
*c-1/a/x*e+1/a^2/x*b*d-1/a^3/x*b^2*c
```

maxima [A] time = 3.09, size = 97, normalized size = 0.93

$$-\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{3}}-\frac{15\left(b^{2}c-abd+a^{2}e\right)x^{4}+3\,a^{2}c-5\left(abc-a^{2}d\right)x^{2}}{15\,a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
- 1/15*(15*(b^2*c - a*b*d + a^2*e)*x^4 + 3*a^2*c - 5*(a*b*c - a^2*d)*x^2)/(
a^3*x^5)
```

mupad [B] time = 1.20, size = 94, normalized size = 0.90

$$-\frac{\frac{c}{5\,a}+\frac{x^2\,\left(a\,d-b\,c\right)}{3\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{a^3}}{x^5}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{7/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x)
```

```
[Out] - (c/(5*a) + (x^2*(a*d - b*c))/(3*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/a^3)
/x^5 - (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(
7/2)*b^(1/2))
```

sympy [A] time = 6.73, size = 167, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^7b}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^7b}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**4*sqrt(-
1/(a**7*b)) + x)/2 + sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3
*c)*log(a**4*sqrt(-1/(a**7*b)) + x)/2 + (-3*a**2*c + x**4*(-15*a**2*e + 15*
a*b*d - 15*b**2*c) + x**2*(-5*a**2*d + 5*a*b*c))/(15*a**3*x**5)
```

3.121 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=137

$$\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{9/2}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{7ax^7}$$

```
[Out] -1/7*c/a/x^7+1/5*(-a*d+b*c)/a^2/x^5+1/3*(-a^2*e+a*b*d-b^2*c)/a^3/x^3+(-a^3*
f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1
/2)/a^(1/2))*b^(1/2)/a^(9/2)
```

Rubi [A] time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{a^4x}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{9/2}}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{bc-ad}{5a^2x^5}-\frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x^3
) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) + (Sqrt[b]*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)}\,dx &= \int \left(\frac{c}{ax^8}+\frac{-bc+ad}{a^2x^6}+\frac{b^2c-abd+a^2e}{a^3x^4}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2}-\frac{b\left(-b^3c+\right.}{a^4}\right.\\ &= -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}+\left(b\left(b^3c-ab^2d\right.\right.\\ &= -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}+\frac{\sqrt{b}\left(b^3c-ab^2d\right.}{}\end{aligned}$$

Mathematica [A] time = 0.12, size = 139, normalized size = 1.01

$$\frac{bc-ad}{5a^2x^5}+\frac{a^2(-e)+abd-b^2c}{3a^3x^3}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{9/2}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/7*c/(a*x^7) + (b*c - a*d)/(5*a^2*x^5) + (-(b^2*c) + a*b*d - a^2*e)/(3*a^
3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (Sqrt[b]*(-(b^3*c) +
 a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

fricas [A] time = 0.65, size = 292, normalized size = 2.13

$$\left[-\frac{105\left(b^3c-ab^2d+a^2be-a^3f\right)x^7\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-210\left(b^3c-ab^2d+a^2be-a^3f\right)x^6+70\left(ab^2c-a^2bd\right.}{210\,a^4x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/210*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*sqrt(-b/a)*log((b*x^2
- 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 210*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x^6 + 70*(a*b^2*c - a^2*b*d + a^3*e)*x^4 + 30*a^3*c - 42*(a^2*b*c - a^3
*d)*x^2)/(a^4*x^7), 1/105*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*sqrt
(b/a)*arctan(x*sqrt(b/a)) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6 - 3
5*(a*b^2*c - a^2*b*d + a^3*e)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x^2)/(a
^4*x^7)]
```

giac [A] time = 0.46, size = 151, normalized size = 1.10

$$\frac{\left(b^4c-ab^3d-a^3bf+a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}+\frac{105\,b^3cx^6-105\,ab^2dx^6-105\,a^3fx^6+105\,a^2bx^6e-35\,ab^2cx^4+35\,a^2}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
4) + 1/105*(105*b^3*c*x^6 - 105*a*b^2*d*x^6 - 105*a^3*f*x^6 + 105*a^2*b*x^6
*e - 35*a*b^2*c*x^4 + 35*a^2*b*d*x^4 - 35*a^3*x^4*e + 21*a^2*b*c*x^2 - 21*a
^3*d*x^2 - 15*a^3*c)/(a^4*x^7)
```

maple [A] time = 0.01, size = 190, normalized size = 1.39

$$-\frac{bf\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{b^2e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}-\frac{b^3d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^3}+\frac{b^4c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^4}-\frac{f}{ax}+\frac{be}{a^2x}-\frac{b^2d}{a^3x}+\frac{b^3c}{a^4x}-\frac{e}{3a\,x^3}+\frac{bd}{3a^2x^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x)
```

```
[Out] -b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+b^2/a^2/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)*e-b^3/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+b^4/a^4/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/7*c/a/x^7-1/5/a/x^5*d+1/5/a^2/x^5*
b*c-1/3/a/x^3*e+1/3/a^2/x^3*b*d-1/3/a^3/x^3*b^2*c-1/a/x*f+1/a^2/x*b*e-1/a^3
/x*b^2*d+1/a^4/x*b^3*c
```

maxima [A] time = 3.03, size = 134, normalized size = 0.98

$$\frac{\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}+\frac{105\left(b^3c-ab^2d+a^2be-a^3f\right)x^6-35\left(ab^2c-a^2bd+a^3e\right)x^4-15\,a^3c}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
4) + 1/105*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6 - 35*(a*b^2*c - a^2
*b*d + a^3*e)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x^2)/(a^4*x^7)
```

mupad [B] time = 0.98, size = 127, normalized size = 0.93

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{9/2}}-\frac{\frac{c}{7\,a}-\frac{x^6\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^4}+\frac{x^2\,(a\,d-b\,c)}{5\,a^2}+\frac{x^4\left(e\,a^2-d\,a\,b+c\,b^2\right)}{3\,a^3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(
9/2) - (c/(7*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4 + (x^2*(a*d
 - b*c))/(5*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(3*a^3))/x^7
```

sympy [B] time = 21.65, size = 301, normalized size = 2.20

$$\frac{\sqrt{-\frac{b}{a^9}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3bf-a^2b^2e+ab^3d-b^4c}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^9}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\cdots\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**5*sqrt(-b/a**
9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f - a**2*b**2*e + a*b**3
*d - b**4*c) + x)/2 - sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)
*log(a**5*sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f -
 a**2*b**2*e + a*b**3*d - b**4*c) + x)/2 + (-15*a**3*c + x**6*(-105*a**3*f
+ 105*a**2*b*e - 105*a*b**2*d + 105*b**3*c) + x**4*(-35*a**3*e + 35*a**2*b*
d - 35*a*b**2*c) + x**2*(-21*a**3*d + 21*a**2*b*c))/(105*a**4*x**7)
```

3.122 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=175

$$\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{11/2}}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5x}+\frac{a^3(-f$$

```
[Out] -1/9*c/a/x^9+1/7*(-a*d+b*c)/a^2/x^7+1/5*(-a^2*e+a*b*d-b^2*c)/a^3/x^5+1/3*(-
a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^3-b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^5/x
-b^(3/2)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)
```

Rubi [A] time = 0.15, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^4x^3}-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^5x}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{11/2}}-\frac{a^2e$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(5*a^3*x^5
) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) - (b*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f))/(a^5*x) - (b^(3/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}\,dx=\int \left(\frac{c}{ax^{10}}+\frac{-bc+ad}{a^2x^8}+\frac{b^2c-abd+a^2e}{a^3x^6}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^4}-\frac{b\left(-b^3c+ab\right.}{}\right.$$

$$=-\frac{c}{9ax^9}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{5a^3x^5}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}-\frac{b\left(b^3c-ab^2d+a^2\right.}{a^5x}$$

$$=-\frac{c}{9ax^9}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{5a^3x^5}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}-\frac{b\left(b^3c-ab^2d+a^2\right.}{a^5x}$$

Mathematica [A] time = 0.14, size = 174, normalized size = 0.99

$$\frac{bc-ad}{7a^2x^7}+\frac{a^2(-e)+abd-b^2c}{5a^3x^5}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{11/2}}+\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5x}+\frac{a^3(-f)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -1/9*c/(a*x^9) + (b*c - a*d)/(7*a^2*x^7) + (-(b^2*c) + a*b*d - a^2*e)/(5*a^
3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(-(b^3*c) + a
*b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(3/2)*(-(b^3*c) + a*b^2*d - a^2*b*e
 + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

fricas [A] time = 0.75, size = 374, normalized size = 2.14

$$\left[-\frac{315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^9\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+630\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^8-210\left(ab^3\right.}{630a^5x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/630*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 630*(b^4*c - a*b^3*d + a^2*b^2*e
 - a^3*b*f)*x^8 - 210*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 70*a^4*
c + 126*(a^2*b^2*c - a^3*b*d + a^4*e)*x^4 - 90*(a^3*b*c - a^4*d)*x^2)/(a^5*
x^9), -1/315*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(b/a)*arc
tan(x*sqrt(b/a)) + 315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^8 - 105*(a
*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 35*a^4*c + 63*(a^2*b^2*c - a^3*
b*d + a^4*e)*x^4 - 45*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9)]
```

giac [A] time = 0.42, size = 201, normalized size = 1.15

$$-\frac{\left(b^5c-ab^4d-a^3b^2f+a^2b^3e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^5}-\frac{315\,b^4cx^8-315\,ab^3dx^8-315\,a^3bfx^8+315\,a^2b^2x^8e-105\,ab^3cx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^5) - 1/315*(315*b^4*c*x^8 - 315*a*b^3*d*x^8 - 315*a^3*b*f*x^8 + 315*a^2*
b^2*x^8*e - 105*a*b^3*c*x^6 + 105*a^2*b^2*d*x^6 + 105*a^4*f*x^6 - 105*a^3*b
*x^6*e + 63*a^2*b^2*c*x^4 - 63*a^3*b*d*x^4 + 63*a^4*x^4*e - 45*a^3*b*c*x^2
+ 45*a^4*d*x^2 + 35*a^4*c)/(a^5*x^9)
```

maple [A] time = 0.01, size = 238, normalized size = 1.36

$$\frac{b^2f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}-\frac{b^3e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^3}+\frac{b^4d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^4}-\frac{b^5c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^5}+\frac{bf}{a^2x}-\frac{b^2e}{a^3x}+\frac{b^3d}{a^4x}-\frac{b^4c}{a^5x}-\frac{f}{3a\,x^3}+\frac{be}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x)
```

```
[Out] b^2/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-b^3/a^3/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*e+b^4/a^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-b^5/a^
5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/9*c/a/x^9-1/7/a/x^7*d+1/7/a^2/x
^7*b*c-1/5/a/x^5*e+1/5/a^2/x^5*b*d-1/5/a^3/x^5*b^2*c-1/3/a/x^3*f+1/3/a^2/x^
3*b*e-1/3/a^3/x^3*b^2*d+1/3/a^4/x^3*b^3*c+1/a^2*b/x*f-1/a^3*b^2/x*e+1/a^4*b
^3/x*d-1/a^5*b^4/x*c
```

maxima [A] time = 3.01, size = 175, normalized size = 1.00

$$-\frac{\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^5}-\frac{315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^8-105\left(ab^3c-a^2b^2d+a^3be-\right.}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^5) - 1/315*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^8 - 105*(a*b^3
*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 35*a^4*c + 63*(a^2*b^2*c - a^3*b*d
+ a^4*e)*x^4 - 45*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9)
```

mupad [B] time = 1.02, size = 161, normalized size = 0.92

$$-\frac{\frac{c}{9\,a}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^4}+\frac{x^2\,\left(a\,d-b\,c\right)}{7\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{5\,a^3}+\frac{b\,x^8\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^5}}{x^9}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+\right.}{a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x)
```

```
[Out] - (c/(9*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^4) + (x^2*(a*d
- b*c))/(7*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(5*a^3) + (b*x^8*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/a^5)/x^9 - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(11/2)
```

sympy [B] time = 32.72, size = 354, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^2f-a^2b^3e+ab^4d-b^5c}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**6*sqrt(-
b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**2*f - a**2*b**
3*e + a*b**4*d - b**5*c) + x)/2 + sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*
b**2*d - b**3*c)*log(a**6*sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d -
 b**3*c)/(a**3*b**2*f - a**2*b**3*e + a*b**4*d - b**5*c) + x)/2 + (-35*a**4
*c + x**8*(315*a**3*b*f - 315*a**2*b**2*e + 315*a*b**3*d - 315*b**4*c) + x*
*6*(-105*a**4*f + 105*a**3*b*e - 105*a**2*b**2*d + 105*a*b**3*c) + x**4*(-6
3*a**4*e + 63*a**3*b*d - 63*a**2*b**2*c) + x**2*(-45*a**4*d + 45*a**3*b*c))
/(315*a**5*x**9)
```

3.123 $\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)} dx$

Optimal. Leaf size=211

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{13/2}}+\frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x}-\frac{b}{}$$

```
[Out] -1/11*c/a/x^11+1/9*(-a*d+b*c)/a^2/x^9+1/7*(-a^2*e+a*b*d-b^2*c)/a^3/x^7+1/5*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^5-1/3*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
/a^5/x^3+b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^6/x+b^(5/2)*(-a^3*f+a^2*b*e-a
*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(13/2)
```

Rubi [A] time = 0.18, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{5a^4x^5}+\frac{b^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^6x}+\frac{b^{5/2}\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^3*x
^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/a^(13/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)} dx &= \int \left(\frac{c}{ax^{12}}+\frac{-bc+ad}{a^2x^{10}}+\frac{b^2c-abd+a^2e}{a^3x^8}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^6}-\frac{b\left(-b^3c+\right.}{}\right.\\
&= -\frac{c}{11ax^{11}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{7a^3x^7}+\frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}-\frac{b\left(b^3c-ab^2d\right.}{3a}\\
&= -\frac{c}{11ax^{11}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{7a^3x^7}+\frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}-\frac{b\left(b^3c-ab^2d\right.}{3a}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 211, normalized size = 1.00

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{13/2}}+\frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x}+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -1/11*c/(a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^3
*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) + (b*(-(b^3*c) + a*
b^2*d - a^2*b*e + a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f))/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/a^(13/2)
```

fricas [A] time = 0.67, size = 458, normalized size = 2.17

$$\left[-\frac{3465\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{11}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-6930\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{10}+2310\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6930*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqrt(-b/a)*lo
g((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 6930*(b^5*c - a*b^4*d + a^2
*b^3*e - a^3*b^2*f)*x^10 + 2310*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)
*x^8 - 1386*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^6 + 630*a^5*c + 990
*(a^3*b^2*c - a^4*b*d + a^5*e)*x^4 - 770*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11),
 1/3465*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqrt(b/a)*arct
an(x*sqrt(b/a)) + 3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^10 - 115
5*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8 + 693*(a^2*b^3*c - a^3*b^
2*d + a^4*b*e - a^5*f)*x^6 - 315*a^5*c - 495*(a^3*b^2*c - a^4*b*d + a^5*e)*
x^4 + 385*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11)]
```

giac [A] time = 0.36, size = 249, normalized size = 1.18

$$\frac{\left(b^6c-ab^5d-a^3b^3f+a^2b^4e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^6}+\frac{3465\,b^5cx^{10}-3465\,ab^4dx^{10}-3465\,a^3b^2fx^{10}+3465\,a^2b^3x^{10}e-115\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^6*c - a*b^5*d - a^3*b^3*f + a^2*b^4*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^6) + 1/3465*(3465*b^5*c*x^10 - 3465*a*b^4*d*x^10 - 3465*a^3*b^2*f*x^10 +
3465*a^2*b^3*x^10*e - 1155*a*b^4*c*x^8 + 1155*a^2*b^3*d*x^8 + 1155*a^4*b*f*
x^8 - 1155*a^3*b^2*x^8*e + 693*a^2*b^3*c*x^6 - 693*a^3*b^2*d*x^6 - 693*a^5*
f*x^6 + 693*a^4*b*x^6*e - 495*a^3*b^2*c*x^4 + 495*a^4*b*d*x^4 - 495*a^5*x^4
*e + 385*a^4*b*c*x^2 - 385*a^5*d*x^2 - 315*a^5*c)/(a^6*x^11)
```

maple [A] time = 0.01, size = 286, normalized size = 1.36

$$-\frac{b^3f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^3}+\frac{b^4e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^4}-\frac{b^5d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^5}+\frac{b^6c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^6}-\frac{b^2f}{a^3x}+\frac{b^3e}{a^4x}-\frac{b^4d}{a^5x}+\frac{b^5c}{a^6x}+\frac{bf}{3a^2x^3}-\frac{b^2e}{3a^3x^3}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x)
```

```
[Out] -b^3/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+b^4/a^4/(a*b)^(1/2)*arctan
(1/(a*b)^(1/2)*b*x)*e-b^5/a^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+b^6/a
^6/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/11*c/a/x^11-1/9/a/x^9*d+1/9/a^
```

```
2/x^9*b*c-1/7/a/x^7*e+1/7/a^2/x^7*b*d-1/7/a^3/x^7*b^2*c-1/5/a/x^5*f+1/5/a^2
/x^5*b*e-1/5/a^3/x^5*b^2*d+1/5/a^4/x^5*b^3*c-1/a^3*b^2/x*f+1/a^4*b^3/x*e-1/
a^5*b^4/x*d+1/a^6*b^5/x*c+1/3/a^2*b/x^3*f-1/3/a^3*b^2/x^3*e+1/3/a^4*b^3/x^3
*d-1/3/a^5*b^4/x^3*c
```

maxima [A] time = 2.95, size = 214, normalized size = 1.01

$$\frac{\left(b^6c-ab^5d+a^2b^4e-a^3b^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^6}+\frac{3465\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{10}-1155\left(ab^4c-a^2b^3d+a^3b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^6*c - a*b^5*d + a^2*b^4*e - a^3*b^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^6) + 1/3465*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^10 - 1155*(
a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8 + 693*(a^2*b^3*c - a^3*b^2*d
 + a^4*b*e - a^5*f)*x^6 - 315*a^5*c - 495*(a^3*b^2*c - a^4*b*d + a^5*e)*x^4
 + 385*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11)
```

mupad [B] time = 0.99, size = 197, normalized size = 0.93

$$\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{13/2}}-\frac{\frac{c}{11\,a}-\frac{x^6\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{5\,a^4}+\frac{x^2\,(a\,d-b\,c)}{9\,a^2}+\frac{x^4\left(e\,a^2-d\,a\,b+c\,b^2\right)}{7\,a^3}+\cdots}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x)
```

```
[Out] (b^(5/2)*atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(
13/2) - (c/(11*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(5*a^4) + (x^
2*(a*d - b*c))/(9*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(7*a^3) + (b*x^8*(b^
3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^5) - (b^2*x^10*(b^3*c - a^3*f - a*b^
2*d + a^2*b*e))/a^6)/x^11
```

sympy [A] time = 84.14, size = 398, normalized size = 1.89

$$\frac{\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2}-\frac{\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**12/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**7*sqrt(-b
**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**3*f - a**2*b**4
*e + a*b**5*d - b**6*c) + x)/2 - sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b
**2*d - b**3*c)*log(a**7*sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d -
b**3*c)/(a**3*b**3*f - a**2*b**4*e + a*b**5*d - b**6*c) + x)/2 + (-315*a**5
*c + x**10*(-3465*a**3*b**2*f + 3465*a**2*b**3*e - 3465*a*b**4*d + 3465*b**
5*c) + x**8*(1155*a**4*b*f - 1155*a**3*b**2*e + 1155*a**2*b**3*d - 1155*a*b
**4*c) + x**6*(-693*a**5*f + 693*a**4*b*e - 693*a**3*b**2*d + 693*a**2*b**3
*c) + x**4*(-495*a**5*e + 495*a**4*b*d - 495*a**3*b**2*c) + x**2*(-385*a**5
*d + 385*a**4*b*c))/(3465*a**6*x**11)
```

3.124 $\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=240

$$\frac{x^7\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{ax\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{2b^6}+\frac{x^3\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{6b^5}-\frac{x^5\left(-11a^3f+\right.}{}$$

[Out] -1/2*a*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c)*x/b^6+1/6*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c)*x^3/b^5-1/10*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c)*x^5/a/b^4+1/7*(-2*a*f+b*e)*x^7/b^3+1/9*f*x^9/b^2+1/2*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^7/a/(b*x^2+a)+1/2*a^(3/2)*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(13/2)

Rubi [A] time = 0.29, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1804, 1585, 1261, 205}

$$\frac{x^7\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{x^5\left(9a^2be-11a^3f-7ab^2d+5b^3c\right)}{10ab^4}+\frac{x^3\left(9a^2be-11a^3f-7ab^2d+5b^3c\right)}{6b^5}-\frac{ax\left(9a^2be-11a^3\right.}{2}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]

[Out] -(a*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x)/(2*b^6) + ((5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x^3)/(6*b^5) - ((5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x^5)/(10*a*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/(9*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(2*a*(a + b*x^2)) + (a^(3/2)*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
```

```
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^5\left(\left(5bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^6\left(5bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{2a\left(a+bx^2\right)} - \frac{\int \left(\frac{a^2\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)}{b^5} - \frac{a\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)}{b^4}\right.}{}$$

$$= -\frac{a\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)x}{2b^6} + \frac{\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)x^3}{6b^5} -$$

$$= -\frac{a\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)x}{2b^6} + \frac{\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)x^3}{6b^5} -$$

Mathematica [A] time = 0.13, size = 227, normalized size = 0.95

$$\frac{x^5\left(3a^2f-2abe+b^2d\right)}{5b^4}+\frac{ax\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)}{b^6}+\frac{x^3\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}-\frac{a^{3/2}\tan^{-1}\left(\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2*
d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^5)/(
5*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/(9*b^2) - ((a^2*b^3*c - a^3*
b^2*d + a^4*b*e - a^5*f)*x)/(2*b^6*(a + b*x^2)) - (a^(3/2)*(-5*b^3*c + 7*a*
b^2*d - 9*a^2*b*e + 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))
```

fricas [A] time = 0.57, size = 572, normalized size = 2.38

$$\left[\frac{140\,b^5fx^{11}+20\left(9\,b^5e-11\,ab^4f\right)x^9+36\left(7\,b^5d-9\,ab^4e+11\,a^2b^3f\right)x^7+84\left(5\,b^5c-7\,ab^4d+9\,a^2b^3e-11\,a^3b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(140*b^5*f*x^11 + 20*(9*b^5*e - 11*a*b^4*f)*x^9 + 36*(7*b^5*d - 9*a
*b^4*e + 11*a^2*b^3*f)*x^7 + 84*(5*b^5*c - 7*a*b^4*d + 9*a^2*b^3*e - 11*a^3
*b^2*f)*x^5 - 420*(5*a*b^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^3
```

```
- 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f + (5*a*b^4*c - 7*a^
2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt
(-a/b) - a)/(b*x^2 + a)) - 630*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*
a^5*f)*x)/(b^7*x^2 + a*b^6), 1/630*(70*b^5*f*x^11 + 10*(9*b^5*e - 11*a*b^4*
f)*x^9 + 18*(7*b^5*d - 9*a*b^4*e + 11*a^2*b^3*f)*x^7 + 42*(5*b^5*c - 7*a*b^
4*d + 9*a^2*b^3*e - 11*a^3*b^2*f)*x^5 - 210*(5*a*b^4*c - 7*a^2*b^3*d + 9*a^
3*b^2*e - 11*a^4*b*f)*x^3 + 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11
*a^5*f + (5*a*b^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^2)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a) - 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11
*a^5*f)*x)/(b^7*x^2 + a*b^6)]
```

giac [A] time = 0.39, size = 252, normalized size = 1.05

$$\frac{\left(5\,a^2b^3c - 7\,a^3b^2d - 11\,a^5f + 9\,a^4be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^6} - \frac{a^2b^3cx - a^3b^2dx - a^5fx + a^4bxe}{2\left(bx^2+a\right)b^6} + \frac{35\,b^{16}fx^9 - 90\,ab^{15}fx^7 + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*a^2*b^3*c - 7*a^3*b^2*d - 11*a^5*f + 9*a^4*b*e)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^6) - 1/2*(a^2*b^3*c*x - a^3*b^2*d*x - a^5*f*x + a^4*b*x*e)/(
(b*x^2 + a)*b^6) + 1/315*(35*b^16*f*x^9 - 90*a*b^15*f*x^7 + 45*b^16*x^7*e +
 63*b^16*d*x^5 + 189*a^2*b^14*f*x^5 - 126*a*b^15*x^5*e + 105*b^16*c*x^3 - 2
10*a*b^15*d*x^3 - 420*a^3*b^13*f*x^3 + 315*a^2*b^14*x^3*e - 630*a*b^15*c*x
+ 945*a^2*b^14*d*x + 1575*a^4*b^12*f*x - 1260*a^3*b^13*x*e)/b^18
```

maple [A] time = 0.01, size = 309, normalized size = 1.29

$$\frac{fx^9}{9b^2} - \frac{2afx^7}{7b^3} + \frac{ex^7}{7b^2} + \frac{3a^2fx^5}{5b^4} - \frac{2aex^5}{5b^3} + \frac{dx^5}{5b^2} - \frac{4a^3fx^3}{3b^5} + \frac{a^2ex^3}{b^4} - \frac{2adx^3}{3b^3} + \frac{cx^3}{3b^2} + \frac{a^5fx}{2\left(bx^2+a\right)b^6} - \frac{11a^5f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^6} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/9*f*x^9/b^2-2/7/b^3*x^7*a*f+1/7/b^2*x^7*e+3/5/b^4*x^5*a^2*f-2/5/b^3*x^5*a
*e+1/5/b^2*x^5*d-4/3/b^5*x^3*a^3*f+1/b^4*x^3*a^2*e-2/3/b^3*x^3*a*d+1/3/b^2*
x^3*c+5/b^6*a^4*f*x-4/b^5*a^3*e*x+3/b^4*a^2*d*x-2/b^3*a*c*x+1/2*a^5/b^6*x/(
b*x^2+a)*f-1/2*a^4/b^5*x/(b*x^2+a)*e+1/2*a^3/b^4*x/(b*x^2+a)*d-1/2*a^2/b^3*
x/(b*x^2+a)*c-11/2*a^5/b^6/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+9/2*a^4/
b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-7/2*a^3/b^4/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*d+5/2*a^2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 3.01, size = 227, normalized size = 0.95

$$-\frac{\left(a^2b^3c - a^3b^2d + a^4be - a^5f\right)x}{2\left(b^7x^2+ab^6\right)} + \frac{\left(5\,a^2b^3c - 7\,a^3b^2d + 9\,a^4be - 11\,a^5f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^6} + \frac{35\,b^4fx^9 + 45\left(b^4e - 2\,ab^3 \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x/(b^7*x^2 + a*b^6) + 1/2*(5
*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*b^6) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e - 2*a*b^3*f)*x^7 + 63*(b^4*d
- 2*a*b^3*e + 3*a^2*b^2*f)*x^5 + 105*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a
^3*b*f)*x^3 - 315*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*x)/b^6
```

mupad [B] time = 0.10, size = 413, normalized size = 1.72

$$x^7\left(\frac{e}{7\,b^2}-\frac{2\,a\,f}{7\,b^3}\right)-x\left(\frac{2\,a\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)}{b}-\frac{a^2\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b^2}\right)-x^5\left(\frac{a^2\,f}{5\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^7*(e/(7*b^2) - (2*a*f)/(7*b^3)) - x*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/
b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b))
/b - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b^2) - x^5
*((a^2*f)/(5*b^4) - d/(5*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(5*b)) + x^3*(c
/(3*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(3*b^2) + (2*a*((a^2*f)/b^4 - d/b^2
+ (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b)) + (f*x^9)/(9*b^2) + (x*((a^5*f)/2
- (a^2*b^3*c)/2 + (a^3*b^2*d)/2 - (a^4*b*e)/2))/(a*b^6 + b^7*x^2) - (a^(3/2
)*atan((a^(3/2)*b^(1/2)*x*(5*b^3*c - 11*a^3*f - 7*a*b^2*d + 9*a^2*b*e))/(11
*a^5*f - 5*a^2*b^3*c + 7*a^3*b^2*d - 9*a^4*b*e))*(5*b^3*c - 11*a^3*f - 7*a*
b^2*d + 9*a^2*b*e))/(2*b^(13/2))
```

sympy [A] time = 3.07, size = 444, normalized size = 1.85

$$x^7\left(-\frac{2af}{7b^3}+\frac{e}{7b^2}\right)+x^5\left(\frac{3a^2f}{5b^4}-\frac{2ae}{5b^3}+\frac{d}{5b^2}\right)+x^3\left(-\frac{4a^3f}{3b^5}+\frac{a^2e}{b^4}-\frac{2ad}{3b^3}+\frac{c}{3b^2}\right)+x\left(\frac{5a^4f}{b^6}-\frac{4a^3e}{b^5}+\frac{3a^2d}{b^4}-\frac{2ac}{b^3}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**7*(-2*a*f/(7*b**3) + e/(7*b**2)) + x**5*(3*a**2*f/(5*b**4) - 2*a*e/(5*b*
*3) + d/(5*b**2)) + x**3*(-4*a**3*f/(3*b**5) + a**2*e/b**4 - 2*a*d/(3*b**3)
 + c/(3*b**2)) + x*(5*a**4*f/b**6 - 4*a**3*e/b**5 + 3*a**2*d/b**4 - 2*a*c/b
**3) + x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(2*a*b**6 + 2*b**7
*x**2) + sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)
*log(-b**6*sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*
c)/(11*a**4*f - 9*a**3*b*e + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 - sqrt(-a**
3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)*log(b**6*sqrt(-a*
*3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)/(11*a**4*f - 9*a
**3*b*e + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 + f*x**9/(9*b**2)
```

3.125 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=202

$$\frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-9a^3f+7a^2be-5ab^2d+3b^3c)}{2b^{11/2}}+\frac{x(-9a^3f+7a^2be-5ab^2d+3b^3c)}{2b^5}-\frac{x^3}{}$$

```
[Out] 1/2*(-9*a^3*f+7*a^2*b*e-5*a*b^2*d+3*b^3*c)*x/b^5-1/6*(-9*a^3*f+7*a^2*b*e-5*
a*b^2*d+3*b^3*c)*x^3/a/b^4+1/5*(-2*a*f+b*e)*x^5/b^3+1/7*f*x^7/b^2+1/2*(c-a*
(a^2*f-a*b*e+b^2*d)/b^3)*x^5/a/(b*x^2+a)-1/2*(-9*a^3*f+7*a^2*b*e-5*a*b^2*d+
3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(11/2)
```

Rubi [A] time = 0.23, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1804, 1585, 1261, 205}

$$\frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}-\frac{x^3(7a^2be-9a^3f-5ab^2d+3b^3c)}{6ab^4}+\frac{x(7a^2be-9a^3f-5ab^2d+3b^3c)}{2b^5}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(7a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*x)/(2*b^5) - ((3*b^3*c - 5*a*b
^2*d + 7*a^2*b*e - 9*a^3*f)*x^3)/(6*a*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) +
(f*x^7)/(7*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^5)/(2*a*(a + b*x
^2)) - (Sqrt[a]*(3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(2*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
```

```
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^3\left(\left(3bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^4\left(3bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int \left(-\frac{a\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)}{b^4}+\frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^2}{b^3}\right.}{2a}$$

$$= \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x}{2b^5} - \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^3}{6ab^4} + \frac{(be}{}$$

$$= \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x}{2b^5} - \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^3}{6ab^4} + \frac{(be}{}$$

Mathematica [A] time = 0.10, size = 187, normalized size = 0.93

$$\frac{x^3\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(9a^3f-7a^2be+5ab^2d-3b^3c\right)}{2b^{11/2}}+\frac{x\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{b^5}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a
^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (f*x^7)/(7*b^2) + ((a*b^
3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/(2*b^5*(a + b*x^2)) + (Sqrt[a]*(-3*b^
3*c + 5*a*b^2*d - 7*a^2*b*e + 9*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(1
1/2))
```

fricas [A] time = 0.71, size = 478, normalized size = 2.37

$$\left[\frac{60\,b^4fx^9+12\left(7\,b^4e-9\,ab^3f\right)x^7+28\left(5\,b^4d-7\,ab^3e+9\,a^2b^2f\right)x^5+140\left(3\,b^4c-5\,ab^3d+7\,a^2b^2e-9\,a^3bf\right)x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*b^4*f*x^9 + 12*(7*b^4*e - 9*a*b^3*f)*x^7 + 28*(5*b^4*d - 7*a*b^3
*e + 9*a^2*b^2*f)*x^5 + 140*(3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)
*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f + (3*b^4*c - 5*a*
b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a
/b) - a)/(b*x^2 + a)) + 210*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)
```

```
*x)/(b^6*x^2 + a*b^5), 1/210*(30*b^4*f*x^9 + 6*(7*b^4*e - 9*a*b^3*f)*x^7 +
14*(5*b^4*d - 7*a*b^3*e + 9*a^2*b^2*f)*x^5 + 70*(3*b^4*c - 5*a*b^3*d + 7*a^
2*b^2*e - 9*a^3*b*f)*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4
*f + (3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)*x^2)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a) + 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)*x)/(
b^6*x^2 + a*b^5)]
```

giac [A] time = 0.42, size = 201, normalized size = 1.00

$$-\frac{\left(3\,ab^3c-5\,a^2b^2d-9\,a^4f+7\,a^3be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}+\frac{ab^3cx-a^2b^2dx-a^4fx+a^3bxe}{2\left(bx^2+a\right)b^5}+\frac{15\,b^{12}fx^7-42\,ab^{11}fx^5+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b^3*c - 5*a^2*b^2*d - 9*a^4*f + 7*a^3*b*e)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^5) + 1/2*(a*b^3*c*x - a^2*b^2*d*x - a^4*f*x + a^3*b*x*e)/((b*x
^2 + a)*b^5) + 1/105*(15*b^12*f*x^7 - 42*a*b^11*f*x^5 + 21*b^12*x^5*e + 35*
b^12*d*x^3 + 105*a^2*b^10*f*x^3 - 70*a*b^11*x^3*e + 105*b^12*c*x - 210*a*b^
11*d*x - 420*a^3*b^9*f*x + 315*a^2*b^10*x*e)/b^14
```

maple [A] time = 0.01, size = 258, normalized size = 1.28

$$\frac{fx^7}{7b^2}-\frac{2afx^5}{5b^3}+\frac{ex^5}{5b^2}+\frac{a^2fx^3}{b^4}-\frac{2aex^3}{3b^3}+\frac{dx^3}{3b^2}-\frac{a^4fx}{2\left(bx^2+a\right)b^5}+\frac{9a^4f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^5}+\frac{a^3ex}{2\left(bx^2+a\right)b^4}-\frac{7a^3e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/7*f*x^7/b^2-2/5/b^3*x^5*a*f+1/5/b^2*x^5*e+1/b^4*x^3*a^2*f-2/3/b^3*x^3*a*e
+1/3/b^2*x^3*d-4/b^5*a^3*f*x+3/b^4*a^2*e*x-2/b^3*a*d*x+1/b^2*c*x-1/2*a^4/b^
5*x/(b*x^2+a)*f+1/2*a^3/b^4*x/(b*x^2+a)*e-1/2*a^2/b^3*x/(b*x^2+a)*d+1/2*a/b
^2*x/(b*x^2+a)*c+9/2*a^4/b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-7/2*a^
3/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+5/2*a^2/b^3/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)*d-3/2*a/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 3.01, size = 183, normalized size = 0.91

$$\frac{\left(ab^3c-a^2b^2d+a^3be-a^4f\right)x}{2\left(b^6x^2+ab^5\right)}-\frac{\left(3\,ab^3c-5\,a^2b^2d+7\,a^3be-9\,a^4f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}+\frac{15\,b^3fx^7+21\left(b^3e-2\,ab^2f\right)x^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x/(b^6*x^2 + a*b^5) - 1/2*(3*a*
b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^5) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - 2*a*b^2*f)*x^5 + 35*(b^3*d - 2*a*
b^2*e + 3*a^2*b*f)*x^3 + 105*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b
^5
```

mupad [B] time = 0.97, size = 288, normalized size = 1.43

$$x^5\left(\frac{e}{5\,b^2}-\frac{2\,a\,f}{5\,b^3}\right)+x\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)-x^3\left(\frac{a^2\,f}{3\,b^4}-\frac{d}{3\,b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b}\right)-\frac{x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^5*(e/(5*b^2) - (2*a*f)/(5*b^3)) + x*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/b^3))/
b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) - x^3*
((a^2*f)/(3*b^4) - d/(3*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(3*b)) - (x*((a^
4*f)/2 + (a^2*b^2*d)/2 - (a*b^3*c)/2 - (a^3*b*e)/2))/(a*b^5 + b^6*x^2) + (f
*x^7)/(7*b^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(3*b^3*c - 9*a^3*f - 5*a*b
^2*d + 7*a^2*b*e))/(9*a^4*f + 5*a^2*b^2*d - 3*a*b^3*c - 7*a^3*b*e))*(3*b^3*
c - 9*a^3*f - 5*a*b^2*d + 7*a^2*b*e))/(2*b^(11/2))
```

sympy [A] time = 4.76, size = 257, normalized size = 1.27

$$x^5\left(-\frac{2af}{5b^3}+\frac{e}{5b^2}\right)+x^3\left(\frac{a^2f}{b^4}-\frac{2ae}{3b^3}+\frac{d}{3b^2}\right)+x\left(-\frac{4a^3f}{b^5}+\frac{3a^2e}{b^4}-\frac{2ad}{b^3}+\frac{c}{b^2}\right)+\frac{x\left(-a^4f+a^3be-a^2b^2d+ab^3c\right)}{2ab^5+2b^6x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**5*(-2*a*f/(5*b**3) + e/(5*b**2)) + x**3*(a**2*f/b**4 - 2*a*e/(3*b**3) +
d/(3*b**2)) + x*(-4*a**3*f/b**5 + 3*a**2*e/b**4 - 2*a*d/b**3 + c/b**2) + x*
(-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(2*a*b**5 + 2*b**6*x**2) - sq
rt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d - 3*b**3*c)*log(-b**5*sqrt
(-a/b**11) + x)/4 + sqrt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d - 3*
b**3*c)*log(b**5*sqrt(-a/b**11) + x)/4 + f*x**7/(7*b**2)
```

3.126 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2} dx$

Optimal. Leaf size=163

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-7a^3f+5a^2be-3ab^2d+b^3c\right)}{2\sqrt{a}\,b^{9/2}}-\frac{x\left(-7a^3f+5a^2be-3ab^2d+b^3c\right)}{2ab^4}+\frac{x^3(be-2}{3b^3}$$

```
[Out] -1/2*(-7*a^3*f+5*a^2*b*e-3*a*b^2*d+b^3*c)*x/a/b^4+1/3*(-2*a*f+b*e)*x^3/b^3+
1/5*f*x^5/b^2+1/2*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^3/a/(b*x^2+a)+1/2*(-7*a^3
*f+5*a^2*b*e-3*a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)/a^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1804, 1585, 1261, 205}

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{x\left(5a^2be-7a^3f-3ab^2d+b^3c\right)}{2ab^4}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(5a^2be-7a^3f-3ab^2d+b^3c\right)}{2\sqrt{a}\,b^{9/2}}+\frac{x^3(be-2af}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] -((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*x)/(2*a*b^4) + ((b*e - 2*a*f)*x
^3)/(3*b^3) + (f*x^5)/(5*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^3)
/(2*a*(a + b*x^2)) + ((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{2a\left(a+bx^2\right)} - \frac{\int \frac{x\left(\left(bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^2\left(bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{2a\left(a+bx^2\right)} - \frac{\int \left(c-\frac{a\left(3b^2d-5abe+7a^2f\right)}{b^3}-\frac{2a(be-2af)x^2}{b^2}-\frac{2afx^4}{b}+\frac{-ab^3c\cdots}{\cdots}\right.}{2ab}$$

$$= -\frac{\left(b^3c-3ab^2d+5a^2be-7a^3f\right)x}{2ab^4} + \frac{(be-2af)x^3}{3b^3} + \frac{fx^5}{5b^2} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right.}{2a\left(a+bx^2\right)}$$

$$= -\frac{\left(b^3c-3ab^2d+5a^2be-7a^3f\right)x}{2ab^4} + \frac{(be-2af)x^3}{3b^3} + \frac{fx^5}{5b^2} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right.}{2a\left(a+bx^2\right)}$$

Mathematica [A] time = 0.09, size = 148, normalized size = 0.91

$$\frac{x\left(3a^2f-2abe+b^2d\right)}{b^4} - \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(7a^3f-5a^2be+3ab^2d-b^3c\right)}{2\sqrt{a}\,b^{9/2}} - \frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2b^4\left(a+bx^2\right)} + \frac{x^3(be-2\cdots}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^5)
/(5*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*b^4*(a + b*x^2)) - ((
-(b^3*c) + 3*a*b^2*d - 5*a^2*b*e + 7*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
*Sqrt[a]*b^(9/2))
```

fricas [A] time = 0.64, size = 418, normalized size = 2.56

$$\left[\frac{12\,ab^4fx^7+4\left(5\,ab^4e-7\,a^2b^3f\right)x^5+20\left(3\,ab^4d-5\,a^2b^3e+7\,a^3b^2f\right)x^3+15\left(ab^3c-3\,a^2b^2d+5\,a^3be-7\,a^4f\cdots\right.}{60\left(ab^6x^2\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a*b^4*f*x^7 + 4*(5*a*b^4*e - 7*a^2*b^3*f)*x^5 + 20*(3*a*b^4*d - 5
*a^2*b^3*e + 7*a^3*b^2*f)*x^3 + 15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a
^4*f + (b^4*c - 3*a*b^3*d + 5*a^2*b^2*e - 7*a^3*b*f)*x^2)*sqrt(-a*b)*log((b
*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 30*(a*b^4*c - 3*a^2*b^3*d + 5*a^3
*b^2*e - 7*a^4*b*f)*x)/(a*b^6*x^2 + a^2*b^5), 1/30*(6*a*b^4*f*x^7 + 2*(5*a*
b^4*e - 7*a^2*b^3*f)*x^5 + 10*(3*a*b^4*d - 5*a^2*b^3*e + 7*a^3*b^2*f)*x^3 +
 15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + (b^4*c - 3*a*b^3*d + 5*a
^2*b^2*e - 7*a^3*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 15*(a*b^4*c -
3*a^2*b^3*d + 5*a^3*b^2*e - 7*a^4*b*f)*x)/(a*b^6*x^2 + a^2*b^5)]
```

giac [A] time = 0.37, size = 152, normalized size = 0.93

$$\frac{\left(b^3c-3\,ab^2d-7\,a^3f+5\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\left(bx^2+a\right)b^4}+\frac{3\,b^8fx^5-10\,ab^7fx^3+5\,b^8x^3e+15}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c - 3*a*b^2*d - 7*a^3*f + 5*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^4) - 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*f*x^5 - 10*a*b^7*f*x^3 + 5*b^8*x^3*e + 15*b^8*d*x + 45*a^2*
b^6*f*x - 30*a*b^7*x*e)/b^10
```

maple [A] time = 0.01, size = 212, normalized size = 1.30

$$\frac{fx^5}{5b^2}-\frac{2afx^3}{3b^3}+\frac{ex^3}{3b^2}+\frac{a^3fx}{2\left(bx^2+a\right)b^4}-\frac{7a^3f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^4}-\frac{a^2ex}{2\left(bx^2+a\right)b^3}+\frac{5a^2e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^3}+\frac{adx}{2\left(bx^2+a\right)b^2}-\frac{3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/5*f*x^5/b^2-2/3/b^3*x^3*a*f+1/3/b^2*x^3*e+3/b^4*a^2*f*x-2/b^3*a*e*x+1/b^2
*d*x+1/2/b^4*x/(b*x^2+a)*a^3*f-1/2/b^3*x/(b*x^2+a)*a^2*e+1/2/b^2*x/(b*x^2+a
)*a*d-1/2/(b*x^2+a)/b*c*x-7/2/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^3
*f+5/2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^2*e-3/2/b^2/(a*b)^(1/2)*
arctan(1/(a*b)^(1/2)*b*x)*a*d+1/2/(a*b)^(1/2)/b*c*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.96, size = 140, normalized size = 0.86

$$-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2\left(b^5x^2+ab^4\right)}+\frac{\left(b^3c-3\,ab^2d+5\,a^2be-7\,a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}+\frac{3\,b^2fx^5+5\left(b^2e-2\,abf\right)x^3+15\left(b\right.}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(b^5*x^2 + a*b^4) + 1/2*(b^3*c -
 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1
/15*(3*b^2*f*x^5 + 5*(b^2*e - 2*a*b*f)*x^3 + 15*(b^2*d - 2*a*b*e + 3*a^2*f)
*x)/b^4
```

mupad [B] time = 1.00, size = 153, normalized size = 0.94

$$x^3\left(\frac{e}{3\,b^2}-\frac{2\,a\,f}{3\,b^3}\right)-x\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)-\frac{x\left(-\frac{f\,a^3}{2}+\frac{e\,a^2\,b}{2}-\frac{d\,a\,b^2}{2}+\frac{c\,b^3}{2}\right)}{b^5\,x^2+a\,b^4}+\frac{f\,x^5}{5\,b^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-7\,f\,a^3+\right.}{2\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^3*(e/(3*b^2) - (2*a*f)/(3*b^3)) - x*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 -
(2*a*f)/b^3))/b) - (x*((b^3*c)/2 - (a^3*f)/2 - (a*b^2*d)/2 + (a^2*b*e)/2))/
(a*b^4 + b^5*x^2) + (f*x^5)/(5*b^2) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - 7
*a^3*f - 3*a*b^2*d + 5*a^2*b*e))/(2*a^(1/2)*b^(9/2))
```

sympy [A] time = 3.04, size = 221, normalized size = 1.36

$$x^3\left(-\frac{2af}{3b^3}+\frac{e}{3b^2}\right)+x\left(\frac{3a^2f}{b^4}-\frac{2ae}{b^3}+\frac{d}{b^2}\right)+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{1}{ab^9}}\,\left(7a^3f-5a^2be+3ab^2d-b^3c\right)1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**3*(-2*a*f/(3*b**3) + e/(3*b**2)) + x*(3*a**2*f/b**4 - 2*a*e/b**3 + d/b**
2) + x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*a*b**4 + 2*b**5*x**2) + s
qrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b**2*d - b**3*c)*log(-a*b**4*
sqrt(-1/(a*b**9)) + x)/4 - sqrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b
**2*d - b**3*c)*log(a*b**4*sqrt(-1/(a*b**9)) + x)/4 + f*x**5/(5*b**2)
```

3.127 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=118

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(5a^3f-3a^2be+ab^2d+b^3c)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

```
[Out] (-2*a*f+b*e)*x/b^3+1/3*f*x^3/b^2+1/2*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x/a/(b*x
^2+a)+1/2*(5*a^3*f-3*a^2*b*e+a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/
2)/b^(7/2)
```

Rubi [A] time = 0.12, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1814, 1153, 205}

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(-3a^2be+5a^3f+ab^2d+b^3c)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x)/(2*a*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx = \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x}{2a\left(a+bx^2\right)} - \frac{\int \frac{-\frac{b^3c+ab^2d-a^2be+a^3f}{b^3}-\frac{2a(be-af)x^2}{b^2}-\frac{2afx^4}{b}}{a+bx^2}\,dx}{2a}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x}{2a\left(a+bx^2\right)} - \frac{\int \left(-\frac{2a(be-2af)}{b^3}-\frac{2afx^2}{b^2}+\frac{-b^3c-ab^2d+3a^2be-5a^3f}{b^3(a+bx^2)}\right)dx}{2a}$$

$$= \frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\int\frac{1}{a+bx}}{2ab^3}$$

$$= \frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\tan^{-1}}{2a^{3/2}b^{7/2}}$$

Mathematica [A] time = 0.10, size = 122, normalized size = 1.03

$$-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2ab^3\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^3f-3a^2be+ab^2d+b^3c\right)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) - ((-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x)/(2*a*b^3*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

fricas [A] time = 0.65, size = 364, normalized size = 3.08

$$\left[\frac{4\,a^2b^3fx^5+4\left(3\,a^2b^3e-5\,a^3b^2f\right)x^3-3\left(ab^3c+a^2b^2d-3\,a^3be+5\,a^4f+\left(b^4c+ab^3d-3\,a^2b^2e+5\,a^3bf\right)x^2\right)\sqrt{}}{12\left(a^2b^5x^2+a^3b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*b^3*f*x^5 + 4*(3*a^2*b^3*e - 5*a^3*b^2*f)*x^3 - 3*(a*b^3*c + a
^2*b^2*d - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*b^3*d - 3*a^2*b^2*e + 5*a^3*b*f
)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*
c - a^2*b^3*d + 3*a^3*b^2*e - 5*a^4*b*f)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2
*a^2*b^3*f*x^5 + 2*(3*a^2*b^3*e - 5*a^3*b^2*f)*x^3 + 3*(a*b^3*c + a^2*b^2*d
 - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*b^3*d - 3*a^2*b^2*e + 5*a^3*b*f)*x^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c - a^2*b^3*d + 3*a^3*b^2*e - 5*a
^4*b*f)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

giac [A] time = 0.40, size = 126, normalized size = 1.07

$$\frac{\left(b^3c+ab^2d+5\,a^3f-3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^3}+\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\left(bx^2+a\right)ab^3}+\frac{b^4fx^3-6\,ab^3fx+3\,b^4xe}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c + a*b^2*d + 5*a^3*f - 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a*b^3) + 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a*b
^3) + 1/3*(b^4*f*x^3 - 6*a*b^3*f*x + 3*b^4*x*e)/b^6
```

maple [A] time = 0.01, size = 177, normalized size = 1.50

$$\frac{f\,x^{3}}{3b^{2}}-\frac{a^{2}f x}{2\left(b\,x^{2}+a\right)b^{3}}+\frac{5a^{2}f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{3}}+\frac{aex}{2\left(b\,x^{2}+a\right)b^{2}}-\frac{3ae\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{2}}+\frac{cx}{2\left(b\,x^{2}+a\right)a}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*f*x^3/b^2-2/b^3*a*f*x+1/b^2*e*x-1/2/b^3*a^2*x/(b*x^2+a)*f+1/2/b^2*a*x/(
b*x^2+a)*e-1/2/b*x/(b*x^2+a)*d+1/2/(b*x^2+a)/a*c*x+5/2/b^3*a^2/(a*b)^(1/2)*
arctan(1/(a*b)^(1/2)*b*x)*f-3/2/b^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
*e+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+1/2/(a*b)^(1/2)/a*c*arctan
(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 117, normalized size = 0.99

$$\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x}{2\left(ab^{4}x^{2}+a^{2}b^{3}\right)}+\frac{bfx^{3}+3\left(be-2\,af\right)x}{3\,b^{3}}+\frac{\left(b^{3}c+ab^{2}d-3\,a^{2}be+5\,a^{3}f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(a*b^4*x^2 + a^2*b^3) + 1/3*(b*f*
x^3 + 3*(b*e - 2*a*f)*x)/b^3 + 1/2*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3)
```

mupad [B] time = 0.10, size = 113, normalized size = 0.96

$$x\left(\frac{e}{b^{2}}-\frac{2\,a\,f}{b^{3}}\right)+\frac{f\,x^{3}}{3\,b^{2}}+\frac{x\,\left(-f\,a^{3}+e\,a^{2}\,b-d\,a\,b^{2}+c\,b^{3}\right)}{2\,a\,\left(b^{4}\,x^{2}+a\,b^{3}\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(5\,f\,a^{3}-3\,e\,a^{2}\,b+d\,a\,b^{2}+c\,b^{3}\right)}{2\,a^{3/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x)
```

```
[Out] x*(e/b^2 - (2*a*f)/b^3) + (f*x^3)/(3*b^2) + (x*(b^3*c - a^3*f - a*b^2*d + a
^2*b*e))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c + 5*a^
3*f + a*b^2*d - 3*a^2*b*e))/(2*a^(3/2)*b^(7/2))
```

sympy [A] time = 2.88, size = 201, normalized size = 1.70

$$x\left(-\frac{2af}{b^{3}}+\frac{e}{b^{2}}\right)+\frac{x\left(-a^{3}f+a^{2}be-ab^{2}d+b^{3}c\right)}{2a^{2}b^{3}+2ab^{4}x^{2}}-\frac{\sqrt{-\frac{1}{a^{3}b^{7}}}\left(5a^{3}f-3a^{2}be+ab^{2}d+b^{3}c\right)\log\left(-a^{2}b^{3}\sqrt{-\frac{1}{a^{3}b^{7}}}+x\right)}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*f/b**3 + e/b**2) + x*(-a**3*f + a**2*b*e - a*b**2*d + b**3*c)/(2*a*
*2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7))*(5*a**3*f - 3*a**2*b*e + a*
b**2*d + b**3*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7)) + x)/4 + sqrt(-1/(a**3
*b**7))*(5*a**3*f - 3*a**2*b*e + a*b**2*d + b**3*c)*log(a**2*b**3*sqrt(-1/(
a**3*b**7)) + x)/4 + f*x**3/(3*b**2)
```

3.128 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=112

$$-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\left(a+bx^2\right)}-\frac{c}{a^2x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-a^2be-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}+\frac{fx}{b^2}$$

```
[Out] -c/a^2/x+f*x/b^2-1/2*(b*c/a-d+a*e/b-a^2*f/b^2)*x/a/(b*x^2+a)-1/2*(3*a^3*f-a
^2*b*e-a*b^2*d+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(5/2)
```

Rubi [A] time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1261, 205}

$$-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be+3a^3f-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}-\frac{c}{a^2x}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(2*a*(
a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)} - \frac{\int \frac{-2c+\left(\frac{bc}{a}-d-\frac{ae}{b}+\frac{a^2f}{b^2}\right)x^2-\frac{2afx^4}{b}}{x^2\left(a+bx^2\right)}\,dx}{2a}$$

$$= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)} - \frac{\int \left(-\frac{2af}{b^2}-\frac{2c}{ax^2}+\frac{3b^3c-ab^2d-a^2be+3a^3f}{ab^2\left(a+bx^2\right)}\right)dx}{2a}$$

$$= -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\int \frac{1}{a+bx^2}\,dx}{2a^2b^2}$$

$$= -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

Mathematica [A] time = 0.07, size = 115, normalized size = 1.03

$$-\frac{c}{a^2x}+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^2b^2\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-a^2be-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^
2*b^2*(a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

fricas [A] time = 0.61, size = 354, normalized size = 3.16

$$\left[\frac{4\,a^3b^2fx^4-4\,a^2b^3c-2\left(3\,ab^4c-a^2b^3d+a^3b^2e-3\,a^4bf\right)x^2-\left(\left(3\,b^4c-ab^3d-a^2b^2e+3\,a^3bf\right)x^3+\left(3\,ab^3c-a^2b\right.\right.}{4\left(a^3b^4x^3+a^4b^3x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^3*b^2*f*x^4 - 4*a^2*b^3*c - 2*(3*a*b^4*c - a^2*b^3*d + a^3*b^2*e
- 3*a^4*b*f)*x^2 - ((3*b^4*c - a*b^3*d - a^2*b^2*e + 3*a^3*b*f)*x^3 + (3*a*
b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)))/(a^3*b^4*x^3 + a^4*b^3*x), 1/2*(2*a^3*b^2*f*x^4 -
2*a^2*b^3*c - (3*a*b^4*c - a^2*b^3*d + a^3*b^2*e - 3*a^4*b*f)*x^2 - ((3*b^4
*c - a*b^3*d - a^2*b^2*e + 3*a^3*b*f)*x^3 + (3*a*b^3*c - a^2*b^2*d - a^3*b*
e + 3*a^4*f)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^4*x^3 + a^4*b^3*x)]
```

giac [A] time = 0.36, size = 122, normalized size = 1.09

$$\frac{fx}{b^2}-\frac{\left(3\,b^3c-ab^2d+3\,a^3f-a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b^2}-\frac{3\,b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e+2\,ab^2c}{2\left(bx^3+ax\right)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d + 3*a^3*f - a^2*b*e)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x
^2*e + 2*a*b^2*c)/((b*x^3 + a*x)*a^2*b^2)
```

maple [A] time = 0.01, size = 165, normalized size = 1.47

$$\frac{afx}{2\left(bx^2+a\right)b^2}-\frac{3af\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^2}+\frac{dx}{2\left(bx^2+a\right)a}+\frac{d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}-\frac{bcx}{2\left(bx^2+a\right)a^2}-\frac{3bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^2}-\frac{e}{2\left(bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] f*x/b^2+1/2*a/b^2*x/(b*x^2+a)*f-1/2/b*x/(b*x^2+a)*e+1/2/a*x/(b*x^2+a)*d-1/2
/(b*x^2+a)/a^2*b*c*x-3/2*a/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+1/2/
b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)*d-3/2/(a*b)^(1/2)/a^2*b*c*arctan(1/(a*b)^(1/2)*b*x)-1/a^2*c/x
```

maxima [A] time = 2.99, size = 117, normalized size = 1.04

$$-\frac{2\,ab^2c+\left(3\,b^3c-ab^2d+a^2be-a^3f\right)x^2}{2\left(a^2b^3x^3+a^3b^2x\right)}+\frac{fx}{b^2}-\frac{\left(3\,b^3c-ab^2d-a^2be+3\,a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b^2*c + (3*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a^2*b^3*x^3 +
 a^3*b^2*x) + f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

mupad [B] time = 1.00, size = 112, normalized size = 1.00

$$\frac{f\,x}{b^2}-\frac{\frac{x^2\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+3\,c\,b^3\right)}{2\,a^2}+\frac{b^2\,c}{a}}{b^3\,x^3+a\,b^2\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3\,f\,a^3-e\,a^2\,b-d\,a\,b^2+3\,c\,b^3\right)}{2\,a^{5/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] (f*x)/b^2 - ((x^2*(3*b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(2*a^2) + (b^2*c)/
a)/(b^3*x^3 + a*b^2*x) - (atan((b^(1/2)*x)/a^(1/2))*(3*b^3*c + 3*a^3*f - a*
b^2*d - a^2*b*e))/(2*a^(5/2)*b^(5/2))
```

sympy [A] time = 9.46, size = 197, normalized size = 1.76

$$\frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^3f-a^2be-ab^2d+3b^3c\right)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^3f-a^2be-ab^2d+3b^3c\right)\log\left(a^3b^2\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*e - a*b**2*d + 3*b**3*c)*log(-a**3*
b**2*sqrt(-1/(a**5*b**5)) + x)/4 - sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*
e - a*b**2*d + 3*b**3*c)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/4 + (-2*a*
b**2*c + x**2*(a**3*f - a**2*b*e + a*b**2*d - 3*b**3*c))/(2*a**3*b**2*x + 2
*a**2*b**3*x**3) + f*x/b**2
```

3.129 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=121

$$\frac{2bc-ad}{a^3x}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}-\frac{c}{3a^2x^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3f+a^2be-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}$$

```
[Out] -1/3*c/a^2/x^3+(-a*d+2*b*c)/a^3/x+1/2*(b^2*c/a^2-b*d/a+e-a*f/b)*x/a/(b*x^2+
a)+1/2*(a^3*f+a^2*b*e-3*a*b^2*d+5*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/
b^(3/2)
```

Rubi [A] time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1261, 205}

$$\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^2be+a^3f-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}+\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a*f
)/b)*x)/(2*a*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}\,dx = \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2+\left(-\frac{b^2c}{a^2}+\frac{bd}{a}-e-\frac{af}{b}\right)x^4}{x^4\left(a+bx^2\right)}dx}{2a}$$

$$= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)} - \frac{\int \left(-\frac{2c}{ax^4}-\frac{2(-2bc+ad)}{a^2x^2}+\frac{-5b^3c+3ab^2d-a^2be-a^3f}{a^2b\left(a+bx^2\right)}\right)dx}{2a}$$

$$= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\int \frac{1}{a+bx}$$

$$= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\tan^{-1}}{2a^{7/2}b^{3/2}}$$

Mathematica [A] time = 0.08, size = 125, normalized size = 1.03

$$\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^3b\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f+a^2be-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) + (2*b*c - a*d)/(a^3*x) - ((-(b^3*c) + a*b^2*d - a^2*b*e +
 a^3*f)*x)/(2*a^3*b*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

fricas [A] time = 0.75, size = 378, normalized size = 3.12

$$\left[-\frac{4\,a^3b^2c-6\left(5\,ab^4c-3\,a^2b^3d+a^3b^2e-a^4bf\right)x^4-4\left(5\,a^2b^3c-3\,a^3b^2d\right)x^2+3\left(\left(5\,b^4c-3\,ab^3d+a^2b^2e+a^3b\right.\right.}{12\left(a^4b^3x^5+a^5b^2x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^3*b^2*c - 6*(5*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4
 - 4*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 + 3*((5*b^4*c - 3*a*b^3*d + a^2*b^2*e
+ a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^3*x^5 + a^5*b^2*x^3
), -1/6*(2*a^3*b^2*c - 3*(5*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^
4 - 2*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 - 3*((5*b^4*c - 3*a*b^3*d + a^2*b^2*e
 + a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)*sqrt(a*b
)*arctan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + a^5*b^2*x^3)]
```

giac [A] time = 0.48, size = 123, normalized size = 1.02

$$\frac{\left(5\,b^3c-3\,ab^2d+a^3f+a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3b}+\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\left(bx^2+a\right)a^3b}+\frac{6\,bcx^2-3\,adx^2-ac}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c - 3*a*b^2*d + a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^3*b) + 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a^3
*b) + 1/3*(6*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^3*x^3)
```

maple [A] time = 0.02, size = 182, normalized size = 1.50

$$\frac{ex}{2\left(b\,x^{2}+a\right)a}+\frac{e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{bdx}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{3bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{2}}+\frac{b^{2}cx}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{5b^{2}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{3}}-\frac{fx}{2\left(b\,x^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b*x/(b*x^2+a)*f+1/2/a*x/(b*x^2+a)*e-1/2/a^2*b*x/(b*x^2+a)*d+1/2/a^3*b^
2*x/(b*x^2+a)*c+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+1/2/a/(a*b)^(
1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)
*b*x)*d+5/2/a^3*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/3*c/a^2/x^3-1
/a^2/x*d+2/a^3/x*b*c
```

maxima [A] time = 3.02, size = 130, normalized size = 1.07

$$\frac{3\left(5\,b^{3}c-3\,ab^{2}d+a^{2}be-a^{3}f\right)x^{4}-2\,a^{2}bc+2\left(5\,ab^{2}c-3\,a^{2}bd\right)x^{2}}{6\left(a^{3}b^{2}x^{5}+a^{4}bx^{3}\right)}+\frac{\left(5\,b^{3}c-3\,ab^{2}d+a^{2}be+a^{3}f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^3*c - 3*a*b^2*d + a^2*b*e - a^3*f)*x^4 - 2*a^2*b*c + 2*(5*a*b^2
*c - 3*a^2*b*d)*x^2)/(a^3*b^2*x^5 + a^4*b*x^3) + 1/2*(5*b^3*c - 3*a*b^2*d +
 a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b)
```

mupad [B] time = 0.13, size = 119, normalized size = 0.98

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(f\,a^{3}+e\,a^{2}\,b-3\,d\,a\,b^{2}+5\,c\,b^{3}\right)}{2\,a^{7/2}\,b^{3/2}}-\frac{\frac{c}{3\,a}+\frac{x^{2}\,\left(3\,a\,d-5\,b\,c\right)}{3\,a^{2}}-\frac{x^{4}\,\left(-f\,a^{3}+e\,a^{2}\,b-3\,d\,a\,b^{2}+5\,c\,b^{3}\right)}{2\,a^{3}\,b}}{b\,x^{5}+a\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(5*b^3*c + a^3*f - 3*a*b^2*d + a^2*b*e))/(2*a^(7
/2)*b^(3/2)) - (c/(3*a) + (x^2*(3*a*d - 5*b*c))/(3*a^2) - (x^4*(5*b^3*c - a
^3*f - 3*a*b^2*d + a^2*b*e))/(2*a^3*b))/(a*x^3 + b*x^5)
```

sympy [A] time = 25.99, size = 212, normalized size = 1.75

$$-\frac{\sqrt{-\frac{1}{a^{7}b^{3}}}\left(a^{3}f+a^{2}be-3ab^{2}d+5b^{3}c\right)\log\left(-a^{4}b\sqrt{-\frac{1}{a^{7}b^{3}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{7}b^{3}}}\left(a^{3}f+a^{2}be-3ab^{2}d+5b^{3}c\right)\log\left(a^{4}b\sqrt{-\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e - 3*a*b**2*d + 5*b**3*c)*log(-a**4
*b*sqrt(-1/(a**7*b**3)) + x)/4 + sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e -
3*a*b**2*d + 5*b**3*c)*log(a**4*b*sqrt(-1/(a**7*b**3)) + x)/4 + (-2*a**2*b*
c + x**4*(-3*a**3*f + 3*a**2*b*e - 9*a*b**2*d + 15*b**3*c) + x**2*(-6*a**2*
b*d + 10*a*b**2*c))/(6*a**4*b*x**3 + 6*a**3*b**2*x**5)
```

3.130 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=152

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{5a^2x^5}-\frac{a^2e-2abd+3b^2c}{a^4x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+3a^2be-5ab^2d+7b^3c\right)}{2a^{9/2}\sqrt{b}}-\frac{x\left(a^3(-f)+a^2be-ab^2d\right.}{2a^4\left(a+bx^2\right)}$$

```
[Out] -1/5*c/a^2/x^5+1/3*(-a*d+2*b*c)/a^3/x^3+(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x-1/2*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^2+a)-1/2*(-a^3*f+3*a^2*b*e-5*a*b^
2*d+7*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1802, 205}

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^2be+a^3(-f)-5ab^2d+7b^3c\right)}{2a^{9/2}\sqrt{b}}-\frac{a^2e-2abd+3b^2c}{a^4x}+\frac{2bc-ad}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) - (3*b^2*c - 2*a*b*d + a^2*e)/(a
^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^4*(a + b*x^2)) - ((7*b
^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/
2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}}{x^6\left(a+bx^2\right)}dx}{2a}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)} - \frac{\int \left(-\frac{2c}{ax^6}-\frac{2(-2bc+ad)}{a^2x^4}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^2}+\frac{7b^3c-5ab^2d+3a}{a^3\left(a+bx^2\right.}\right.}{2a}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)}-\frac{\left(7b^3c-5a\right.}{}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)}-\frac{\left(7b^3c-5a\right.}{}$$

Mathematica [A] time = 0.09, size = 151, normalized size = 0.99

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{5a^2x^5}+\frac{a^2(-e)+2abd-3b^2c}{a^4x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-3a^2be+5ab^2d-7b^3c\right)}{2a^{9/2}\sqrt{b}}+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -1/5*c/(a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(a^4*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^4*(a + b*x^2)) +
 ((-7*b^3*c + 5*a*b^2*d - 3*a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
2*a^(9/2)*Sqrt[b])
```

fricas [A] time = 0.71, size = 438, normalized size = 2.88

$$\left[-\frac{30\left(7\,ab^4c-5\,a^2b^3d+3\,a^3b^2e-a^4bf\right)x^6+12\,a^4bc+20\left(7\,a^2b^3c-5\,a^3b^2d+3\,a^4be\right)x^4-4\left(7\,a^3b^2c-5\,a^4bd\right)x^2}{60\left(a^5b^2x^7+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(30*(7*a*b^4*c - 5*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^6 + 12*a^4*b
*c + 20*(7*a^2*b^3*c - 5*a^3*b^2*d + 3*a^4*b*e)*x^4 - 4*(7*a^3*b^2*c - 5*a^
4*b*d)*x^2 - 15*((7*b^4*c - 5*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^7 + (7*a*b
^3*c - 5*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^2*x^7 + a^6*b*x^5), -1/30*(15*(7*a*b^4*c
 - 5*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^6 + 6*a^4*b*c + 10*(7*a^2*b^3*c -
 5*a^3*b^2*d + 3*a^4*b*e)*x^4 - 2*(7*a^3*b^2*c - 5*a^4*b*d)*x^2 + 15*((7*b^
4*c - 5*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^7 + (7*a*b^3*c - 5*a^2*b^2*d + 3
*a^3*b*e - a^4*f)*x^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^2*x^7 + a^6*
b*x^5)]
```

giac [A] time = 0.49, size = 151, normalized size = 0.99

$$-\frac{\left(7\,b^3c-5\,ab^2d-a^3f+3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\left(bx^2+a\right)a^4}-\frac{45\,b^2cx^4-30\,abdx^4+15\,a^2x^4e-}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*b^3*c - 5*a*b^2*d - a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a^4) - 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a^
4) - 1/15*(45*b^2*c*x^4 - 30*a*b*d*x^4 + 15*a^2*x^4*e - 10*a*b*c*x^2 + 5*a^
2*d*x^2 + 3*a^2*c)/(a^4*x^5)
```

maple [A] time = 0.02, size = 219, normalized size = 1.44

$$\frac{fx}{2\left(b\,x^{2}+a\right)a}+\frac{f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{bex}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{3be\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{2}}+\frac{b^{2}dx}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{5b^{2}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{3}}-\frac{}{2\left(b\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a*x/(b*x^2+a)*f-1/2/a^2*x/(b*x^2+a)*b*e+1/2/a^3*x/(b*x^2+a)*b^2*d-1/2/a
^4*x/(b*x^2+a)*b^3*c+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-3/2/a^2/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b*e+5/2/a^3/(a*b)^(1/2)*arctan(1/(a*b
)^(1/2)*b*x)*b^2*d-7/2/a^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^3*c-1/5*
c/a^2/x^5-1/3/a^2/x^3*d+2/3/a^3/x^3*b*c-1/a^2/x*e+2/a^3/x*b*d-3/a^4/x*b^2*c
```

maxima [A] time = 2.99, size = 151, normalized size = 0.99

$$-\frac{15\left(7\,b^{3}c-5\,ab^{2}d+3\,a^{2}be-a^{3}f\right)x^{6}+10\left(7\,ab^{2}c-5\,a^{2}bd+3\,a^{3}e\right)x^{4}+6\,a^{3}c-2\left(7\,a^{2}bc-5\,a^{3}d\right)x^{2}}{30\left(a^{4}bx^{7}+a^{5}x^{5}\right)}-\frac{\left(7\,b^{3}c-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*x^6 + 10*(7*a*b^2*c - 5
*a^2*b*d + 3*a^3*e)*x^4 + 6*a^3*c - 2*(7*a^2*b*c - 5*a^3*d)*x^2)/(a^4*b*x^7
 + a^5*x^5) - 1/2*(7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^4)
```

mupad [B] time = 1.00, size = 145, normalized size = 0.95

$$-\frac{\frac{c}{5\,a}+\frac{x^{6}\left(-f\,a^{3}+3\,e\,a^{2}\,b-5\,d\,a\,b^{2}+7\,c\,b^{3}\right)}{2\,a^{4}}+\frac{x^{2}\left(5\,a\,d-7\,b\,c\right)}{15\,a^{2}}+\frac{x^{4}\left(3\,e\,a^{2}-5\,d\,a\,b+7\,c\,b^{2}\right)}{3\,a^{3}}}{b\,x^{7}+a\,x^{5}}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^{3}+3\,e\,a^{2}\,b-5\,d\,a\,b^{2}\right.}{2\,a^{9/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x)
```

```
[Out] - (c/(5*a) + (x^6*(7*b^3*c - a^3*f - 5*a*b^2*d + 3*a^2*b*e))/(2*a^4) + (x^2
*(5*a*d - 7*b*c))/(15*a^2) + (x^4*(7*b^2*c + 3*a^2*e - 5*a*b*d))/(3*a^3))/(
a*x^5 + b*x^7) - (atan((b^(1/2)*x)/a^(1/2))*(7*b^3*c - a^3*f - 5*a*b^2*d +
3*a^2*b*e))/(2*a^(9/2)*b^(1/2))
```

sympy [A] time = 32.02, size = 226, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{a^{9}b}}\left(a^{3}f-3a^{2}be+5ab^{2}d-7b^{3}c\right)\log\left(-a^{5}\sqrt{-\frac{1}{a^{9}b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{9}b}}\left(a^{3}f-3a^{2}be+5ab^{2}d-7b^{3}c\right)\log\left(a^{5}\sqrt{-\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**2*d - 7*b**3*c)*log(-a**5*
sqrt(-1/(a**9*b)) + x)/4 + sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**
2*d - 7*b**3*c)*log(a**5*sqrt(-1/(a**9*b)) + x)/4 + (-6*a**3*c + x**6*(15*a
**3*f - 45*a**2*b*e + 75*a*b**2*d - 105*b**3*c) + x**4*(-30*a**3*e + 50*a**
2*b*d - 70*a*b**2*c) + x**2*(-10*a**3*d + 14*a**2*b*c))/(30*a**5*x**5 + 30*
a**4*b*x**7)
```

3.131 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=189

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{7a^2x^7}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^3f+5a^2be-7ab^2d+9b^3c\right)}{2a^{11/2}}+\frac{bx\left(a^3(-f)+a^2be-\right.}{2a^5\left(a+bx\right.}$$

```
[Out] -1/7*c/a^2/x^7+1/5*(-a*d+2*b*c)/a^3/x^5+1/3*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x^
3+(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x+1/2*b*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*x/a^5/(b*x^2+a)+1/2*(-3*a^3*f+5*a^2*b*e-7*a*b^2*d+9*b^3*c)*arctan(x*
b^(1/2)/a^(1/2))*b^(1/2)/a^(11/2)
```

Rubi [A] time = 0.29, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1802, 205}

$$\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^5\left(a+bx^2\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{a^5x}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^2be-3a^3f-7ab^2d+\right.}{2a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/(3
*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^5*(a + b*x^2)) + (Sqrt[b]*(9*b^3*c - 7*
a*b^2*d + 5*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^5\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{b\left(b^3c-\right.}{}}{x^8\left(a+bx^2\right)}}{2a}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^5\left(a+bx^2\right)} - \frac{\int\left(-\frac{2c}{ax^8}-\frac{2(-2bc+ad)}{a^2x^6}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^4}-\frac{2\left(-4b^3c+3ab^2d\right.}{a^4x}\right.}{2a}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-a\right.}{2a}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-a\right.}{2a}$$

Mathematica [A] time = 0.11, size = 190, normalized size = 1.01

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{7a^2x^7}+\frac{a^2(-e)+2abd-3b^2c}{3a^4x^3}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)}{2a^{11/2}}-\frac{bx\left(a^3f-a^2be+ab^2d\right.}{2a^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -1/7*c/(a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(3*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) - (b*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^5*(a + b*x^2)) - (Sqrt[b]*(-9*b^3
*c + 7*a*b^2*d - 5*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11
/2))
```

fricas [A] time = 0.62, size = 488, normalized size = 2.58

$$\left[\frac{210\left(9b^4c-7ab^3d+5a^2b^2e-3a^3bf\right)x^8+140\left(9ab^3c-7a^2b^2d+5a^3be-3a^4f\right)x^6-60a^4c-28\left(9a^2b^2c-7a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(210*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^8 + 140*(9*a*
b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6 - 60*a^4*c - 28*(9*a^2*b^2*c
 - 7*a^3*b*d + 5*a^4*e)*x^4 + 12*(9*a^3*b*c - 7*a^4*d)*x^2 - 105*((9*b^4*c
- 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^9 + (9*a*b^3*c - 7*a^2*b^2*d + 5*a
^3*b*e - 3*a^4*f)*x^7)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)))/(a^5*b*x^9 + a^6*x^7), 1/210*(105*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*
e - 3*a^3*b*f)*x^8 + 70*(9*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6
 - 30*a^4*c - 14*(9*a^2*b^2*c - 7*a^3*b*d + 5*a^4*e)*x^4 + 6*(9*a^3*b*c - 7
*a^4*d)*x^2 + 105*((9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^9 + (9
*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^7)*sqrt(b/a)*arctan(x*sqrt(
b/a)))/(a^5*b*x^9 + a^6*x^7)]
```

giac [A] time = 0.43, size = 201, normalized size = 1.06

$$\frac{\left(9b^4c-7ab^3d-3a^3bf+5a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^5}+\frac{b^4cx-ab^3dx-a^3bfx+a^2b^2xe}{2\left(bx^2+a\right)a^5}+\frac{420b^3cx^6-315ab^2dx^6-105}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*b^4*c - 7*a*b^3*d - 3*a^3*b*f + 5*a^2*b^2*e)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^5) + 1/2*(b^4*c*x - a*b^3*d*x - a^3*b*f*x + a^2*b^2*x*e)/((b*x^
2 + a)*a^5) + 1/105*(420*b^3*c*x^6 - 315*a*b^2*d*x^6 - 105*a^3*f*x^6 + 210*
a^2*b*x^6*e - 105*a*b^2*c*x^4 + 70*a^2*b*d*x^4 - 35*a^3*x^4*e + 42*a^2*b*c*
x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^5*x^7)
```

maple [A] time = 0.02, size = 268, normalized size = 1.42

$$-\frac{bfx}{2\left(bx^2+a\right)a^2}-\frac{3bf\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^2}+\frac{b^2ex}{2\left(bx^2+a\right)a^3}+\frac{5b^2e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^3}-\frac{b^3dx}{2\left(bx^2+a\right)a^4}-\frac{7b^3d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x)
```

```
[Out] -1/2*b/a^2*x/(b*x^2+a)*f+1/2*b^2/a^3*x/(b*x^2+a)*e-1/2*b^3/a^4*x/(b*x^2+a)*
d+1/2*b^4/a^5*x/(b*x^2+a)*c-3/2*b/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
*f+5/2*b^2/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-7/2*b^3/a^4/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)*d+9/2*b^4/a^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
)*b*x)*c-1/7*c/a^2/x^7-1/5/a^2/x^5*d+2/5/a^3/x^5*b*c-1/3/a^2/x^3*e+2/3/a^3/
x^3*b*d-1/a^4/x^3*b^2*c-1/a^2/x*f+2/a^3/x*b*e-3/a^4/x*b^2*d+4/a^5/x*b^3*c
```

maxima [A] time = 3.05, size = 194, normalized size = 1.03

$$\frac{105\left(9\,b^4c-7\,ab^3d+5\,a^2b^2e-3\,a^3bf\right)x^8+70\left(9\,ab^3c-7\,a^2b^2d+5\,a^3be-3\,a^4f\right)x^6-30\,a^4c-14\left(9\,a^2b^2c-7\right.}{210\left(a^5bx^9+a^6x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/210*(105*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^8 + 70*(9*a*b^
3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6 - 30*a^4*c - 14*(9*a^2*b^2*c -
 7*a^3*b*d + 5*a^4*e)*x^4 + 6*(9*a^3*b*c - 7*a^4*d)*x^2)/(a^5*b*x^9 + a^6*x
^7) + 1/2*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 0.99, size = 181, normalized size = 0.96

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{2\,a^{11/2}}-\frac{\frac{c}{7\,a}-\frac{x^6\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{3\,a^4}+\frac{x^2\,(7\,a\,d-9\,b\,c)}{35\,a^2}+\frac{x^4\,(5\,e}{}}{b\,x^9+a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*b
*e))/(2*a^(11/2)) - (c/(7*a) - (x^6*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*
b*e))/(3*a^4) + (x^2*(7*a*d - 9*b*c))/(35*a^2) + (x^4*(9*b^2*c + 5*a^2*e -
7*a*b*d))/(15*a^3) - (b*x^8*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*b*e))/(2
*a^5))/(a*x^7 + b*x^9)
```

sympy [B] time = 99.02, size = 394, normalized size = 2.08

$$\frac{\sqrt{-\frac{b}{a^{11}}}\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)\log\left(-\frac{a^6\sqrt{-\frac{b}{a^{11}}}\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)}{3a^3bf-5a^2b^2e+7ab^3d-9b^4c}+x\right)}{4}-\frac{\sqrt{-\frac{b}{a^{11}}}\left(3a^3f-5a^2be+7ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**11)*(3*a**3*f - 5*a**2*b*e + 7*a*b**2*d - 9*b**3*c)*log(-a**6*sq
rt(-b/a**11)*(3*a**3*f - 5*a**2*b*e + 7*a*b**2*d - 9*b**3*c)/(3*a**3*b*f -
5*a**2*b**2*e + 7*a*b**3*d - 9*b**4*c) + x)/4 - sqrt(-b/a**11)*(3*a**3*f -
5*a**2*b*e + 7*a*b**2*d - 9*b**3*c)*log(a**6*sqrt(-b/a**11)*(3*a**3*f - 5*a
**2*b*e + 7*a*b**2*d - 9*b**3*c)/(3*a**3*b*f - 5*a**2*b**2*e + 7*a*b**3*d -
 9*b**4*c) + x)/4 + (-30*a**4*c + x**8*(-315*a**3*b*f + 525*a**2*b**2*e - 7
35*a*b**3*d + 945*b**4*c) + x**6*(-210*a**4*f + 350*a**3*b*e - 490*a**2*b**
2*d + 630*a*b**3*c) + x**4*(-70*a**4*e + 98*a**3*b*d - 126*a**2*b**2*c) + x
**2*(-42*a**4*d + 54*a**3*b*c))/(210*a**6*x**7 + 210*a**5*b*x**9)
```

3.132 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=230

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{9a^2x^9}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-5a^3f+7a^2be-9ab^2d+11b^3c\right)}{2a^{13/2}}-\frac{b^2x\left(a^3(-f)+a^2be\right.}{2a^6\left(a+b\right.}$$

```
[Out] -1/9*c/a^2/x^9+1/7*(-a*d+2*b*c)/a^3/x^7+1/5*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x^
5+1/3*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^3-b*(-2*a^3*f+3*a^2*b*e-4*
a*b^2*d+5*b^3*c)/a^6/x-1/2*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^6/(b*x^2+
a)-1/2*b^(3/2)*(-5*a^3*f+7*a^2*b*e-9*a*b^2*d+11*b^3*c)*arctan(x*b^(1/2)/a^(
1/2))/a^(13/2)
```

Rubi [A] time = 0.38, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1802, 205}

$$-\frac{b^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^6\left(a+bx^2\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{3a^5x^3}-\frac{b\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{a^6x}-\frac{b^{3/2}\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e)/(5
*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) - (b*(5*b
^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*x) - (b^2*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x)/(2*a^6*(a + b*x^2)) - (b^(3/2)*(11*b^3*c - 9*a*b^2*d +
7*a^2*b*e - 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^6\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{2b}{}}{x^{10}\left(a+bx^2\right)}}{2a}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^6\left(a+bx^2\right)} - \frac{\int\left(-\frac{2c}{ax^{10}}-\frac{2(-2bc+ad)}{a^2x^8}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^6}-\frac{2\left(-4b^3c+3a\right.}{}\right.}{}$$

$$= -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}-\frac{b\left(5b^3c-\right.}{}$$

$$= -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}-\frac{b\left(5b^3c-\right.}{}$$

Mathematica [A] time = 0.12, size = 230, normalized size = 1.00

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{9a^2x^9}+\frac{a^2(-e)+2abd-3b^2c}{5a^4x^5}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^3f-7a^2be+9ab^2d-11b^3c\right)}{2a^{13/2}}+\frac{b^2x\left(a^3f-a^2be+ab\right.}{2a^6\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -1/9*c/(a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*(
-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f)*x)/(2*a^6*(a + b*x^2)) + (b^(3/2)*(-11*b^3*c + 9*a*
b^2*d - 7*a^2*b*e + 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

fricas [A] time = 0.67, size = 582, normalized size = 2.53

$$\left[-\frac{630\left(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f\right)x^{10}+420\left(11\,ab^4c-9\,a^2b^3d+7\,a^3b^2e-5\,a^4bf\right)x^8-84\left(11\,a^2b^3c-9\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/1260*(630*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 420
*(11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^8 - 84*(11*a^2*b^3*
c - 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f)*x^6 + 140*a^5*c + 36*(11*a^3*b^2*c -
 9*a^4*b*d + 7*a^5*e)*x^4 - 20*(11*a^4*b*c - 9*a^5*d)*x^2 + 315*((11*b^5*c
- 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^11 + (11*a*b^4*c - 9*a^2*b^3*d +
 7*a^3*b^2*e - 5*a^4*b*f)*x^9)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(a^6*b*x^11 + a^7*x^9), -1/630*(315*(11*b^5*c - 9*a*b^4*d +
 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 210*(11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^
2*e - 5*a^4*b*f)*x^8 - 42*(11*a^2*b^3*c - 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f
)*x^6 + 70*a^5*c + 18*(11*a^3*b^2*c - 9*a^4*b*d + 7*a^5*e)*x^4 - 10*(11*a^4
*b*c - 9*a^5*d)*x^2 + 315*((11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*
f)*x^11 + (11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^9)*sqrt(b/
a)*arctan(x*sqrt(b/a)))/(a^6*b*x^11 + a^7*x^9)]
```

giac [A] time = 0.38, size = 252, normalized size = 1.10

$$-\frac{\left(11\,b^5c - 9\,ab^4d - 5\,a^3b^2f + 7\,a^2b^3e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^6} - \frac{b^5cx - ab^4dx - a^3b^2fx + a^2b^3xe}{2\left(bx^2+a\right)a^6} - \frac{1575\,b^4cx^8 - 1260\,ab^3 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(11*b^5*c - 9*a*b^4*d - 5*a^3*b^2*f + 7*a^2*b^3*e)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a^6) - 1/2*(b^5*c*x - a*b^4*d*x - a^3*b^2*f*x + a^2*b^3*x*e)/
((b*x^2 + a)*a^6) - 1/315*(1575*b^4*c*x^8 - 1260*a*b^3*d*x^8 - 630*a^3*b*f*
x^8 + 945*a^2*b^2*x^8*e - 420*a*b^3*c*x^6 + 315*a^2*b^2*d*x^6 + 105*a^4*f*x
^6 - 210*a^3*b*x^6*e + 189*a^2*b^2*c*x^4 - 126*a^3*b*d*x^4 + 63*a^4*x^4*e -
 90*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^6*x^9)
```

maple [A] time = 0.02, size = 318, normalized size = 1.38

$$\frac{b^2fx}{2\left(b\,x^2+a\right)a^3} + \frac{5b^2f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^3} - \frac{b^3ex}{2\left(b\,x^2+a\right)a^4} - \frac{7b^3e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^4} + \frac{b^4dx}{2\left(b\,x^2+a\right)a^5} + \frac{9b^4d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^5} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a^3*b^2*x/(b*x^2+a)*f-1/2/a^4*b^3*x/(b*x^2+a)*e+1/2/a^5*b^4*x/(b*x^2+a)
*d-1/2/a^6*b^5*x/(b*x^2+a)*c+5/2/a^3*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)*f-7/2/a^4*b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+9/2/a^5*b^4/(a*b)
^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-11/2/a^6*b^5/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)*c-1/9*c/a^2/x^9-1/7/a^2/x^7*d+2/7/a^3/x^7*b*c-1/5/a^2/x^5*e+2/5/
a^3/x^5*b*d-3/5/a^4/x^5*b^2*c-1/3/a^2/x^3*f+2/3/a^3/x^3*b*e-1/a^4/x^3*b^2*d
+4/3/a^5/x^3*b^3*c+2*b/a^3/x*f-3*b^2/a^4/x*e+4*b^3/a^5/x*d-5*b^4/a^6/x*c
```

maxima [A] time = 2.99, size = 238, normalized size = 1.03

$$-\frac{315\left(11\,b^5c - 9\,ab^4d + 7\,a^2b^3e - 5\,a^3b^2f\right)x^{10} + 210\left(11\,ab^4c - 9\,a^2b^3d + 7\,a^3b^2e - 5\,a^4bf\right)x^8 - 42\left(11\,a^2b^3c - \cdots\right.}{630\left(a^6bx^{11} + a^7\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/630*(315*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 210*(
11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^8 - 42*(11*a^2*b^3*c
- 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f)*x^6 + 70*a^5*c + 18*(11*a^3*b^2*c - 9*
a^4*b*d + 7*a^5*e)*x^4 - 10*(11*a^4*b*c - 9*a^5*d)*x^2)/(a^6*b*x^11 + a^7*x
^9) - 1/2*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*arctan(b*x/sqr
t(a*b))/(sqrt(a*b)*a^6)
```

mupad [B] time = 1.01, size = 219, normalized size = 0.95

$$-\frac{\frac{c}{9\,a} - \frac{x^6\left(-5\,f\,a^3+7\,e\,a^2\,b-9\,d\,a\,b^2+11\,c\,b^3\right)}{15\,a^4} + \frac{x^2\,(9\,a\,d-11\,b\,c)}{63\,a^2} + \frac{x^4\left(7\,e\,a^2-9\,d\,a\,b+11\,c\,b^2\right)}{35\,a^3} + \frac{b\,x^8\left(-5\,f\,a^3+7\,e\,a^2\,b-9\,d\,a\,b^2+11\,c\,b^3\right)}{3\,a^5} + \frac{b^2\,x \cdots}{\cdots}}{b\,x^{11}+a\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x)
```

```
[Out] - (c/(9*a) - (x^6*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(15*a^4) +
(x^2*(9*a*d - 11*b*c))/(63*a^2) + (x^4*(11*b^2*c + 7*a^2*e - 9*a*b*d))/(35*
a^3) + (b*x^8*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(3*a^5) + (b^2*
x^10*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(2*a^6))/(a*x^9 + b*x^11
) - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*
a^2*b*e))/(2*a^(13/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.133 $\int \frac{x^8\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=287

$$\frac{x^9\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{a^2x\left(-17a^3f+13a^2be-9ab^2d+5b^3c\right)}{8b^7\left(a+bx^2\right)}-\frac{ax\left(-63a^3f+43a^2be-27ab^2d+15b^3c\right)}{4b^7}+\frac{x^3\left(\right.}{}$$

```
[Out] -1/4*a*(-63*a^3*f+43*a^2*b*e-27*a*b^2*d+15*b^3*c)*x/b^7+1/6*(-23*a^3*f+15*a
^2*b*e-9*a*b^2*d+5*b^3*c)*x^3/b^6-1/20*(-29*a^3*f+17*a^2*b*e-9*a*b^2*d+5*b^
3*c)*x^5/a/b^5+1/7*(-3*a*f+b*e)*x^7/b^4+1/9*f*x^9/b^3+1/4*(c-a*(a^2*f-a*b*e
+b^2*d)/b^3)*x^9/a/(b*x^2+a)^2-1/8*a^2*(-17*a^3*f+13*a^2*b*e-9*a*b^2*d+5*b^
3*c)*x/b^7/(b*x^2+a)+1/8*a^(3/2)*(-143*a^3*f+99*a^2*b*e-63*a*b^2*d+35*b^3*c
)*arctan(x*b^(1/2)/a^(1/2))/b^(15/2)
```

Rubi [A] time = 0.49, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1804, 1585, 1257, 1810, 205}

$$\frac{x^9\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x^5\left(17a^2be-29a^3f-9ab^2d+5b^3c\right)}{20ab^5}+\frac{x^3\left(15a^2be-23a^3f-9ab^2d+5b^3c\right)}{6b^6}-\frac{a^2x\left(13a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] -(a*(15*b^3*c - 27*a*b^2*d + 43*a^2*b*e - 63*a^3*f)*x)/(4*b^7) + ((5*b^3*c
- 9*a*b^2*d + 15*a^2*b*e - 23*a^3*f)*x^3)/(6*b^6) - ((5*b^3*c - 9*a*b^2*d +
 17*a^2*b*e - 29*a^3*f)*x^5)/(20*a*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*
x^9)/(9*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^9)/(4*a*(a + b*x^2)
^2) - (a^2*(5*b^3*c - 9*a*b^2*d + 13*a^2*b*e - 17*a^3*f)*x)/(8*b^7*(a + b*x
^2)) + (a^(3/2)*(35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(8*b^(15/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^8\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^7\left(\left(5bc-9ad+\frac{9a^2e}{b}-\frac{9a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^8\left(5bc-9ad+\frac{9a^2e}{b}-\frac{9a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{a^2\left(5b^3c-9ab^2d+13a^2be-17a^3f\right)x}{8b^7\left(a+bx^2\right)} + \frac{\int \frac{a^3\left(5b^3c-9ab^2d\right.}{}}{}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{a^2\left(5b^3c-9ab^2d+13a^2be-17a^3f\right)x}{8b^7\left(a+bx^2\right)} + \frac{\int\left(-2a^2\left(15b^3\right.\right.}{}$$

$$= -\frac{a\left(15b^3c-27ab^2d+43a^2be-63a^3f\right)x}{4b^7} + \frac{\left(5b^3c-9ab^2d+15a^2be-23a^3f\right)x^3}{6b^6}$$

$$= -\frac{a\left(15b^3c-27ab^2d+43a^2be-63a^3f\right)x}{4b^7} + \frac{\left(5b^3c-9ab^2d+15a^2be-23a^3f\right)x^3}{6b^6}$$

Mathematica [A] time = 0.19, size = 272, normalized size = 0.95

$$\frac{x^5\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{a^2x\left(25a^3f-21a^2be+17ab^2d-13b^3c\right)}{8b^7\left(a+bx^2\right)}+\frac{a^3x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4b^7\left(a+bx^2\right)^2}+\frac{ax\left(15a^3f-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b^
2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^5
)/(5*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^9)/(9*b^3) + (a^3*(b^3*c - a
```

```
*b^2*d + a^2*b*e - a^3*f)*x)/(4*b^7*(a + b*x^2)^2) + (a^2*(-13*b^3*c + 17*a
*b^2*d - 21*a^2*b*e + 25*a^3*f)*x)/(8*b^7*(a + b*x^2)) - (a^(3/2)*(-35*b^3*
c + 63*a*b^2*d - 99*a^2*b*e + 143*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^
(15/2))
```

fricas [A] time = 0.58, size = 762, normalized size = 2.66

$$\left[\frac{560\,b^{6}fx^{13}+80\left(9\,b^{6}e-13\,ab^{5}f\right)x^{11}+16\left(63\,b^{6}d-99\,ab^{5}e+143\,a^{2}b^{4}f\right)x^{9}+48\left(35\,b^{6}c-63\,ab^{5}d+99\,a^{2}b^{4}e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/5040*(560*b^6*f*x^13 + 80*(9*b^6*e - 13*a*b^5*f)*x^11 + 16*(63*b^6*d - 9
9*a*b^5*e + 143*a^2*b^4*f)*x^9 + 48*(35*b^6*c - 63*a*b^5*d + 99*a^2*b^4*e -
 143*a^3*b^3*f)*x^7 - 336*(35*a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a
^4*b^2*f)*x^5 - 1050*(35*a^2*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*
b*f)*x^3 - 315*(35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f + (35*
a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*
c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 -
 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(35*a^3*b^3*c - 63*a^4*b^2*d + 99
*a^5*b*e - 143*a^6*f)*x)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7), 1/2520*(280*b^6
*f*x^13 + 40*(9*b^6*e - 13*a*b^5*f)*x^11 + 8*(63*b^6*d - 99*a*b^5*e + 143*a
^2*b^4*f)*x^9 + 24*(35*b^6*c - 63*a*b^5*d + 99*a^2*b^4*e - 143*a^3*b^3*f)*x
^7 - 168*(35*a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^5 - 5
25*(35*a^2*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^3 + 315*(35
*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f + (35*a*b^5*c - 63*a^2*b
^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*c - 63*a^3*b^3*d +
 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(
35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f)*x)/(b^9*x^4 + 2*a*b^8
*x^2 + a^2*b^7)]
```

giac [A] time = 0.40, size = 301, normalized size = 1.05

$$\frac{\left(35\,a^{2}b^{3}c-63\,a^{3}b^{2}d-143\,a^{5}f+99\,a^{4}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{7}}-\frac{13\,a^{2}b^{4}cx^{3}-17\,a^{3}b^{3}dx^{3}-25\,a^{5}bfx^{3}+21\,a^{4}b^{2}x^{3}e+}{8\left(bx^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*a^2*b^3*c - 63*a^3*b^2*d - 143*a^5*f + 99*a^4*b*e)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^7) - 1/8*(13*a^2*b^4*c*x^3 - 17*a^3*b^3*d*x^3 - 25*a^5*b
*f*x^3 + 21*a^4*b^2*x^3*e + 11*a^3*b^3*c*x - 15*a^4*b^2*d*x - 23*a^6*f*x +
19*a^5*b*x*e)/((b*x^2 + a)^2*b^7) + 1/315*(35*b^24*f*x^9 - 135*a*b^23*f*x^7
 + 45*b^24*x^7*e + 63*b^24*d*x^5 + 378*a^2*b^22*f*x^5 - 189*a*b^23*x^5*e +
105*b^24*c*x^3 - 315*a*b^23*d*x^3 - 1050*a^3*b^21*f*x^3 + 630*a^2*b^22*x^3*
e - 945*a*b^23*c*x + 1890*a^2*b^22*d*x + 4725*a^4*b^20*f*x - 3150*a^3*b^21*
x*e)/b^27
```

maple [A] time = 0.02, size = 394, normalized size = 1.37

$$\frac{f\,x^{9}}{9b^{3}}-\frac{3af\,x^{7}}{7b^{4}}+\frac{e\,x^{7}}{7b^{3}}+\frac{25a^{5}f\,x^{3}}{8\left(b\,x^{2}+a\right)^{2}b^{6}}-\frac{21a^{4}e\,x^{3}}{8\left(b\,x^{2}+a\right)^{2}b^{5}}+\frac{17a^{3}d\,x^{3}}{8\left(b\,x^{2}+a\right)^{2}b^{4}}-\frac{13a^{2}c\,x^{3}}{8\left(b\,x^{2}+a\right)^{2}b^{3}}+\frac{6a^{2}f\,x^{5}}{5b^{5}}-\frac{3ae\,x^{5}}{5b^{4}}+\frac{d\,x}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/3/b^3*x^3*c-3/7/b^4*x^7*a*f+6/5/b^5*x^5*a^2*f-3/5/b^4*x^5*a*e-10/3/b^6*x^
3*a^3*f+2/b^5*x^3*a^2*e-1/b^4*x^3*a*d+15/b^7*a^4*f*x-10/b^6*a^3*e*x+6/b^5*a
^2*d*x-3/b^4*a*c*x+1/9*f*x^9/b^3+25/8*a^5/b^6/(b*x^2+a)^2*x^3*f+23/8*a^6/b^
7/(b*x^2+a)^2*f*x-19/8*a^5/b^6/(b*x^2+a)^2*e*x+15/8*a^4/b^5/(b*x^2+a)^2*d*x
-11/8*a^3/b^4/(b*x^2+a)^2*c*x-143/8*a^5/b^7/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
)*b*x)*f+99/8*a^4/b^6/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-63/8*a^3/b^5/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+35/8*a^2/b^4/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)*c+1/7/b^3*x^7*e+1/5/b^3*x^5*d-21/8*a^4/b^5/(b*x^2+a)^2*x^3*
e+17/8*a^3/b^4/(b*x^2+a)^2*x^3*d-13/8*a^2/b^3/(b*x^2+a)^2*x^3*c
```

maxima [A] time = 3.10, size = 281, normalized size = 0.98

$$-\frac{\left(13\,a^2b^4c-17\,a^3b^3d+21\,a^4b^2e-25\,a^5bf\right)x^3+\left(11\,a^3b^3c-15\,a^4b^2d+19\,a^5be-23\,a^6f\right)x}{8\left(b^9x^4+2\,ab^8x^2+a^2b^7\right)}+\frac{\left(35\,a^2b^3c-63\,a^3b^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*a^2*b^4*c - 17*a^3*b^3*d + 21*a^4*b^2*e - 25*a^5*b*f)*x^3 + (11*a
^3*b^3*c - 15*a^4*b^2*d + 19*a^5*b*e - 23*a^6*f)*x)/(b^9*x^4 + 2*a*b^8*x^2
+ a^2*b^7) + 1/8*(35*a^2*b^3*c - 63*a^3*b^2*d + 99*a^4*b*e - 143*a^5*f)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e - 3*a*
b^3*f)*x^7 + 63*(b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^5 + 105*(b^4*c - 3*a*b^
3*d + 6*a^2*b^2*e - 10*a^3*b*f)*x^3 - 315*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3
*b*e - 15*a^4*f)*x)/b^7
```

mupad [B] time = 1.00, size = 506, normalized size = 1.76

$$x^7\left(\frac{e}{7\,b^3}-\frac{3\,a\,f}{7\,b^4}\right)+x^3\left(\frac{c}{3\,b^3}-\frac{a^3\,f}{3\,b^6}-\frac{a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)+\frac{x\left(\frac{23\,f\,a^6}{8}-\frac{19\,e\,a^5\,b}{8}+\frac{15\,d\,a^4\,b^2}{8}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x^7*(e/(7*b^3) - (3*a*f)/(7*b^4)) + x^3*(c/(3*b^3) - (a^3*f)/(3*b^6) - (a^2
*(e/b^3 - (3*a*f)/b^4))/b^2 + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*
a*f)/b^4))/b))/b) + (x*((23*a^6*f)/8 - (11*a^3*b^3*c)/8 + (15*a^4*b^2*d)/8
- (19*a^5*b*e)/8) - x^3*((13*a^2*b^4*c)/8 - (17*a^3*b^3*d)/8 + (21*a^4*b^2*
e)/8 - (25*a^5*b*f)/8))/(a^2*b^7 + b^9*x^4 + 2*a*b^8*x^2) - x*((3*a*(c/b^3
- (a^3*f)/b^6 - (3*a^2*(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d
/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b))/b - (3*a^2*((3*a^2*f)/b^5 - d/b^
3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b^2 + (a^3*(e/b^3 - (3*a*f)/b^4))/b^3)
- x^5*((3*a^2*f)/(5*b^5) - d/(5*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(5*b)) +
 (f*x^9)/(9*b^3) - (a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(35*b^3*c - 143*a^3*f -
 63*a*b^2*d + 99*a^2*b*e))/(143*a^5*f - 35*a^2*b^3*c + 63*a^3*b^2*d - 99*a^
4*b*e))*(35*b^3*c - 143*a^3*f - 63*a*b^2*d + 99*a^2*b*e))/(8*b^(15/2))
```

sympy [A] time = 16.43, size = 503, normalized size = 1.75

$$x^7\left(-\frac{3af}{7b^4}+\frac{e}{7b^3}\right)+x^5\left(\frac{6a^2f}{5b^5}-\frac{3ae}{5b^4}+\frac{d}{5b^3}\right)+x^3\left(-\frac{10a^3f}{3b^6}+\frac{2a^2e}{b^5}-\frac{ad}{b^4}+\frac{c}{3b^3}\right)+x\left(\frac{15a^4f}{b^7}-\frac{10a^3e}{b^6}+\frac{6a^2d}{b^5}-\frac{3a}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**7*(-3*a*f/(7*b**4) + e/(7*b**3)) + x**5*(6*a**2*f/(5*b**5) - 3*a*e/(5*b*
*4) + d/(5*b**3)) + x**3*(-10*a**3*f/(3*b**6) + 2*a**2*e/b**5 - a*d/b**4 +
c/(3*b**3)) + x*(15*a**4*f/b**7 - 10*a**3*e/b**6 + 6*a**2*d/b**5 - 3*a*c/b*
*4) + sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c
)*log(-b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*
b**3*c)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/16 -
 sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c)*log
(b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c
)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/16 + (x**3
*(25*a**5*b*f - 21*a**4*b**2*e + 17*a**3*b**3*d - 13*a**2*b**4*c) + x*(23*a
**6*f - 19*a**5*b*e + 15*a**4*b**2*d - 11*a**3*b**3*c))/(8*a**2*b**7 + 16*a
*b**8*x**2 + 8*b**9*x**4) + f*x**9/(9*b**3)
```

3.134 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=247

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-99a^3f+63a^2be-35ab^2d+15b^3c\right)}{8b^{13/2}}+\frac{ax\left(-15a^3f+11a^2be-7ab^2d+3\right.}{8b^6\left(a+bx^2\right)}$$

```
[Out] 1/2*(-21*a^3*f+13*a^2*b*e-7*a*b^2*d+3*b^3*c)*x/b^6-1/12*(-27*a^3*f+15*a^2*b
*e-7*a*b^2*d+3*b^3*c)*x^3/a/b^5+1/5*(-3*a*f+b*e)*x^5/b^4+1/7*f*x^7/b^3+1/4*
(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^7/a/(b*x^2+a)^2+1/8*a*(-15*a^3*f+11*a^2*b*e
-7*a*b^2*d+3*b^3*c)*x/b^6/(b*x^2+a)-1/8*(-99*a^3*f+63*a^2*b*e-35*a*b^2*d+15
*b^3*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(13/2)
```

Rubi [A] time = 0.41, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1804, 1585, 1257, 1810, 205}

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x^3\left(15a^2be-27a^3f-7ab^2d+3b^3c\right)}{12ab^5}+\frac{ax\left(11a^2be-15a^3f-7ab^2d+3b^3c\right)}{8b^6\left(a+bx^2\right)}+\frac{x\left(13a^2be-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((3*b^3*c - 7*a*b^2*d + 13*a^2*b*e - 21*a^3*f)*x)/(2*b^6) - ((3*b^3*c - 7*a
*b^2*d + 15*a^2*b*e - 27*a^3*f)*x^3)/(12*a*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^
4) + (f*x^7)/(7*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(4*a*(a
+ b*x^2)^2) + (a*(3*b^3*c - 7*a*b^2*d + 11*a^2*b*e - 15*a^3*f)*x)/(8*b^6*(a
 + b*x^2)) - (Sqrt[a]*(15*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^5\left(\left(3bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^6\left(3bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} + \frac{a\left(3b^3c-7ab^2d+11a^2be-15a^3f\right)x}{8b^6\left(a+bx^2\right)} + \int \frac{-a^2\left(3b^3c-7a\right.}{}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} + \frac{a\left(3b^3c-7ab^2d+11a^2be-15a^3f\right)x}{8b^6\left(a+bx^2\right)} + \frac{\int \left(4a\left(3b^3c\right.\right.}{}$$

$$= \frac{\left(3b^3c-7ab^2d+13a^2be-21a^3f\right)x}{2b^6} - \frac{\left(3b^3c-7ab^2d+15a^2be-27a^3f\right)x^3}{12ab^5} +$$

$$= \frac{\left(3b^3c-7ab^2d+13a^2be-21a^3f\right)x}{2b^6} - \frac{\left(3b^3c-7ab^2d+15a^2be-27a^3f\right)x^3}{12ab^5} +$$

Mathematica [A] time = 0.15, size = 232, normalized size = 0.94

$$\frac{x^3\left(6a^2f-3abe+b^2d\right)}{3b^5} + \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(99a^3f-63a^2be+35ab^2d-15b^3c\right)}{8b^{13/2}} + \frac{ax\left(-21a^3f+17a^2be-13ab^2d\right.}{8b^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*
a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (f*x^7)/(7*b^3) + (a^2*
(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*b^6*(a + b*x^2)^2) + (a*(9*b^3
*c - 13*a*b^2*d + 17*a^2*b*e - 21*a^3*f)*x)/(8*b^6*(a + b*x^2)) + (Sqrt[a]*
```

```
(-15*b^3*c + 35*a*b^2*d - 63*a^2*b*e + 99*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(8*b^(13/2))
```

fricas [A] time = 0.64, size = 668, normalized size = 2.70

$$\left[\frac{240\,b^5fx^{11}+48\left(7\,b^5e-11\,ab^4f\right)x^9+16\left(35\,b^5d-63\,ab^4e+99\,a^2b^3f\right)x^7+112\left(15\,b^5c-35\,ab^4d+63\,a^2b^3e-9\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1680*(240*b^5*f*x^11 + 48*(7*b^5*e - 11*a*b^4*f)*x^9 + 16*(35*b^5*d - 63
*a*b^4*e + 99*a^2*b^3*f)*x^7 + 112*(15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e -
99*a^3*b^2*f)*x^5 + 350*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*a^4*
b*f)*x^3 - 105*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f + (15*b
^5*c - 35*a*b^4*d + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a
^2*b^3*d + 63*a^3*b^2*e - 99*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sq
rt(-a/b) - a)/(b*x^2 + a)) + 210*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*a^4*b*e
- 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6), 1/840*(120*b^5*f*x^11 + 2
4*(7*b^5*e - 11*a*b^4*f)*x^9 + 8*(35*b^5*d - 63*a*b^4*e + 99*a^2*b^3*f)*x^7
 + 56*(15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^5 + 175*(15*a
*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*a^4*b*f)*x^3 - 105*(15*a^2*b^3*c
- 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f + (15*b^5*c - 35*a*b^4*d + 63*a^2*b^
3*e - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*
a^4*b*f)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(15*a^2*b^3*c - 35*a^
3*b^2*d + 63*a^4*b*e - 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)]
```

giac [A] time = 0.47, size = 250, normalized size = 1.01

$$-\frac{\left(15\,ab^3c-35\,a^2b^2d-99\,a^4f+63\,a^3be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^6}+\frac{9\,ab^4cx^3-13\,a^2b^3dx^3-21\,a^4bfx^3+17\,a^3b^2x^3e+7\,a^2b^3}{8\left(bx^2+a\right)^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*a*b^3*c - 35*a^2*b^2*d - 99*a^4*f + 63*a^3*b*e)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^6) + 1/8*(9*a*b^4*c*x^3 - 13*a^2*b^3*d*x^3 - 21*a^4*b*f*x^
3 + 17*a^3*b^2*x^3*e + 7*a^2*b^3*c*x - 11*a^3*b^2*d*x - 19*a^5*f*x + 15*a^4
*b*x*e)/((b*x^2 + a)^2*b^6) + 1/105*(15*b^18*f*x^7 - 63*a*b^17*f*x^5 + 21*b
^18*x^5*e + 35*b^18*d*x^3 + 210*a^2*b^16*f*x^3 - 105*a*b^17*x^3*e + 105*b^1
8*c*x - 315*a*b^17*d*x - 1050*a^3*b^15*f*x + 630*a^2*b^16*x*e)/b^21
```

maple [A] time = 0.02, size = 343, normalized size = 1.39

$$\frac{fx^7}{7b^3}-\frac{21a^4fx^3}{8\left(bx^2+a\right)^2b^5}+\frac{17a^3ex^3}{8\left(bx^2+a\right)^2b^4}-\frac{13a^2dx^3}{8\left(bx^2+a\right)^2b^3}+\frac{9acx^3}{8\left(bx^2+a\right)^2b^2}-\frac{3afx^5}{5b^4}+\frac{ex^5}{5b^3}-\frac{19a^5fx}{8\left(bx^2+a\right)^2b^6}+\frac{15}{8\left(bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/7*f*x^7/b^3-3/5/b^4*x^5*a*f+1/5/b^3*x^5*e+2/b^5*x^3*a^2*f-1/b^4*x^3*a*e+1
/3/b^3*x^3*d-10/b^6*a^3*f*x+6/b^5*a^2*e*x-3/b^4*a*d*x+1/b^3*c*x-21/8*a^4/b^
5/(b*x^2+a)^2*x^3*f+17/8*a^3/b^4/(b*x^2+a)^2*x^3*e-13/8*a^2/b^3/(b*x^2+a)^2
```

```
*x^3*d+9/8*a/b^2/(b*x^2+a)^2*x^3*c-19/8*a^5/b^6/(b*x^2+a)^2*f*x+15/8*a^4/b^
5/(b*x^2+a)^2*e*x-11/8*a^3/b^4/(b*x^2+a)^2*d*x+7/8*a^2/b^3/(b*x^2+a)^2*c*x+
99/8*a^4/b^6/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-63/8*a^3/b^5/(a*b)^(1/
2)*arctan(1/(a*b)^(1/2)*b*x)*e+35/8*a^2/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
)*b*x)*d-15/8*a/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.99, size = 237, normalized size = 0.96

$$\frac{\left(9\,ab^4c-13\,a^2b^3d+17\,a^3b^2e-21\,a^4bf\right)x^3+\left(7\,a^2b^3c-11\,a^3b^2d+15\,a^4be-19\,a^5f\right)x}{8\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)}-\frac{\left(15\,ab^3c-35\,a^2b^2d+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*a*b^4*c - 13*a^2*b^3*d + 17*a^3*b^2*e - 21*a^4*b*f)*x^3 + (7*a^2*b^
3*c - 11*a^3*b^2*d + 15*a^4*b*e - 19*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2
*b^6) - 1/8*(15*a*b^3*c - 35*a^2*b^2*d + 63*a^3*b*e - 99*a^4*f)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^6) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - 3*a*b^2*f)*x
^5 + 35*(b^3*d - 3*a*b^2*e + 6*a^2*b*f)*x^3 + 105*(b^3*c - 3*a*b^2*d + 6*a^
2*b*e - 10*a^3*f)*x)/b^6
```

mupad [B] time = 0.11, size = 348, normalized size = 1.41

$$x^5\left(\frac{e}{5\,b^3}-\frac{3\,a\,f}{5\,b^4}\right)+x\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)-x^3\left(\frac{a^2\,f}{b^5}-\frac{d}{3\,b^3}+\frac{a\left(\frac{e}{b^3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x^5*(e/(5*b^3) - (3*a*f)/(5*b^4)) + x*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3
- (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b
^4))/b))/b) - x^3*((a^2*f)/b^5 - d/(3*b^3) + (a*(e/b^3 - (3*a*f)/b^4))/b) -
 (x*((19*a^5*f)/8 - (7*a^2*b^3*c)/8 + (11*a^3*b^2*d)/8 - (15*a^4*b*e)/8) +
x^3*((13*a^2*b^3*d)/8 - (17*a^3*b^2*e)/8 - (9*a*b^4*c)/8 + (21*a^4*b*f)/8))
/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2) + (f*x^7)/(7*b^3) + (a^(1/2)*atan((a^(1/
2)*b^(1/2)*x*(15*b^3*c - 99*a^3*f - 35*a*b^2*d + 63*a^2*b*e))/(99*a^4*f + 3
5*a^2*b^2*d - 15*a*b^3*c - 63*a^3*b*e))*(15*b^3*c - 99*a^3*f - 35*a*b^2*d +
 63*a^2*b*e))/(8*b^(13/2))
```

sympy [A] time = 18.22, size = 316, normalized size = 1.28

$$x^5\left(-\frac{3af}{5b^4}+\frac{e}{5b^3}\right)+x^3\left(\frac{2a^2f}{b^5}-\frac{ae}{b^4}+\frac{d}{3b^3}\right)+x\left(-\frac{10a^3f}{b^6}+\frac{6a^2e}{b^5}-\frac{3ad}{b^4}+\frac{c}{b^3}\right)-\frac{\sqrt{-\frac{a}{b^{13}}}\left(99a^3f-63a^2be+35ab^2\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**5*(-3*a*f/(5*b**4) + e/(5*b**3)) + x**3*(2*a**2*f/b**5 - a*e/b**4 + d/(3
*b**3)) + x*(-10*a**3*f/b**6 + 6*a**2*e/b**5 - 3*a*d/b**4 + c/b**3) - sqrt(
-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b**2*d - 15*b**3*c)*log(-b**6*sqr
t(-a/b**13) + x)/16 + sqrt(-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b**2*d
 - 15*b**3*c)*log(b**6*sqrt(-a/b**13) + x)/16 + (x**3*(-21*a**4*b*f + 17*a*
*3*b**2*e - 13*a**2*b**3*d + 9*a*b**4*c) + x*(-19*a**5*f + 15*a**4*b*e - 11
*a**3*b**2*d + 7*a**2*b**3*c))/(8*a**2*b**6 + 16*a*b**7*x**2 + 8*b**8*x**4)
 + f*x**7/(7*b**3)
```

3.135 $\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=207

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{8\sqrt{a}\,b^{11/2}}-\frac{x\left(-13a^3f+9a^2be-5ab^2d+b^3c\right)}{8b^5\left(a+bx^2\right)}-\frac{x\left(\right.}{}$$

```
[Out] -1/4*(-25*a^3*f+13*a^2*b*e-5*a*b^2*d+b^3*c)*x/a/b^5+1/3*(-3*a*f+b*e)*x^3/b^
4+1/5*f*x^5/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^5/a/(b*x^2+a)^2-1/8*(-1
3*a^3*f+9*a^2*b*e-5*a*b^2*d+b^3*c)*x/b^5/(b*x^2+a)+1/8*(-63*a^3*f+35*a^2*b*
e-15*a*b^2*d+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)/a^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1804, 1585, 1257, 1810, 205}

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(9a^2be-13a^3f-5ab^2d+b^3c\right)}{8b^5\left(a+bx^2\right)}-\frac{x\left(13a^2be-25a^3f-5ab^2d+b^3c\right)}{4ab^5}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(35a^2b\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] -((b^3*c - 5*a*b^2*d + 13*a^2*b*e - 25*a^3*f)*x)/(4*a*b^5) + ((b*e - 3*a*f)
*x^3)/(3*b^4) + (f*x^5)/(5*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^
5)/(4*a*(a + b*x^2)^2) - ((b^3*c - 5*a*b^2*d + 9*a^2*b*e - 13*a^3*f)*x)/(8*
b^5*(a + b*x^2)) + ((3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^3\left(\left(bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^4\left(bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c-5ab^2d+9a^2be-13a^3f\right)x}{8b^5\left(a+bx^2\right)} + \frac{\int \frac{a\left(b^3c-5ab^2d+9a^2\right.}{}}{}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c-5ab^2d+9a^2be-13a^3f\right)x}{8b^5\left(a+bx^2\right)} + \frac{\int\left(-2\left(b^3c-5a\right.\right.}{}$$

$$= -\frac{\left(b^3c-5ab^2d+13a^2be-25a^3f\right)x}{4ab^5} + \frac{(be-3af)x^3}{3b^4} + \frac{fx^5}{5b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a\right.}{b^3}\right.}{4a\left(a+bx^2\right.}$$

$$= -\frac{\left(b^3c-5ab^2d+13a^2be-25a^3f\right)x}{4ab^5} + \frac{(be-3af)x^3}{3b^4} + \frac{fx^5}{5b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a\right.}{b^3}\right.}{4a\left(a+bx^2\right.}$$

Mathematica [A] time = 0.18, size = 176, normalized size = 0.85

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{8\sqrt{a}\,b^{11/2}}+\frac{x\left(945a^4f-525a^3b\left(e-3fx^2\right)+a^2b^2\left(225d-875ex^2+50\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(945*a^4*f - 525*a^3*b*(e - 3*f*x^2) + a^2*b^2*(225*d - 875*e*x^2 + 504*
f*x^4) - a*b^3*(45*c - 375*d*x^2 + 280*e*x^4 + 72*f*x^6) + b^4*x^2*(-75*c +
 8*(15*d*x^2 + 5*e*x^4 + 3*f*x^6))))/(120*b^5*(a + b*x^2)^2) + ((3*b^3*c -
15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]
*b^(11/2))
```

fricas [A] time = 0.50, size = 614, normalized size = 2.97

$$\left[\frac{48\,ab^5fx^9+16\left(5\,ab^5e-9\,a^2b^4f\right)x^7+16\left(15\,ab^5d-35\,a^2b^4e+63\,a^3b^3f\right)x^5-50\left(3\,ab^5c-15\,a^2b^4d+35\,a^3b^3e-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*a*b^5*f*x^9 + 16*(5*a*b^5*e - 9*a^2*b^4*f)*x^7 + 16*(15*a*b^5*d
- 35*a^2*b^4*e + 63*a^3*b^3*f)*x^5 - 50*(3*a*b^5*c - 15*a^2*b^4*d + 35*a^3*
b^3*e - 63*a^4*b^2*f)*x^3 + 15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e - 6
3*a^5*f + (3*b^5*c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*(3*a
*b^4*c - 15*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*sqrt(-a*b)*log((b*x
^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 30*(3*a^2*b^4*c - 15*a^3*b^3*d + 35
*a^4*b^2*e - 63*a^5*b*f)*x)/(a*b^8*x^4 + 2*a^2*b^7*x^2 + a^3*b^6), 1/120*(2
4*a*b^5*f*x^9 + 8*(5*a*b^5*e - 9*a^2*b^4*f)*x^7 + 8*(15*a*b^5*d - 35*a^2*b^
4*e + 63*a^3*b^3*f)*x^5 - 25*(3*a*b^5*c - 15*a^2*b^4*d + 35*a^3*b^3*e - 63*
a^4*b^2*f)*x^3 + 15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e - 63*a^5*f + (
3*b^5*c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c - 15
*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
) - 15*(3*a^2*b^4*c - 15*a^3*b^3*d + 35*a^4*b^2*e - 63*a^5*b*f)*x)/(a*b^8*x
^4 + 2*a^2*b^7*x^2 + a^3*b^6)]
```

giac [A] time = 0.51, size = 200, normalized size = 0.97

$$\frac{\left(3\,b^3c-15\,ab^2d-63\,a^3f+35\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^5}-\frac{5\,b^4cx^3-9\,ab^3dx^3-17\,a^3bfx^3+13\,a^2b^2x^3e+3\,ab^3cx-7\,a^2}{8\left(bx^2+a\right)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*c - 15*a*b^2*d - 63*a^3*f + 35*a^2*b*e)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*b^5) - 1/8*(5*b^4*c*x^3 - 9*a*b^3*d*x^3 - 17*a^3*b*f*x^3 + 13*a^2*
b^2*x^3*e + 3*a*b^3*c*x - 7*a^2*b^2*d*x - 15*a^4*f*x + 11*a^3*b*x*e)/((b*x^
2 + a)^2*b^5) + 1/15*(3*b^12*f*x^5 - 15*a*b^11*f*x^3 + 5*b^12*x^3*e + 15*b^
12*d*x + 90*a^2*b^10*f*x - 45*a*b^11*x*e)/b^15
```

maple [A] time = 0.01, size = 294, normalized size = 1.42

$$\frac{17a^3f\,x^3}{8\left(b\,x^2+a\right)^2b^4}-\frac{13a^2e\,x^3}{8\left(b\,x^2+a\right)^2b^3}+\frac{9ad\,x^3}{8\left(b\,x^2+a\right)^2b^2}-\frac{5c\,x^3}{8\left(b\,x^2+a\right)^2b}+\frac{f\,x^5}{5b^3}+\frac{15a^4f\,x}{8\left(b\,x^2+a\right)^2b^5}-\frac{11a^3ex}{8\left(b\,x^2+a\right)^2b^4}+\frac{}{8\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/5*f*x^5/b^3-1/b^4*x^3*a*f+1/3/b^3*x^3*e+6/b^5*a^2*f*x-3/b^4*a*e*x+1/b^3*d
*x+17/8/b^4/(b*x^2+a)^2*x^3*a^3*f-13/8/b^3/(b*x^2+a)^2*x^3*a^2*e+9/8/b^2/(b
*x^2+a)^2*x^3*a*d-5/8/b/(b*x^2+a)^2*x^3*c+15/8/b^5/(b*x^2+a)^2*a^4*f*x-11/8
/b^4/(b*x^2+a)^2*a^3*e*x+7/8/b^3/(b*x^2+a)^2*a^2*d*x-3/8/b^2/(b*x^2+a)^2*a*
c*x-63/8/b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^3*f+35/8/b^4/(a*b)^(1/
2)*arctan(1/(a*b)^(1/2)*b*x)*a^2*e-15/8/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
)*b*x)*a*d+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 3.04, size = 193, normalized size = 0.93

$$-\frac{\left(5\,b^{4}c-9\,ab^{3}d+13\,a^{2}b^{2}e-17\,a^{3}bf\right)x^{3}+\left(3\,ab^{3}c-7\,a^{2}b^{2}d+11\,a^{3}be-15\,a^{4}f\right)x}{8\left(b^{7}x^{4}+2\,ab^{6}x^{2}+a^{2}b^{5}\right)}+\frac{\left(3\,b^{3}c-15\,ab^{2}d+35\,a^{2}be\right.}{8\,\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^4*c - 9*a*b^3*d + 13*a^2*b^2*e - 17*a^3*b*f)*x^3 + (3*a*b^3*c -
7*a^2*b^2*d + 11*a^3*b*e - 15*a^4*f)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) +
 1/8*(3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^5) + 1/15*(3*b^2*f*x^5 + 5*(b^2*e - 3*a*b*f)*x^3 + 15*(b^2*d -
3*a*b*e + 6*a^2*f)*x)/b^5
```

mupad [B] time = 0.95, size = 206, normalized size = 1.00

$$x^{3}\left(\frac{e}{3\,b^{3}}-\frac{a\,f}{b^{4}}\right)-x\left(\frac{3\,a^{2}\,f}{b^{5}}-\frac{d}{b^{3}}+\frac{3\,a\,\left(\frac{e}{b^{3}}-\frac{3\,a\,f}{b^{4}}\right)}{b}\right)-\frac{x^{3}\left(-\frac{17\,f\,a^{3}\,b}{8}+\frac{13\,e\,a^{2}\,b^{2}}{8}-\frac{9\,d\,a\,b^{3}}{8}+\frac{5\,c\,b^{4}}{8}\right)-x\left(\frac{15\,f\,a^{4}}{8}-\frac{11\,e\,a}{8}\right.}{a^{2}\,b^{5}+2\,a\,b^{6}\,x^{2}+b^{7}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x^3*(e/(3*b^3) - (a*f)/b^4) - x*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a
*f)/b^4))/b) - (x^3*((5*b^4*c)/8 + (13*a^2*b^2*e)/8 - (9*a*b^3*d)/8 - (17*a
^3*b*f)/8) - x*((15*a^4*f)/8 + (7*a^2*b^2*d)/8 - (3*a*b^3*c)/8 - (11*a^3*b*
e)/8))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) + (f*x^5)/(5*b^3) + (atan((b^(1/2)
*x)/a^(1/2))*(3*b^3*c - 63*a^3*f - 15*a*b^2*d + 35*a^2*b*e))/(8*a^(1/2)*b^(
11/2))
```

sympy [A] time = 17.84, size = 280, normalized size = 1.35

$$x^{3}\left(-\frac{af}{b^{4}}+\frac{e}{3b^{3}}\right)+x\left(\frac{6a^{2}f}{b^{5}}-\frac{3ae}{b^{4}}+\frac{d}{b^{3}}\right)+\frac{\sqrt{-\frac{1}{ab^{11}}}\left(63a^{3}f-35a^{2}be+15ab^{2}d-3b^{3}c\right)\log\left(-ab^{5}\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**3*(-a*f/b**4 + e/(3*b**3)) + x*(6*a**2*f/b**5 - 3*a*e/b**4 + d/b**3) + s
qrt(-1/(a*b**11))*(63*a**3*f - 35*a**2*b*e + 15*a*b**2*d - 3*b**3*c)*log(-a
*b**5*sqrt(-1/(a*b**11)) + x)/16 - sqrt(-1/(a*b**11))*(63*a**3*f - 35*a**2*
b*e + 15*a*b**2*d - 3*b**3*c)*log(a*b**5*sqrt(-1/(a*b**11)) + x)/16 + (x**3
*(17*a**3*b*f - 13*a**2*b**2*e + 9*a*b**3*d - 5*b**4*c) + x*(15*a**4*f - 11
*a**3*b*e + 7*a**2*b**2*d - 3*a*b**3*c))/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*
b**7*x**4) + f*x**5/(5*b**3)
```

3.136 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=167

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(11a^3f-7a^2be+3ab^2d+b^3c\right)}{8ab^4\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(35a^3f-15a^2be+3ab^2d+b^3c\right)}{8a^{3/2}b^{9/2}}+\frac{x(be-3}{b^4}$$

```
[Out] (-3*a*f+b*e)*x/b^4+1/3*f*x^3/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^3/a/(b
*x^2+a)^2-1/8*(11*a^3*f-7*a^2*b*e+3*a*b^2*d+b^3*c)*x/a/b^4/(b*x^2+a)+1/8*(3
5*a^3*f-15*a^2*b*e+3*a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(9/
2)
```

Rubi [A] time = 0.26, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1804, 1585, 1257, 1153, 205}

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(-7a^2be+11a^3f+3ab^2d+b^3c\right)}{8ab^4\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-15a^2be+35a^3f+3ab^2d+b^3c\right)}{8a^{3/2}b^{9/2}}+\frac{x(be}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^3)/(3*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x^3)/(4*a*(a + b*x^2)^2) - ((b^3*c + 3*a*b^2*d - 7*a^2*b*e + 11*a^3*f
)*x)/(8*a*b^4*(a + b*x^2)) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
```

```
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x\left(-\left(bc+3ad-\frac{3a^2e}{b}+\frac{3a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^2\left(-bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\int \frac{b^3c+3ab^2d-7a^2be}{}}{}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\int \left(8a(be-3af)\right.}{}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^3}{3b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right.}{8ab^4\left(a+bx^2\right)}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^3}{3b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right.}{8ab^4\left(a+bx^2\right)}$$

Mathematica [A] time = 0.13, size = 156, normalized size = 0.93

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(35a^3f-15a^2be+3ab^2d+b^3c\right)}{8a^{3/2}b^{9/2}} + \frac{x\left(-105a^4f+5a^3b\left(9e-35fx^2\right)+a^2b^2\left(-9d+75ex^2-56fx^4\right)\right.}{24ab^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(-105*a^4*f + 3*b^4*c*x^2 + 5*a^3*b*(9*e - 35*f*x^2) + a^2*b^2*(-9*d + 7
5*e*x^2 - 56*f*x^4) + a*b^3*(-3*c - 15*d*x^2 + 24*e*x^4 + 8*f*x^6)))/(24*a*
b^4*(a + b*x^2)^2) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

fricas [A] time = 0.74, size = 555, normalized size = 3.32

$$\left[\frac{16\,a^2b^4fx^7+16\left(3\,a^2b^4e-7\,a^3b^3f\right)x^5+2\left(3\,ab^5c-15\,a^2b^4d+75\,a^3b^3e-175\,a^4b^2f\right)x^3-3\left(a^2b^3c+3\,a^3b^2d-15\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^2*b^4*f*x^7 + 16*(3*a^2*b^4*e - 7*a^3*b^3*f)*x^5 + 2*(3*a*b^5*c
 - 15*a^2*b^4*d + 75*a^3*b^3*e - 175*a^4*b^2*f)*x^3 - 3*(a^2*b^3*c + 3*a^3*
b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5*c + 3*a*b^4*d - 15*a^2*b^3*e + 35*a^3*
b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*f)*x^2)*sqr
t(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(a^2*b^4*c + 3*a^
3*b^3*d - 15*a^4*b^2*e + 35*a^5*b*f)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*
b^5), 1/24*(8*a^2*b^4*f*x^7 + 8*(3*a^2*b^4*e - 7*a^3*b^3*f)*x^5 + (3*a*b^5*
c - 15*a^2*b^4*d + 75*a^3*b^3*e - 175*a^4*b^2*f)*x^3 + 3*(a^2*b^3*c + 3*a^3
*b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5*c + 3*a*b^4*d - 15*a^2*b^3*e + 35*a^3
*b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*f)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) - 3*(a^2*b^4*c + 3*a^3*b^3*d - 15*a^4*b^2*e +
 35*a^5*b*f)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5)]
```

giac [A] time = 0.45, size = 173, normalized size = 1.04

$$\frac{\left(b^3c+3\,ab^2d+35\,a^3f-15\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^4}+\frac{b^4cx^3-5\,ab^3dx^3-13\,a^3bfx^3+9\,a^2b^2x^3e-ab^3cx-3\,a^2b^2dx-}{8\left(bx^2+a\right)^2ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b^3*c + 3*a*b^2*d + 35*a^3*f - 15*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a*b^4) + 1/8*(b^4*c*x^3 - 5*a*b^3*d*x^3 - 13*a^3*b*f*x^3 + 9*a^2*b^2*
x^3*e - a*b^3*c*x - 3*a^2*b^2*d*x - 11*a^4*f*x + 7*a^3*b*x*e)/((b*x^2 + a)^
2*a*b^4) + 1/3*(b^6*f*x^3 - 9*a*b^5*f*x + 3*b^6*x*e)/b^9
```

maple [A] time = 0.01, size = 259, normalized size = 1.55

$$-\frac{13a^2f\,x^3}{8\left(b\,x^2+a\right)^2b^3}+\frac{9ae\,x^3}{8\left(b\,x^2+a\right)^2b^2}+\frac{c\,x^3}{8\left(b\,x^2+a\right)^2a}-\frac{5d\,x^3}{8\left(b\,x^2+a\right)^2b}-\frac{11a^3fx}{8\left(b\,x^2+a\right)^2b^4}+\frac{7a^2ex}{8\left(b\,x^2+a\right)^2b^3}-\frac{3adx}{8\left(b\,x^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/3*f*x^3/b^3-3/b^4*a*f*x+1/b^3*e*x-13/8/b^3/(b*x^2+a)^2*x^3*a^2*f+9/8/b^2/
(b*x^2+a)^2*x^3*a*e-5/8/b/(b*x^2+a)^2*x^3*d+1/8/(b*x^2+a)^2/a*x^3*c-11/8/b^
4/(b*x^2+a)^2*a^3*f*x+7/8/b^3/(b*x^2+a)^2*a^2*e*x-3/8/b^2/(b*x^2+a)^2*a*d*x
-1/8/b/(b*x^2+a)^2*c*x+35/8/b^4*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f
-15/8/b^3*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+3/8/b^2/(a*b)^(1/2)*arc
tan(1/(a*b)^(1/2)*b*x)*d+1/8/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 3.02, size = 169, normalized size = 1.01

$$\frac{\left(b^4c-5\,ab^3d+9\,a^2b^2e-13\,a^3bf\right)x^3-\left(ab^3c+3\,a^2b^2d-7\,a^3be+11\,a^4f\right)x}{8\left(ab^6x^4+2\,a^2b^5x^2+a^3b^4\right)}+\frac{bfx^3+3\left(be-3\,af\right)x}{3\,b^4}+\frac{\left(b^3c+3\,ab^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^4*c - 5*a*b^3*d + 9*a^2*b^2*e - 13*a^3*b*f)*x^3 - (a*b^3*c + 3*a^2*
b^2*d - 7*a^3*b*e + 11*a^4*f)*x)/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4) + 1/
3*(b*f*x^3 + 3*(b*e - 3*a*f)*x)/b^4 + 1/8*(b^3*c + 3*a*b^2*d - 15*a^2*b*e +
 35*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

mupad [B] time = 1.02, size = 163, normalized size = 0.98

$$x\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)-\frac{x\,\left(\frac{11\,f\,a^3}{8}-\frac{7\,e\,a^2\,b}{8}+\frac{3\,d\,a\,b^2}{8}+\frac{c\,b^3}{8}\right)-\frac{x^3\,\left(-13\,f\,a^3\,b+9\,e\,a^2\,b^2-5\,d\,a\,b^3+c\,b^4\right)}{8\,a}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+\frac{f\,x^3}{3\,b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(35\,f\,a^3\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x*(e/b^3 - (3*a*f)/b^4) - (x*((b^3*c)/8 + (11*a^3*f)/8 + (3*a*b^2*d)/8 - (7
*a^2*b*e)/8) - (x^3*(b^4*c + 9*a^2*b^2*e - 5*a*b^3*d - 13*a^3*b*f))/(8*a))/
(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + (f*x^3)/(3*b^3) + (atan((b^(1/2)*x)/a^(
1/2))*(b^3*c + 35*a^3*f + 3*a*b^2*d - 15*a^2*b*e))/(8*a^(3/2)*b^(9/2))
```

sympy [A] time = 13.07, size = 260, normalized size = 1.56

$$x\left(-\frac{3af}{b^4}+\frac{e}{b^3}\right)-\frac{\sqrt{-\frac{1}{a^3b^9}}\left(35a^3f-15a^2be+3ab^2d+b^3c\right)\log\left(-a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^9}}\left(35a^3f-15a^2be\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x*(-3*a*f/b**4 + e/b**3) - sqrt(-1/(a**3*b**9))*(35*a**3*f - 15*a**2*b*e +
3*a*b**2*d + b**3*c)*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/16 + sqrt(-1/
(a**3*b**9))*(35*a**3*f - 15*a**2*b*e + 3*a*b**2*d + b**3*c)*log(a**2*b**4*
sqrt(-1/(a**3*b**9)) + x)/16 + (x**3*(-13*a**3*b*f + 9*a**2*b**2*e - 5*a*b*
*3*d + b**4*c) + x*(-11*a**4*f + 7*a**3*b*e - 3*a**2*b**2*d - a*b**3*c))/(8
*a**3*b**4 + 16*a**2*b**5*x**2 + 8*a*b**6*x**4) + f*x**3/(3*b**3)
```

3.137 $\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=147

$$\frac{x\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{x\left(9a^3f-5a^2be+ab^2d+3b^3c\right)}{8a^2b^3\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+3a^2be+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}}+\frac{fx}{b^3}$$

```
[Out] f*x/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x/a/(b*x^2+a)^2+1/8*(9*a^3*f-5*a^
2*b*e+a*b^2*d+3*b^3*c)*x/a^2/b^3/(b*x^2+a)+1/8*(-15*a^3*f+3*a^2*b*e+a*b^2*d
+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(7/2)
```

Rubi [A] time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1814, 1157, 388, 205}

$$\frac{x\left(-5a^2be+9a^3f+ab^2d+3b^3c\right)}{8a^2b^3\left(a+bx^2\right)}+\frac{x\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^2be-15a^3f+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}}+\frac{fx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (f*x)/b^3 + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x)/(4*a*(a + b*x^2)^2) +
 ((3*b^3*c + a*b^2*d - 5*a^2*b*e + 9*a^3*f)*x)/(8*a^2*b^3*(a + b*x^2)) + ((
3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^3}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{-\frac{3b^3c+ab^2d-a^2be+a^3f}{b^3}-\frac{4a(be-af)x^2}{b^2}-\frac{4afx^4}{b}}{\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)} + \frac{\int \frac{\frac{3b^3c+ab^2d+3a^2be-7a^3f}{b^3}+\frac{8a^2}{b^{\cdots}}\cdots}{a+bx^2}}{8a^2}$$

$$= \frac{fx}{b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)} + \frac{\left(3b^3c+ab^2d+3\cdots\right.}{\cdots}$$

$$= \frac{fx}{b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)} + \frac{\left(3b^3c+ab^2d+3\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.12, size = 141, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-15a^3f+3a^2be+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}} + \frac{x\left(15a^4f+a^3b\left(25fx^2-3e\right)-a^2b^2\left(d+5ex^2-8fx^4\right)+ab^3\left(5\cdots\right.\right.}{8a^2b^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (x*(15*a^4*f + 3*b^4*c*x^2 + a*b^3*(5*c + d*x^2) + a^3*b*(-3*e + 25*f*x^2)
- a^2*b^2*(d + 5*e*x^2 - 8*f*x^4)))/(8*a^2*b^3*(a + b*x^2)^2) + ((3*b^3*c +
 a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^
(7/2))
```

fricas [A] time = 0.81, size = 504, normalized size = 3.43

$$\left[\frac{16\,a^3b^3fx^5+2\left(3\,ab^5c+a^2b^4d-5\,a^3b^3e+25\,a^4b^2f\right)x^3+\left(3\,a^2b^3c+a^3b^2d+3\,a^4be-15\,a^5f+\left(3\,b^5c+ab^4d\cdots\right.\right.}{1\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^3*b^3*f*x^5 + 2*(3*a*b^5*c + a^2*b^4*d - 5*a^3*b^3*e + 25*a^4*b
^2*f)*x^3 + (3*a^2*b^3*c + a^3*b^2*d + 3*a^4*b*e - 15*a^5*f + (3*b^5*c + a*
b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c + a^2*b^3*d + 3*a^3*
b^2*e - 15*a^4*b*f)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)) + 2*(5*a^2*b^4*c - a^3*b^3*d - 3*a^4*b^2*e + 15*a^5*b*f)*x)/(a^3*b^6
*x^4 + 2*a^4*b^5*x^2 + a^5*b^4), 1/8*(8*a^3*b^3*f*x^5 + (3*a*b^5*c + a^2*b^
4*d - 5*a^3*b^3*e + 25*a^4*b^2*f)*x^3 + (3*a^2*b^3*c + a^3*b^2*d + 3*a^4*b*
e - 15*a^5*f + (3*b^5*c + a*b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*x^4 + 2*(3*
a*b^4*c + a^2*b^3*d + 3*a^3*b^2*e - 15*a^4*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(
a*b)*x/a) + (5*a^2*b^4*c - a^3*b^3*d - 3*a^4*b^2*e + 15*a^5*b*f)*x)/(a^3*b^
6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4)]
```

giac [A] time = 0.46, size = 149, normalized size = 1.01

$$\frac{fx}{b^3}+\frac{\left(3\,b^3c+ab^2d-15\,a^3f+3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^3}+\frac{3\,b^4cx^3+ab^3dx^3+9\,a^3bfx^3-5\,a^2b^2x^3e+5\,ab^3cx-a^2b^2dx}{8\left(bx^2+a\right)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] f*x/b^3 + 1/8*(3*b^3*c + a*b^2*d - 15*a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c*x^3 + a*b^3*d*x^3 + 9*a^3*b*f*x^3 -
5*a^2*b^2*x^3*e + 5*a*b^3*c*x - a^2*b^2*d*x + 7*a^4*f*x - 3*a^3*b*x*e)/((b*
x^2 + a)^2*a^2*b^3)
```

maple [A] time = 0.01, size = 234, normalized size = 1.59

$$\frac{9afx^3}{8\left(bx^2+a\right)^2b^2}+\frac{dx^3}{8\left(bx^2+a\right)^2a}+\frac{3bcx^3}{8\left(bx^2+a\right)^2a^2}-\frac{5ex^3}{8\left(bx^2+a\right)^2b}+\frac{7a^2fx}{8\left(bx^2+a\right)^2b^3}-\frac{3aex}{8\left(bx^2+a\right)^2b^2}+\frac{5cx}{8\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] f*x/b^3+9/8/b^2/(b*x^2+a)^2*x^3*a*f-5/8/b/(b*x^2+a)^2*x^3*e+1/8/(b*x^2+a)^2
/a*x^3*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c+7/8/b^3/(b*x^2+a)^2*a^2*f*x-3/8/b^2/(b
*x^2+a)^2*a*e*x-1/8/b/(b*x^2+a)^2*d*x+5/8/(b*x^2+a)^2/a*x*c-15/8/b^3*a/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
)*b*x)*e+1/8/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+3/8/a^2/(a*b)^(1/2
)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 3.00, size = 154, normalized size = 1.05

$$\frac{\left(3\,b^4c+ab^3d-5\,a^2b^2e+9\,a^3bf\right)x^3+\left(5\,ab^3c-a^2b^2d-3\,a^3be+7\,a^4f\right)x}{8\left(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3\right)}+\frac{fx}{b^3}+\frac{\left(3\,b^3c+ab^2d+3\,a^2be-15\,a^3f\right)a}{8\,\sqrt{ab}\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^4*c + a*b^3*d - 5*a^2*b^2*e + 9*a^3*b*f)*x^3 + (5*a*b^3*c - a^2*b
^2*d - 3*a^3*b*e + 7*a^4*f)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) + f*
x/b^3 + 1/8*(3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a^2*b^3)
```

mupad [B] time = 1.05, size = 148, normalized size = 1.01

$$\frac{\frac{x\left(7\,f\,a^3-3\,e\,a^2\,b-d\,a\,b^2+5\,c\,b^3\right)}{8\,a}+\frac{x^3\left(9\,f\,a^3\,b-5\,e\,a^2\,b^2+d\,a\,b^3+3\,c\,b^4\right)}{8\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{f\,x}{b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-15\,f\,a^3+3\,e\,a^2\,b+d\,a\,b^2+3\,c\,b^3\right)}{8\,a^{5/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x)
```

```
[Out] ((x*(5*b^3*c + 7*a^3*f - a*b^2*d - 3*a^2*b*e))/(8*a) + (x^3*(3*b^4*c - 5*a^
2*b^2*e + a*b^3*d + 9*a^3*b*f))/(8*a^2))/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2)
+ (f*x)/b^3 + (atan((b^(1/2)*x)/a^(1/2))*(3*b^3*c - 15*a^3*f + a*b^2*d + 3*
a^2*b*e))/(8*a^(5/2)*b^(7/2))
```

<u>**sympy**</u> **[A]** time = 10.09, size = 243, normalized size = 1.65

$$\frac{\sqrt{-\frac{1}{a^5b^7}}\left(15a^3f-3a^2be-ab^2d-3b^3c\right)\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{a^5b^7}}\left(15a^3f-3a^2be-ab^2d-3b^3c\right)\log\left(\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*a**2*b*e - a*b**2*d - 3*b**3*c)*log(-a*
*3*b**3*sqrt(-1/(a**5*b**7)) + x)/16 - sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*
a**2*b*e - a*b**2*d - 3*b**3*c)*log(a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/16
+ (x**3*(9*a**3*b*f - 5*a**2*b**2*e + a*b**3*d + 3*b**4*c) + x*(7*a**4*f -
3*a**3*b*e - a**2*b**2*d + 5*a*b**3*c))/(8*a**4*b**3 + 16*a**3*b**4*x**2 +
8*a**2*b**5*x**4) + f*x/b**3
```

3.138 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{c}{a^3x}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(5a^3f-a^2be-3ab^2d+7b^3c\right)}{8a^3b^2\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-3a^3f-a^2be-3ab^2d+15b^3c\right)}{8a^{7/2}b^{5/2}}$$

```
[Out] -c/a^3/x-1/4*(b*c/a-d+a*e/b-a^2*f/b^2)*x/a/(b*x^2+a)^2-1/8*(5*a^3*f-a^2*b*e
-3*a*b^2*d+7*b^3*c)*x/a^3/b^2/(b*x^2+a)-1/8*(-3*a^3*f-a^2*b*e-3*a*b^2*d+15*
b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(5/2)
```

Rubi [A] time = 0.18, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1805, 1259, 453, 205}

$$-\frac{x\left(-a^2be+5a^3f-3ab^2d+7b^3c\right)}{8a^3b^2\left(a+bx^2\right)}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a\left(a+bx^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-a^2be-3a^3f-3ab^2d+15b^3c\right)}{8a^{7/2}b^{5/2}}-\frac{c}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(4*a*(a + b*x^2)^2
) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a + b*x^2)) -
 ((15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(7/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int\frac{-4c+\left(\frac{3bc}{a}-3d-\frac{ae}{b}+\frac{a^2f}{b^2}\right)x^2-\frac{4afx^4}{b}}{x^2\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)}+\frac{\int\frac{8ab^2c-\left(7b^3c-3ab^2d-a^2b\ldots\right.}{x^2\left(a+bx^2\right)}}{8a^3b^2}$$

$$= -\frac{c}{a^3x}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)}-\frac{\left(15b^3c-3ab^2\ldots\right.}{\ldots}$$

$$= -\frac{c}{a^3x}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)}-\frac{\left(15b^3c-3ab^2\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.13, size = 155, normalized size = 1.01

$$-\frac{c}{a^3x}-\frac{x\left(5a^3f-a^2be-3ab^2d+7b^3c\right)}{8a^3b^2\left(a+bx^2\right)}+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{4a^2b^2\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f+a^2be+3ab^2d-15\ldots\right)}{8a^{7/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^2*b^2*(a + b
*x^2)^2) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a + b*
x^2)) + ((-15*b^3*c + 3*a*b^2*d + a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(7/2)*b^(5/2))
```

fricas [A] time = 0.63, size = 517, normalized size = 3.38

$$\left[-\frac{16\,a^3b^3c+2\left(15\,ab^5c-3\,a^2b^4d-a^3b^3e+5\,a^4b^2f\right)x^4+2\left(25\,a^2b^4c-5\,a^3b^3d+a^4b^2e+3\,a^5bf\right)x^2-\left(\left(15\,b^5c\ldots\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a^3*b^3*c + 2*(15*a*b^5*c - 3*a^2*b^4*d - a^3*b^3*e + 5*a^4*b^2*
f)*x^4 + 2*(25*a^2*b^4*c - 5*a^3*b^3*d + a^4*b^2*e + 3*a^5*b*f)*x^2 - ((15*
b^5*c - 3*a*b^4*d - a^2*b^3*e - 3*a^3*b^2*f)*x^5 + 2*(15*a*b^4*c - 3*a^2*b^
3*d - a^3*b^2*e - 3*a^4*b*f)*x^3 + (15*a^2*b^3*c - 3*a^3*b^2*d - a^4*b*e -
3*a^5*f)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*
b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x), -1/8*(8*a^3*b^3*c + (15*a*b^5*c - 3*a
^2*b^4*d - a^3*b^3*e + 5*a^4*b^2*f)*x^4 + (25*a^2*b^4*c - 5*a^3*b^3*d + a^4
```

```
*b^2*e + 3*a^5*b*f)*x^2 + ((15*b^5*c - 3*a*b^4*d - a^2*b^3*e - 3*a^3*b^2*f)
*x^5 + 2*(15*a*b^4*c - 3*a^2*b^3*d - a^3*b^2*e - 3*a^4*b*f)*x^3 + (15*a^2*b
^3*c - 3*a^3*b^2*d - a^4*b*e - 3*a^5*f)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))
/(a^4*b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x)]
```

giac [A] time = 0.45, size = 153, normalized size = 1.00

$$-\frac{c}{a^3x}-\frac{\left(15\,b^3c-3\,ab^2d-3\,a^3f-a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3b^2}-\frac{7\,b^4cx^3-3\,ab^3dx^3+5\,a^3bfx^3-a^2b^2x^3e+9\,ab^3cx-5\,a^2b^2}{8\left(bx^2+a\right)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -c/(a^3*x) - 1/8*(15*b^3*c - 3*a*b^2*d - 3*a^3*f - a^2*b*e)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^3*b^2) - 1/8*(7*b^4*c*x^3 - 3*a*b^3*d*x^3 + 5*a^3*b*f*x
^3 - a^2*b^2*x^3*e + 9*a*b^3*c*x - 5*a^2*b^2*d*x + 3*a^4*f*x + a^3*b*x*e)/(
(b*x^2 + a)^2*a^3*b^2)
```

maple [A] time = 0.01, size = 237, normalized size = 1.55

$$\frac{e\,x^3}{8\left(b\,x^2+a\right)^2a}+\frac{3bd\,x^3}{8\left(b\,x^2+a\right)^2a^2}-\frac{7b^2c\,x^3}{8\left(b\,x^2+a\right)^2a^3}-\frac{5f\,x^3}{8\left(b\,x^2+a\right)^2b}-\frac{3af\,x}{8\left(b\,x^2+a\right)^2b^2}+\frac{5dx}{8\left(b\,x^2+a\right)^2a}-\frac{9bcx}{8\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -5/8/(b*x^2+a)^2/b*x^3*f+1/8/a/(b*x^2+a)^2*x^3*e+3/8/a^2/(b*x^2+a)^2*b*x^3*
d-7/8/a^3/(b*x^2+a)^2*b^2*x^3*c-3/8*a/(b*x^2+a)^2/b^2*x*f-1/8/(b*x^2+a)^2/b
*x*e+5/8/a/(b*x^2+a)^2*x*d-9/8/a^2/(b*x^2+a)^2*b*x*c+3/8/b^2/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x)*f+1/8/a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+3
/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-15/8/a^3*b/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)*c-c/a^3/x
```

maxima [A] time = 2.99, size = 161, normalized size = 1.05

$$-\frac{8\,a^2b^2c+\left(15\,b^4c-3\,ab^3d-a^2b^2e+5\,a^3bf\right)x^4+\left(25\,ab^3c-5\,a^2b^2d+a^3be+3\,a^4f\right)x^2}{8\left(a^3b^4x^5+2\,a^4b^3x^3+a^5b^2x\right)}-\frac{\left(15\,b^3c-3\,ab^2d-a^2be\right.}{8\,\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^2*b^2*c + (15*b^4*c - 3*a*b^3*d - a^2*b^2*e + 5*a^3*b*f)*x^4 + (2
5*a*b^3*c - 5*a^2*b^2*d + a^3*b*e + 3*a^4*f)*x^2)/(a^3*b^4*x^5 + 2*a^4*b^3*
x^3 + a^5*b^2*x) - 1/8*(15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

mupad [B] time = 1.09, size = 149, normalized size = 0.97

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3\,f\,a^3+e\,a^2\,b+3\,d\,a\,b^2-15\,c\,b^3\right)}{8\,a^{7/2}\,b^{5/2}}-\frac{\frac{c}{a}+\frac{x^4\left(5\,f\,a^3-e\,a^2\,b-3\,d\,a\,b^2+15\,c\,b^3\right)}{8\,a^3\,b}+\frac{x^2\left(3\,f\,a^3+e\,a^2\,b-5\,d\,a\,b^2+25\,c\,b^3\right)}{8\,a^2\,b^2}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(3*a^3*f - 15*b^3*c + 3*a*b^2*d + a^2*b*e))/(8*a
^(7/2)*b^(5/2)) - (c/a + (x^4*(15*b^3*c + 5*a^3*f - 3*a*b^2*d - a^2*b*e))/(
8*a^3*b) + (x^2*(25*b^3*c + 3*a^3*f - 5*a*b^2*d + a^2*b*e))/(8*a^2*b^2))/(a
^2*x + b^2*x^5 + 2*a*b*x^3)
```

sympy [A] time = 26.56, size = 250, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^7b^5}}\left(3a^3f+a^2be+3ab^2d-15b^3c\right)\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^7b^5}}\left(3a^3f+a^2be+3ab^2d-15b^3c\right)\log}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**7*b**5))*(3*a**3*f + a**2*b*e + 3*a*b**2*d - 15*b**3*c)*log(-a
**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16 + sqrt(-1/(a**7*b**5))*(3*a**3*f + a*
*2*b*e + 3*a*b**2*d - 15*b**3*c)*log(a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16
 + (-8*a**2*b**2*c + x**4*(-5*a**3*b*f + a**2*b**2*e + 3*a*b**3*d - 15*b**4
*c) + x**2*(-3*a**4*f - a**3*b*e + 5*a**2*b**2*d - 25*a*b**3*c))/(8*a**5*b*
*2*x + 16*a**4*b**3*x**3 + 8*a**3*b**4*x**5)
```

3.139 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=168

$$\frac{3bc-ad}{a^4x}-\frac{c}{3a^3x^3}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3f+3a^2be-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}}+\frac{x\left(a^3f+3a^2be-7ab^2d+\right.}{8a^4b\left(a+bx^2\right)}$$

```
[Out] -1/3*c/a^3/x^3+(-a*d+3*b*c)/a^4/x+1/4*(b^2*c/a^2-b*d/a+e-a*f/b)*x/a/(b*x^2+
a)^2+1/8*(a^3*f+3*a^2*b*e-7*a*b^2*d+11*b^3*c)*x/a^4/b/(b*x^2+a)+1/8*(a^3*f+
3*a^2*b*e-15*a*b^2*d+35*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(3/2)
```

Rubi [A] time = 0.24, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1805, 1259, 1261, 205}

$$\frac{x\left(3a^2be+a^3f-7ab^2d+11b^3c\right)}{8a^4b\left(a+bx^2\right)}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3a^2be+a^3f-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}}+\frac{3bc-ad}{a^4x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -c/(3*a^3*x^3) + (3*b*c - a*d)/(a^4*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a*f
)/b)*x)/(4*a*(a + b*x^2)^2) + ((11*b^3*c - 7*a*b^2*d + 3*a^2*b*e + a^3*f)*x
)/(8*a^4*b*(a + b*x^2)) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*f)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
```

```
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2+\left(-\frac{3b^2c}{a^2}+\frac{3bd}{a}-3e-\frac{af}{b}\right)x^4}{x^4\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)} - \frac{\int \frac{-8a^2b^2c+8ab^2(2bc-ad)x^2}{x^4}}{}$$

$$= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)} - \frac{\int \left(-\frac{8ab^2c}{x^4}+\frac{8b^2(3bc-ad)}{x^2}\right.}{}$$

$$= -\frac{c}{3a^3x^3} + \frac{3bc-ad}{a^4x} + \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)} +$$

$$= -\frac{c}{3a^3x^3} + \frac{3bc-ad}{a^4x} + \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)} +$$

Mathematica [A] time = 0.16, size = 169, normalized size = 1.01

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f+3a^2be-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}} + \frac{-3a^4fx^4+a^3b\left(3x^2\left(-8d+5ex^2+fx^4\right)-8c\right)+a^2b^2x^2\left(56c-\right.}{24a^4bx^3\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (-3*a^4*f*x^4 + 105*b^4*c*x^6 + 5*a*b^3*x^4*(35*c - 9*d*x^2) + a^2*b^2*x^2*
(56*c - 75*d*x^2 + 9*e*x^4) + a^3*b*(-8*c + 3*x^2*(-8*d + 5*e*x^2 + f*x^4))
)/(24*a^4*b*x^3*(a + b*x^2)^2) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*
f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

fricas [A] time = 0.80, size = 570, normalized size = 3.39

$$\left[-\frac{16\,a^4b^2c-6\left(35\,ab^5c-15\,a^2b^4d+3\,a^3b^3e+a^4b^2f\right)x^6-2\left(175\,a^2b^4c-75\,a^3b^3d+15\,a^4b^2e-3\,a^5bf\right)x^4-16}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a^4*b^2*c - 6*(35*a*b^5*c - 15*a^2*b^4*d + 3*a^3*b^3*e + a^4*b^2
*f)*x^6 - 2*(175*a^2*b^4*c - 75*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^4 -
 16*(7*a^3*b^3*c - 3*a^4*b^2*d)*x^2 + 3*((35*b^5*c - 15*a*b^4*d + 3*a^2*b^3
*e + a^3*b^2*f)*x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*a^3*b^2*e + a^4*b*f)
*x^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*f)*x^3)*sqrt(-a*b)*lo
```

```
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^4*x^7 + 2*a^6*b^3*x^5 +
 a^7*b^2*x^3), -1/24*(8*a^4*b^2*c - 3*(35*a*b^5*c - 15*a^2*b^4*d + 3*a^3*b^
3*e + a^4*b^2*f)*x^6 - (175*a^2*b^4*c - 75*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5
*b*f)*x^4 - 8*(7*a^3*b^3*c - 3*a^4*b^2*d)*x^2 - 3*((35*b^5*c - 15*a*b^4*d +
 3*a^2*b^3*e + a^3*b^2*f)*x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*a^3*b^2*e
+ a^4*b*f)*x^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*f)*x^3)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^4*x^7 + 2*a^6*b^3*x^5 + a^7*b^2*x^3)]
```

giac [A] time = 0.45, size = 170, normalized size = 1.01

$$\frac{\left(35\,b^3c-15\,ab^2d+a^3f+3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4b}+\frac{11\,b^4cx^3-7\,ab^3dx^3+a^3bfx^3+3\,a^2b^2x^3e+13\,ab^3cx-9\,a^2b^2dx}{8\left(bx^2+a\right)^2a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*b^3*c - 15*a*b^2*d + a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^4*b) + 1/8*(11*b^4*c*x^3 - 7*a*b^3*d*x^3 + a^3*b*f*x^3 + 3*a^2*b^2*
x^3*e + 13*a*b^3*c*x - 9*a^2*b^2*d*x - a^4*f*x + 5*a^3*b*x*e)/((b*x^2 + a)^
2*a^4*b) + 1/3*(9*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^4*x^3)
```

maple [A] time = 0.02, size = 264, normalized size = 1.57

$$\frac{f\,x^3}{8\left(b\,x^2+a\right)^2a}+\frac{3be\,x^3}{8\left(b\,x^2+a\right)^2a^2}-\frac{7b^2d\,x^3}{8\left(b\,x^2+a\right)^2a^3}+\frac{11b^3c\,x^3}{8\left(b\,x^2+a\right)^2a^4}+\frac{5ex}{8\left(b\,x^2+a\right)^2a}-\frac{9bdx}{8\left(b\,x^2+a\right)^2a^2}+\frac{13b^2cx}{8\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x)
```

```
[Out] 1/8/a/(b*x^2+a)^2*x^3*f+3/8/a^2/(b*x^2+a)^2*x^3*b*e-7/8/a^3/(b*x^2+a)^2*x^3
*b^2*d+11/8/a^4/(b*x^2+a)^2*x^3*b^3*c-1/8/(b*x^2+a)^2/b*x*f+5/8/a/(b*x^2+a)
^2*x*e-9/8/a^2/(b*x^2+a)^2*b*x*d+13/8/a^3/(b*x^2+a)^2*b^2*x*c+1/8/a/b/(a*b)
^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)
*b*x)*e-15/8/a^3*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+35/8/a^4*b^2/(a*
b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/3*c/a^3/x^3-1/a^3/x*d+3/a^4/x*b*c
```

maxima [A] time = 3.07, size = 181, normalized size = 1.08

$$\frac{3\left(35\,b^4c-15\,ab^3d+3\,a^2b^2e+a^3bf\right)x^6-8\,a^3bc+\left(175\,ab^3c-75\,a^2b^2d+15\,a^3be-3\,a^4f\right)x^4+8\left(7\,a^2b^2c-3\,a^3b\right.}{24\left(a^4b^3x^7+2\,a^5b^2x^5+a^6bx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(35*b^4*c - 15*a*b^3*d + 3*a^2*b^2*e + a^3*b*f)*x^6 - 8*a^3*b*c + (
175*a*b^3*c - 75*a^2*b^2*d + 15*a^3*b*e - 3*a^4*f)*x^4 + 8*(7*a^2*b^2*c - 3
*a^3*b*d)*x^2)/(a^4*b^3*x^7 + 2*a^5*b^2*x^5 + a^6*b*x^3) + 1/8*(35*b^3*c -
15*a*b^2*d + 3*a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b)
```

mupad [B] time = 1.03, size = 166, normalized size = 0.99

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(f\,a^3+3\,e\,a^2\,b-15\,d\,a\,b^2+35\,c\,b^3\right)}{8\,a^{9/2}\,b^{3/2}}-\frac{\frac{c}{3\,a}-\frac{x^6\,\left(f\,a^3+3\,e\,a^2\,b-15\,d\,a\,b^2+35\,c\,b^3\right)}{8\,a^4}+\frac{x^2\,\left(3\,a\,d-7\,b\,c\right)}{3\,a^2}-\frac{x^4\,\left(-3\,f\,a^3+15\,e\,a^2\right.}{2}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(35*b^3*c + a^3*f - 15*a*b^2*d + 3*a^2*b*e))/(8*
a^(9/2)*b^(3/2)) - (c/(3*a) - (x^6*(35*b^3*c + a^3*f - 15*a*b^2*d + 3*a^2*b
*e))/(8*a^4) + (x^2*(3*a*d - 7*b*c))/(3*a^2) - (x^4*(175*b^3*c - 3*a^3*f -
75*a*b^2*d + 15*a^2*b*e))/(24*a^3*b))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5)
```

sympy [A] time = 70.49, size = 270, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^9b^3}}\left(a^3f+3a^2be-15ab^2d+35b^3c\right)\log\left(-a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^9b^3}}\left(a^3f+3a^2be-15ab^2d+35b^3c\right)\log}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**9*b**3))*(a**3*f + 3*a**2*b*e - 15*a*b**2*d + 35*b**3*c)*log(-
a**5*b*sqrt(-1/(a**9*b**3)) + x)/16 + sqrt(-1/(a**9*b**3))*(a**3*f + 3*a**2
*b*e - 15*a*b**2*d + 35*b**3*c)*log(a**5*b*sqrt(-1/(a**9*b**3)) + x)/16 + (
-8*a**3*b*c + x**6*(3*a**3*b*f + 9*a**2*b**2*e - 45*a*b**3*d + 105*b**4*c)
+ x**4*(-3*a**4*f + 15*a**3*b*e - 75*a**2*b**2*d + 175*a*b**3*c) + x**2*(-2
4*a**3*b*d + 56*a**2*b**2*c))/(24*a**6*b*x**3 + 48*a**5*b**2*x**5 + 24*a**4
*b**3*x**7)
```

3.140 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=196

$$\frac{3bc-ad}{3a^4x^3}-\frac{c}{5a^3x^5}-\frac{a^2e-3abd+6b^2c}{a^5x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^3f+15a^2be-35ab^2d+63b^3c\right)}{8a^{11/2}\sqrt{b}}-\frac{x\left(-3a^3f+7a^2be-11a\cdots\right.}{8a^5\left(a+bx^2\cdots\right.}$$

```
[Out] -1/5*c/a^3/x^5+1/3*(-a*d+3*b*c)/a^4/x^3+(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x-1/4*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^2+a)^2-1/8*(-3*a^3*f+7*a^2*b*e-11
*a*b^2*d+15*b^3*c)*x/a^5/(b*x^2+a)-1/8*(-3*a^3*f+15*a^2*b*e-35*a*b^2*d+63*b
^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)/b^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1802, 205}

$$-\frac{x\left(7a^2be-3a^3f-11ab^2d+15b^3c\right)}{8a^5\left(a+bx^2\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(15a^2be-3a^3f-35ab^2d+6\cdots\right.}{8a^{11/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) - (6*b^2*c - 3*a*b*d + a^2*e)/(a
^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^4*(a + b*x^2)^2) - ((1
5*b^3*c - 11*a*b^2*d + 7*a^2*b*e - 3*a^3*f)*x)/(8*a^5*(a + b*x^2)) - ((63*b
^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(11/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{3\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}}{x^6\left(a+bx^2\right)^2}dx}{4a}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\left(15b^3c-11ab^2d+7a^2be-3a^3f\right)x}{8a^5\left(a+bx^2\right)} + \frac{\int \frac{8c-8\left(\frac{2bc}{a}-\cdots\right)}{\cdots}}{\cdots}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\left(15b^3c-11ab^2d+7a^2be-3a^3f\right)x}{8a^5\left(a+bx^2\right)} + \frac{\int\left(\frac{8c}{ax^6}+\cdots\right)}{\cdots}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{3a^4x^3} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\left(15b^3c\cdots\right.}{\cdots}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{3a^4x^3} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\left(15b^3c\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.12, size = 196, normalized size = 1.00

$$\frac{3bc-ad}{3a^4x^3} - \frac{c}{5a^3x^5} + \frac{a^2(-e)+3abd-6b^2c}{a^5x} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-15a^2be+35ab^2d-63b^3c\right)}{8a^{11/2}\sqrt{b}} + \frac{x\left(3a^3f-7a^2be+\cdots\right.}{8a^5\left(a+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -1/5*c/(a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) + (-6*b^2*c + 3*a*b*d - a^2*e)
/(a^5*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^4*(a + b*x^2)^2)
 + ((-15*b^3*c + 11*a*b^2*d - 7*a^2*b*e + 3*a^3*f)*x)/(8*a^5*(a + b*x^2)) +
 ((-63*b^3*c + 35*a*b^2*d - 15*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(8*a^(11/2)*Sqrt[b])
```

fricas [A] time = 0.57, size = 628, normalized size = 3.20

$$\left[-\frac{30\left(63ab^5c-35a^2b^4d+15a^3b^3e-3a^4b^2f\right)x^8+48a^5bc+50\left(63a^2b^4c-35a^3b^3d+15a^4b^2e-3a^5bf\right)x^6\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(30*(63*a*b^5*c - 35*a^2*b^4*d + 15*a^3*b^3*e - 3*a^4*b^2*f)*x^8 +
48*a^5*b*c + 50*(63*a^2*b^4*c - 35*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^
6 + 16*(63*a^3*b^3*c - 35*a^4*b^2*d + 15*a^5*b*e)*x^4 - 16*(9*a^4*b^2*c - 5
*a^5*b*d)*x^2 - 15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*b^2*f)*x^
9 + 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x^7 + (63*a^2*
b^3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*sqrt(-a*b)*log((b*x^2 - 2
*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*b*x^5),
 -1/120*(15*(63*a*b^5*c - 35*a^2*b^4*d + 15*a^3*b^3*e - 3*a^4*b^2*f)*x^8 +
24*a^5*b*c + 25*(63*a^2*b^4*c - 35*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^
6 + 8*(63*a^3*b^3*c - 35*a^4*b^2*d + 15*a^5*b*e)*x^4 - 8*(9*a^4*b^2*c - 5*a
```

```
^5*b*d)*x^2 + 15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*b^2*f)*x^9
+ 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x^7 + (63*a^2*b^
3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*b*x^5)]
```

giac [A] time = 0.44, size = 198, normalized size = 1.01

$$-\frac{\left(63\,b^3c-35\,ab^2d-3\,a^3f+15\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}-\frac{15\,b^4cx^3-11\,ab^3dx^3-3\,a^3bfx^3+7\,a^2b^2x^3e+17\,ab^3cx-13}{8\left(bx^2+a\right)^2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(63*b^3*c - 35*a*b^2*d - 3*a^3*f + 15*a^2*b*e)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^5) - 1/8*(15*b^4*c*x^3 - 11*a*b^3*d*x^3 - 3*a^3*b*f*x^3 + 7*a^2
*b^2*x^3*e + 17*a*b^3*c*x - 13*a^2*b^2*d*x - 5*a^4*f*x + 9*a^3*b*x*e)/((b*x
^2 + a)^2*a^5) - 1/15*(90*b^2*c*x^4 - 45*a*b*d*x^4 + 15*a^2*x^4*e - 15*a*b*
c*x^2 + 5*a^2*d*x^2 + 3*a^2*c)/(a^5*x^5)
```

maple [A] time = 0.02, size = 300, normalized size = 1.53

$$\frac{3bf\,x^3}{8\left(b\,x^2+a\right)^2a^2}-\frac{7b^2e\,x^3}{8\left(b\,x^2+a\right)^2a^3}+\frac{11b^3d\,x^3}{8\left(b\,x^2+a\right)^2a^4}-\frac{15b^4c\,x^3}{8\left(b\,x^2+a\right)^2a^5}+\frac{5fx}{8\left(b\,x^2+a\right)^2a}-\frac{9bex}{8\left(b\,x^2+a\right)^2a^2}+\frac{13b^2dx}{8\left(b\,x^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x)
```

```
[Out] 3/8/a^2/(b*x^2+a)^2*x^3*b*f-7/8/a^3/(b*x^2+a)^2*x^3*b^2*e+11/8/a^4/(b*x^2+a
)^2*x^3*b^3*d-15/8/a^5/(b*x^2+a)^2*x^3*b^4*c+5/8/a/(b*x^2+a)^2*f*x-9/8/a^2/
(b*x^2+a)^2*b*e*x+13/8/a^3/(b*x^2+a)^2*b^2*d*x-17/8/a^4/(b*x^2+a)^2*b^3*c*x
+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-15/8/a^3/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)*b*e+35/8/a^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^2
*d-63/8/a^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^3*c-1/5*c/a^3/x^5-1/3/a
^3/x^3*d+1/a^4/x^3*b*c-1/a^3/x*e+3/a^4/x*b*d-6/a^5/x*b^2*c
```

maxima [A] time = 2.98, size = 202, normalized size = 1.03

$$-\frac{15\left(63\,b^4c-35\,ab^3d+15\,a^2b^2e-3\,a^3bf\right)x^8+25\left(63\,ab^3c-35\,a^2b^2d+15\,a^3be-3\,a^4f\right)x^6+24\,a^4c+8\left(63\,a^2b^2\right.}{120\left(a^5b^2x^9+2\,a^6bx^7+a^7x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*(63*b^4*c - 35*a*b^3*d + 15*a^2*b^2*e - 3*a^3*b*f)*x^8 + 25*(63*
a*b^3*c - 35*a^2*b^2*d + 15*a^3*b*e - 3*a^4*f)*x^6 + 24*a^4*c + 8*(63*a^2*b
^2*c - 35*a^3*b*d + 15*a^4*e)*x^4 - 8*(9*a^3*b*c - 5*a^4*d)*x^2)/(a^5*b^2*x
^9 + 2*a^6*b*x^7 + a^7*x^5) - 1/8*(63*b^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a
^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 1.04, size = 192, normalized size = 0.98

$$-\frac{\frac{c}{5\,a}+\frac{5\,x^6\left(-3\,f\,a^3+15\,e\,a^2\,b-35\,d\,a\,b^2+63\,c\,b^3\right)}{24\,a^4}+\frac{x^2\,(5\,a\,d-9\,b\,c)}{15\,a^2}+\frac{x^4\left(15\,e\,a^2-35\,d\,a\,b+63\,c\,b^2\right)}{15\,a^3}+\frac{b\,x^8\left(-3\,f\,a^3+15\,e\,a^2\,b-35\,d\,a\,b^2+63\,c\,b^3\right)}{8\,a^5}\quad a}{a^2\,x^5+2\,a\,b\,x^7+b^2\,x^9}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x)
```

```
[Out] - (c/(5*a) + (5*x^6*(63*b^3*c - 3*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(24*a^4
) + (x^2*(5*a*d - 9*b*c))/(15*a^2) + (x^4*(63*b^2*c + 15*a^2*e - 35*a*b*d))
/(15*a^3) + (b*x^8*(63*b^3*c - 3*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(8*a^5))
/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (atan((b^(1/2)*x)/a^(1/2))*(63*b^3*c - 3
*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(8*a^(11/2)*b^(1/2))
```

sympy [**A**] time = 150.75, size = 284, normalized size = 1.45

$$-\frac{\sqrt{-\frac{1}{a^{11}b}}\left(3a^3f-15a^2be+35ab^2d-63b^3c\right)\log\left(-a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^{11}b}}\left(3a^3f-15a^2be+35ab^2d-63b^3c\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**11*b))*(3*a**3*f - 15*a**2*b*e + 35*a*b**2*d - 63*b**3*c)*log(
-a**6*sqrt(-1/(a**11*b)) + x)/16 + sqrt(-1/(a**11*b))*(3*a**3*f - 15*a**2*b
*e + 35*a*b**2*d - 63*b**3*c)*log(a**6*sqrt(-1/(a**11*b)) + x)/16 + (-24*a*
*4*c + x**8*(45*a**3*b*f - 225*a**2*b**2*e + 525*a*b**3*d - 945*b**4*c) + x
**6*(75*a**4*f - 375*a**3*b*e + 875*a**2*b**2*d - 1575*a*b**3*c) + x**4*(-1
20*a**4*e + 280*a**3*b*d - 504*a**2*b**2*c) + x**2*(-40*a**4*d + 72*a**3*b*
c))/(120*a**7*x**5 + 240*a**6*b*x**7 + 120*a**5*b**2*x**9)
```

3.141 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^3}\,dx$

Optimal. Leaf size=234

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+35a^2be-63ab^2d+99b^3c\right)}{8a^{13/2}}+\frac{bx\left(-7a^3f+11a^2b\right.}{8a^6\left(a\right.}$$

```
[Out] -1/7*c/a^3/x^7+1/5*(-a*d+3*b*c)/a^4/x^5+1/3*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x^
3+(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x+1/4*b*(-a^3*f+a^2*b*e-a*b^2*d
+b^3*c)*x/a^5/(b*x^2+a)^2+1/8*b*(-7*a^3*f+11*a^2*b*e-15*a*b^2*d+19*b^3*c)*x
/a^6/(b*x^2+a)+1/8*(-15*a^3*f+35*a^2*b*e-63*a*b^2*d+99*b^3*c)*arctan(x*b^(1
/2)/a^(1/2))*b^(1/2)/a^(13/2)
```

Rubi [A] time = 0.49, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1802, 205}

$$\frac{bx\left(11a^2be-7a^3f-15ab^2d+19b^3c\right)}{8a^6\left(a+bx^2\right)}+\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^5\left(a+bx^2\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{a^6x}+\frac{\sqrt{b}\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/(3
*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c - 15*a
*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*c
- 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13
/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{4\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\cdots}{x^8\left(a+bx^2\right)^2}}{4a}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} + \frac{b\left(19b^3c-15ab^2d+11a^2be-7a^3f\right)x}{8a^6\left(a+bx^2\right)} + \int \frac{8c-8(\cdots}{\cdots}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} + \frac{b\left(19b^3c-15ab^2d+11a^2be-7a^3f\right)x}{8a^6\left(a+bx^2\right)} + \int\left(\frac{8c}{ax^8}+\cdots\right.$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c\cdots\right.}{\cdots}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.15, size = 234, normalized size = 1.00

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+35a^2be-63ab^2d+99b^3c\right)}{8a^{13/2}}+\frac{bx\left(-7a^3f+11\cdots\right.}{8a^6\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -1/7*c/(a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/
(3*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c - 15
*a*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*
c - 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(
13/2))
```

fricas [A] time = 0.85, size = 678, normalized size = 2.90

$$\left[\frac{210\left(99\,b^5c-63\,ab^4d+35\,a^2b^3e-15\,a^3b^2f\right)x^{10}+350\left(99\,ab^4c-63\,a^2b^3d+35\,a^3b^2e-15\,a^4bf\right)x^8+112\left(99\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1680*(210*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 3
50*(99*a*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 112*(99*a^
2*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 240*a^5*c - 16*(99*a^
3*b^2*c - 63*a^4*b*d + 35*a^5*e)*x^4 + 48*(11*a^4*b*c - 7*a^5*d)*x^2 - 105*
((99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*b^4*c
 - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63*a^3*b
^2*d + 35*a^4*b*e - 15*a^5*f)*x^7)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
```

```
 - a)/(b*x^2 + a)))/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7), 1/840*(105*(99*
b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 175*(99*a*b^4*c -
63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 56*(99*a^2*b^3*c - 63*a^3*b
^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 120*a^5*c - 8*(99*a^3*b^2*c - 63*a^4*b*
d + 35*a^5*e)*x^4 + 24*(11*a^4*b*c - 7*a^5*d)*x^2 + 105*((99*b^5*c - 63*a*b
^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*b^4*c - 63*a^2*b^3*d + 3
5*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e -
 15*a^5*f)*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^2*x^11 + 2*a^7*b*x^9
+ a^8*x^7)]
```

giac [A] time = 0.47, size = 250, normalized size = 1.07

$$\frac{\left(99\,b^4c-63\,ab^3d-15\,a^3bf+35\,a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^6}+\frac{19\,b^5cx^3-15\,ab^4dx^3-7\,a^3b^2fx^3+11\,a^2b^3x^3e+21\,ab^4cx}{8\left(bx^2+a\right)^2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(99*b^4*c - 63*a*b^3*d - 15*a^3*b*f + 35*a^2*b^2*e)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*c*x^3 - 15*a*b^4*d*x^3 - 7*a^3*b^2*f*x^3 +
 11*a^2*b^3*x^3*e + 21*a*b^4*c*x - 17*a^2*b^3*d*x - 9*a^4*b*f*x + 13*a^3*b^
2*x*e)/((b*x^2 + a)^2*a^6) + 1/105*(1050*b^3*c*x^6 - 630*a*b^2*d*x^6 - 105*
a^3*f*x^6 + 315*a^2*b*x^6*e - 210*a*b^2*c*x^4 + 105*a^2*b*d*x^4 - 35*a^3*x^
4*e + 63*a^2*b*c*x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^6*x^7)
```

maple [A] time = 0.02, size = 351, normalized size = 1.50

$$-\frac{7b^2f\,x^3}{8\left(b\,x^2+a\right)^2a^3}+\frac{11b^3e\,x^3}{8\left(b\,x^2+a\right)^2a^4}-\frac{15b^4d\,x^3}{8\left(b\,x^2+a\right)^2a^5}+\frac{19b^5c\,x^3}{8\left(b\,x^2+a\right)^2a^6}-\frac{9bf\,x}{8\left(b\,x^2+a\right)^2a^2}+\frac{13b^2ex}{8\left(b\,x^2+a\right)^2a^3}-\frac{17b^3}{8\left(b\,x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x)
```

```
[Out] -7/8/a^3*b^2/(b*x^2+a)^2*x^3*f+11/8/a^4*b^3/(b*x^2+a)^2*x^3*e-15/8/a^5*b^4/
(b*x^2+a)^2*x^3*d+19/8/a^6*b^5/(b*x^2+a)^2*x^3*c-9/8/a^2*b/(b*x^2+a)^2*f*x+
13/8/a^3*b^2/(b*x^2+a)^2*e*x-17/8/a^4*b^3/(b*x^2+a)^2*d*x+21/8/a^5*b^4/(b*x
^2+a)^2*c*x-15/8/a^3*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+35/8/a^4*b^2
/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-63/8/a^5*b^3/(a*b)^(1/2)*arctan(1/
(a*b)^(1/2)*b*x)*d+99/8/a^6*b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/7
*c/a^3/x^7-1/5/a^3/x^5*d+3/5/a^4/x^5*b*c-1/3/a^3/x^3*e+1/a^4/x^3*b*d-2/a^5/
x^3*b^2*c-1/a^3/x*f+3/a^4/x*b*e-6/a^5/x*b^2*d+10/a^6/x*b^3*c
```

maxima [A] time = 3.03, size = 247, normalized size = 1.06

$$\frac{105\left(99\,b^5c-63\,ab^4d+35\,a^2b^3e-15\,a^3b^2f\right)x^{10}+175\left(99\,ab^4c-63\,a^2b^3d+35\,a^3b^2e-15\,a^4bf\right)x^8+56\left(99\,a^2b^3c\right.}{840\left(a^6b^2x^{11}+2\,a^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/840*(105*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 175
*(99*a*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 56*(99*a^2*b
^3*c - 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 120*a^5*c - 8*(99*a^3*b^
2*c - 63*a^4*b*d + 35*a^5*e)*x^4 + 24*(11*a^4*b*c - 7*a^5*d)*x^2)/(a^6*b^2*
x^11 + 2*a^7*b*x^9 + a^8*x^7) + 1/8*(99*b^4*c - 63*a*b^3*d + 35*a^2*b^2*e -
 15*a^3*b*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

mupad [B] time = 1.05, size = 230, normalized size = 0.98

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{8\,a^{13/2}}-\frac{\frac{c}{7\,a}-\frac{x^6\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{15\,a^4}+\frac{x^2\,(7\,a\,d-11\,b\,c}{35\,a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(99*b^3*c - 15*a^3*f - 63*a*b^2*d + 35*a
^2*b*e))/(8*a^(13/2)) - (c/(7*a) - (x^6*(99*b^3*c - 15*a^3*f - 63*a*b^2*d +
 35*a^2*b*e))/(15*a^4) + (x^2*(7*a*d - 11*b*c))/(35*a^2) + (x^4*(99*b^2*c +
 35*a^2*e - 63*a*b*d))/(105*a^3) - (5*b*x^8*(99*b^3*c - 15*a^3*f - 63*a*b^2
*d + 35*a^2*b*e))/(24*a^5) - (b^2*x^10*(99*b^3*c - 15*a^3*f - 63*a*b^2*d +
35*a^2*b*e))/(8*a^6))/(a^2*x^7 + b^2*x^11 + 2*a*b*x^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.142 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=277

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-35a^3f+63a^2be-99ab^2d+143b^3c\right)}{8a^{15/2}}-\frac{b^2x\left(-11a^3f+15\right.}{8a^7}$$

```
[Out] -1/9*c/a^3/x^9+1/7*(-a*d+3*b*c)/a^4/x^7+1/5*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x^
5+1/3*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^3-b*(-3*a^3*f+6*a^2*b*e-1
0*a*b^2*d+15*b^3*c)/a^7/x-1/4*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^6/(b*x
^2+a)^2-1/8*b^2*(-11*a^3*f+15*a^2*b*e-19*a*b^2*d+23*b^3*c)*x/a^7/(b*x^2+a)-
1/8*b^(3/2)*(-35*a^3*f+63*a^2*b*e-99*a*b^2*d+143*b^3*c)*arctan(x*b^(1/2)/a^
(1/2))/a^(15/2)
```

Rubi [A] time = 0.60, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1805, 1802, 205}

$$-\frac{b^2x\left(15a^2be-11a^3f-19ab^2d+23b^3c\right)}{8a^7\left(a+bx^2\right)}-\frac{b^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^6\left(a+bx^2\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{3a^6x^3}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/(5
*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) - (b*(15
*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x)/(4*a^6*(a + b*x^2)^2) - (b^2*(23*b^3*c - 19*a*b^2*d
+ 15*a^2*b*e - 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) - (b^(3/2)*(143*b^3*c - 99*
a*b^2*d + 63*a^2*b*e - 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(15/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^6\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{4\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}}{x^{10}(a+bx}}{4a}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^6\left(a+bx^2\right)^2} - \frac{b^2\left(23b^3c-19ab^2d+15a^2be-11a^3f\right)x}{8a^7\left(a+bx^2\right)} + \frac{\int -}{}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^6\left(a+bx^2\right)^2} - \frac{b^2\left(23b^3c-19ab^2d+15a^2be-11a^3f\right)x}{8a^7\left(a+bx^2\right)} + \frac{\int \left(\right.}{}$$

$$= -\frac{c}{9a^3x^9} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} - \frac{b\left(15b\right.}{}$$

$$= -\frac{c}{9a^3x^9} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} - \frac{b\left(15b\right.}{}$$

Mathematica [A] time = 0.16, size = 276, normalized size = 1.00

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(35a^3f-63a^2be+99ab^2d-143b^3c\right)}{8a^{15/2}}+\frac{b^2x\left(11a^3f-15\right.}{8a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] -1/9*c/(a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/
(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b*(
-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3*c) + a
*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^6*(a + b*x^2)^2) + (b^2*(-23*b^3*c + 19*a
*b^2*d - 15*a^2*b*e + 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) + (b^(3/2)*(-143*b^3
*c + 99*a*b^2*d - 63*a^2*b*e + 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(15/2))
```

fricas [A] time = 0.51, size = 772, normalized size = 2.79

$$\left[-\frac{630\left(143\,b^6c-99\,ab^5d+63\,a^2b^4e-35\,a^3b^3f\right)x^{12}+1050\left(143\,ab^5c-99\,a^2b^4d+63\,a^3b^3e-35\,a^4b^2f\right)x^{10}+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/5040*(630*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 +
 1050*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 336
*(143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 560*a^6*c
 - 48*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 16*(143*
a^4*b^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 80*(13*a^5*b*c - 9*a^6*d)*x^2 + 31
5*((143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^13 + 2*(143*a*b
```

```
^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^11 + (143*a^2*b^4*c -
99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^9)*sqrt(-b/a)*log((b*x^2 + 2*a*
x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9), -1
/2520*(315*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 + 52
5*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 168*(14
3*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 280*a^6*c - 2
4*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 8*(143*a^4*b
^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 40*(13*a^5*b*c - 9*a^6*d)*x^2 + 315*((1
43*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^13 + 2*(143*a*b^5*c
- 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^11 + (143*a^2*b^4*c - 99*a^
3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^9)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a
^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9)]
```

giac [A] time = 0.40, size = 301, normalized size = 1.09

$$-\frac{\left(143\,b^5c-99\,ab^4d-35\,a^3b^2f+63\,a^2b^3e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^7}-\frac{23\,b^6cx^3-19\,ab^5dx^3-11\,a^3b^3fx^3+15\,a^2b^4x^3e+25\,a}{8\left(bx^2+a\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(143*b^5*c - 99*a*b^4*d - 35*a^3*b^2*f + 63*a^2*b^3*e)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^7) - 1/8*(23*b^6*c*x^3 - 19*a*b^5*d*x^3 - 11*a^3*b^3*f*
x^3 + 15*a^2*b^4*x^3*e + 25*a*b^5*c*x - 21*a^2*b^4*d*x - 13*a^4*b^2*f*x + 1
7*a^3*b^3*x*e)/((b*x^2 + a)^2*a^7) - 1/315*(4725*b^4*c*x^8 - 3150*a*b^3*d*x
^8 - 945*a^3*b*f*x^8 + 1890*a^2*b^2*x^8*e - 1050*a*b^3*c*x^6 + 630*a^2*b^2*
d*x^6 + 105*a^4*f*x^6 - 315*a^3*b*x^6*e + 378*a^2*b^2*c*x^4 - 189*a^3*b*d*x
^4 + 63*a^4*x^4*e - 135*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^7*x^9)
```

maple [A] time = 0.02, size = 401, normalized size = 1.45

$$\frac{11b^3f\,x^3}{8\left(b\,x^2+a\right)^2a^4}-\frac{15b^4e\,x^3}{8\left(b\,x^2+a\right)^2a^5}+\frac{19b^5d\,x^3}{8\left(b\,x^2+a\right)^2a^6}-\frac{23b^6c\,x^3}{8\left(b\,x^2+a\right)^2a^7}+\frac{13b^2fx}{8\left(b\,x^2+a\right)^2a^3}-\frac{17b^3ex}{8\left(b\,x^2+a\right)^2a^4}+\frac{21b^4d}{8\left(b\,x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x)
```

```
[Out] 3/5/a^4/x^5*b*d-6/5/a^5/x^5*b^2*c+1/a^4/x^3*b*e-2/a^5/x^3*b^2*d+10/3/a^6/x^
3*b^3*c+3*b/a^4/x*f-6*b^2/a^5/x*e+10*b^3/a^6/x*d-15*b^4/a^7/x*c+3/7/a^4/x^7
*b*c-1/9*c/a^3/x^9+19/8/a^6*b^5/(b*x^2+a)^2*x^3*d-23/8/a^7*b^6/(b*x^2+a)^2*
x^3*c+13/8/a^3*b^2/(b*x^2+a)^2*f*x-17/8/a^4*b^3/(b*x^2+a)^2*e*x+21/8/a^5*b^
4/(b*x^2+a)^2*d*x-25/8/a^6*b^5/(b*x^2+a)^2*c*x+35/8/a^4*b^2/(a*b)^(1/2)*arc
tan(1/(a*b)^(1/2)*b*x)*f-63/8/a^5*b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
*e+99/8/a^6*b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-143/8/a^7*b^5/(a*b)
^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/7/a^3/x^7*d-1/5/a^3/x^5*e-1/3/a^3/x^3*
f+11/8/a^4*b^3/(b*x^2+a)^2*x^3*f-15/8/a^5*b^4/(b*x^2+a)^2*x^3*e
```

maxima [A] time = 3.09, size = 291, normalized size = 1.05

$$-\frac{315\left(143\,b^6c-99\,ab^5d+63\,a^2b^4e-35\,a^3b^3f\right)x^{12}+525\left(143\,ab^5c-99\,a^2b^4d+63\,a^3b^3e-35\,a^4b^2f\right)x^{10}+168\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2520*(315*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 +
525*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 168*(
143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 280*a^6*c -
 24*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 8*(143*a^4
*b^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 40*(13*a^5*b*c - 9*a^6*d)*x^2)/(a^7*b
^2*x^13 + 2*a^8*b*x^11 + a^9*x^9) - 1/8*(143*b^5*c - 99*a*b^4*d + 63*a^2*b^
3*e - 35*a^3*b^2*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7)
```

mupad [B] time = 1.07, size = 268, normalized size = 0.97

$$-\frac{\frac{c}{9\,a}-\frac{x^6\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{105\,a^4}+\frac{x^2\,(9\,a\,d-13\,b\,c)}{63\,a^2}+\frac{x^4\,\left(63\,e\,a^2-99\,d\,a\,b+143\,c\,b^2\right)}{315\,a^3}+\frac{b\,x^8\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\ldots\right.}{15\,a^5}}{a^2\,x^9+2\,a\,b\,x^{11}+b^2\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x)
```

```
[Out] - (c/(9*a) - (x^6*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(105*a^
4) + (x^2*(9*a*d - 13*b*c))/(63*a^2) + (x^4*(143*b^2*c + 63*a^2*e - 99*a*b*
d))/(315*a^3) + (b*x^8*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(1
5*a^5) + (5*b^2*x^10*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(24*
a^6) + (b^3*x^12*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(8*a^7))
/(a^2*x^9 + b^2*x^13 + 2*a*b*x^11) - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(14
3*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(8*a^(15/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.143 $\int \frac{x^5\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$\frac{\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{\left(a+bx^2\right)^{5/2}\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(-5a^3f+4a^2be\right.}{3b^6}$$

```
[Out] -1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*(b*x^2+a)^(3/2)/b^6+1/5*(-10*
a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*(b*x^2+a)^(5/2)/b^6+1/7*(10*a^2*f-4*a*b*e+
b^2*d)*(b*x^2+a)^(7/2)/b^6+1/9*(-5*a*f+b*e)*(b*x^2+a)^(9/2)/b^6+1/11*f*(b*x
^2+a)^(11/2)/b^6+a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^6
```

Rubi [A] time = 0.25, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1799, 1620}

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^2be+\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2\left(c+dx+ex^2+fx^3\right)}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^5\sqrt{a+bx}}+\frac{a\left(-2b^3c+3ab^2d-4a^2be+5a\right.}{b^5}\right.\right.\\ &= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^6}-\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)\left(a+\right.}{3b^6}\end{aligned}$$

Mathematica [A] time = 0.17, size = 158, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-1280a^5f+128a^4b\left(11e+5fx^2\right)-16a^3b^2\left(99d+44ex^2+30fx^4\right)+8a^2b^3\left(231c+99dx^2+66ex^4+5\right.\right.}{3465b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(99
*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f*x
^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*(69
3*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

fricas [A] time = 0.72, size = 177, normalized size = 0.83

$$\left(315\, b^5 f x^{10} + 35\left(11\, b^5 e - 10\, a b^4 f\right) x^8 + 5\left(99\, b^5 d - 88\, a b^4 e + 80\, a^2 b^3 f\right) x^6 + 1848\, a^2 b^3 c - 1584\, a^3 b^2 d + 1408\, a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b^5*d - 88*
a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*b*
e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d + 176*a^2*b^3*e - 160*a^3*b^2*f
)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x^2)*
sqrt(b*x^2 + a)/b^6
```

giac [A] time = 0.45, size = 264, normalized size = 1.23

$$\frac{\left(a^2 b^3 c - a^3 b^2 d - a^5 f + a^4 b e\right)\sqrt{b x^2 + a}}{b^6} + \frac{693\left(b x^2 + a\right)^{\frac{5}{2}} b^3 c - 2310\left(b x^2 + a\right)^{\frac{3}{2}} a b^3 c + 495\left(b x^2 + a\right)^{\frac{7}{2}} b^2 d - 207$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*sqrt(b*x^2 + a)/b^6 + 1/3465*(693
*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 495*(b*x^2 + a)
^(7/2)*b^2*d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*
b^2*d + 315*(b*x^2 + a)^(11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2
 + a)^(7/2)*a^2*f - 6930*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a
^4*f + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 + a)^(7/2)*a*b*e + 4158*(b*x
^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/2)*a^3*b*e)/b^6
```

maple [A] time = 0.01, size = 193, normalized size = 0.90

$$-\frac{\sqrt{b x^2 + a}\,\left(-315 f x^{10} b^5 + 350 a b^4 f x^8 - 385 b^5 e x^8 - 400 a^2 b^3 f x^6 + 440 a b^4 e x^6 - 495 b^5 d x^6 + 480 a^3 b^2 f x^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3465*(b*x^2+a)^(1/2)*(-315*b^5*f*x^10+350*a*b^4*f*x^8-385*b^5*e*x^8-400*
a^2*b^3*f*x^6+440*a*b^4*e*x^6-495*b^5*d*x^6+480*a^3*b^2*f*x^4-528*a^2*b^3*e
*x^4+594*a*b^4*d*x^4-693*b^5*c*x^4-640*a^4*b*f*x^2+704*a^3*b^2*e*x^2-792*a^
2*b^3*d*x^2+924*a*b^4*c*x^2+1280*a^5*f-1408*a^4*b*e+1584*a^3*b^2*d-1848*a^2
*b^3*c)/b^6
```

maxima [A] time = 1.40, size = 347, normalized size = 1.62

$$\frac{\sqrt{b x^2 + a}\, f x^{10}}{11\, b} + \frac{\sqrt{b x^2 + a}\, e x^8}{9\, b} - \frac{10\sqrt{b x^2 + a}\, a f x^8}{99\, b^2} + \frac{\sqrt{b x^2 + a}\, d x^6}{7\, b} - \frac{8\sqrt{b x^2 + a}\, a e x^6}{63\, b^2} + \frac{80\sqrt{b x^2 + a}\, a^2 f x^6}{693\, b^3} + \sqrt{b x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/11*sqrt(b*x^2 + a)*f*x^10/b + 1/9*sqrt(b*x^2 + a)*e*x^8/b - 10/99*sqrt(b*
x^2 + a)*a*f*x^8/b^2 + 1/7*sqrt(b*x^2 + a)*d*x^6/b - 8/63*sqrt(b*x^2 + a)*a
*e*x^6/b^2 + 80/693*sqrt(b*x^2 + a)*a^2*f*x^6/b^3 + 1/5*sqrt(b*x^2 + a)*c*x
^4/b - 6/35*sqrt(b*x^2 + a)*a*d*x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*e*x^4/
b^3 - 32/231*sqrt(b*x^2 + a)*a^3*f*x^4/b^4 - 4/15*sqrt(b*x^2 + a)*a*c*x^2/b
^2 + 8/35*sqrt(b*x^2 + a)*a^2*d*x^2/b^3 - 64/315*sqrt(b*x^2 + a)*a^3*e*x^2/
b^4 + 128/693*sqrt(b*x^2 + a)*a^4*f*x^2/b^5 + 8/15*sqrt(b*x^2 + a)*a^2*c/b^
3 - 16/35*sqrt(b*x^2 + a)*a^3*d/b^4 + 128/315*sqrt(b*x^2 + a)*a^4*e/b^5 - 2
56/693*sqrt(b*x^2 + a)*a^5*f/b^6
```

mupad [B] time = 1.19, size = 186, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{x^6\,\left(400\,f\,a^2\,b^3-440\,e\,a\,b^4+495\,d\,b^5\right)}{3465\,b^6}-\frac{1280\,f\,a^5-1408\,e\,a^4\,b+1584\,d\,a^3\,b^2-1848\,c\,a^2\,b^3}{3465\,b^6}+\frac{x^4\,\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^6*(495*b^5*d + 400*a^2*b^3*f - 440*a*b^4*e))/(3465*b^
6) - (1280*a^5*f - 1848*a^2*b^3*c + 1584*a^3*b^2*d - 1408*a^4*b*e)/(3465*b^
6) + (x^4*(693*b^5*c + 528*a^2*b^3*e - 480*a^3*b^2*f - 594*a*b^4*d))/(3465*
b^6) + (f*x^10)/(11*b) + (x^8*(385*b^5*e - 350*a*b^4*f))/(3465*b^6) - (4*a*
x^2*(231*b^3*c - 160*a^3*f - 198*a*b^2*d + 176*a^2*b*e))/(3465*b^5))
```

sympy [A] time = 4.68, size = 442, normalized size = 2.07

$$\begin{cases} -\frac{256a^5f\sqrt{a+bx^2}}{693b^6}+\frac{128a^4e\sqrt{a+bx^2}}{315b^5}+\frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5}-\frac{16a^3d\sqrt{a+bx^2}}{35b^4}-\frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4}-\frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4}+\frac{8a^2c\sqrt{a+bx^2}}{15b^3}+\frac{8a^2dx}{} \\ \frac{\frac{cx^6}{6}+\frac{dx^8}{8}+\frac{ex^{10}}{10}+\frac{fx^{12}}{12}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b*
x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*d*
sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4) - 3
2*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b
**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b
*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2
*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*e
*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99*b**2) +
 c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e*x**8*sqr
t(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)), ((c*x**6/
6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

3.144 $\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=167

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{\left(a+bx^2\right)^{3/2}\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}-\frac{a\sqrt{a+bx^2}\left(a^3(-f)+a^2be-a\right.}{b^5}$$

```
[Out] 1/3*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*(b*x^2+a)^(3/2)/b^5+1/5*(6*a^2*f-3
*a*b*e+b^2*d)*(b*x^2+a)^(5/2)/b^5+1/7*(-4*a*f+b*e)*(b*x^2+a)^(7/2)/b^5+1/9*
f*(b*x^2+a)^(9/2)/b^5-a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^5
```

Rubi [A] time = 0.19, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1799, 1620}

$$\frac{\left(a+bx^2\right)^{3/2}\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{3b^5}-\frac{a\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^5}+\frac{\left(a+bx^2\right)^{5/2}\left(6a^2f-3a\right.}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x\left(c+dx+ex^2+fx^3\right)}{\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^4\sqrt{a+bx}}+\frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)}{b^4}\right.\right.\\
&= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^5}+\frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)\left(a+\right.}{3b^5}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 122, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(128a^4f-16a^3b\left(9e+4fx^2\right)+24a^2b^2\left(7d+3ex^2+2fx^4\right)-2ab^3\left(105c+42dx^2+27ex^4+20fx^6\right)\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d +
3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b^4
*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)
```

fricas [A] time = 0.66, size = 134, normalized size = 0.80

$$\frac{\left(35\,b^4fx^8 + 5\left(9\,b^4e - 8\,ab^3f\right)x^6 - 210\,ab^3c + 168\,a^2b^2d - 144\,a^3be + 128\,a^4f + 3\left(21\,b^4d - 18\,ab^3e + 16\,a^2b^2f\right)x^4}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c + 168*a^2*b
^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3*e + 16*a^2*b^2*f)*x
^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e - 64*a^3*b*f)*x^2)*sqrt(b*x^2 +
 a)/b^5
```

giac [A] time = 0.51, size = 197, normalized size = 1.18

$$-\frac{\left(ab^3c - a^2b^2d - a^4f + a^3be\right)\sqrt{bx^2+a}}{b^5} + \frac{105\left(bx^2+a\right)^{\frac{3}{2}}b^3c + 63\left(bx^2+a\right)^{\frac{5}{2}}b^2d - 210\left(bx^2+a\right)^{\frac{3}{2}}ab^2d + 35\left(bx^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*sqrt(b*x^2 + a)/b^5 + 1/315*(105*(
b*x^2 + a)^(3/2)*b^3*c + 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)
*a*b^2*d + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)*a*f + 378*(b*x^2
+ a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f + 45*(b*x^2 + a)^(7/2)*b*e -
 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*x^2 + a)^(3/2)*a^2*b*e)/b^5
```

maple [A] time = 0.01, size = 145, normalized size = 0.87

$$\frac{\sqrt{b\,x^2+a}\,\left(35f\,x^8b^4 - 40a\,b^3f\,x^6 + 45b^4e\,x^6 + 48a^2b^2f\,x^4 - 54a\,b^3e\,x^4 + 63b^4d\,x^4 - 64a^3bf\,x^2 + 72a^2b^2e\,x^2 - 84\right.}{315b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(1/2)*(35*b^4*f*x^8-40*a*b^3*f*x^6+45*b^4*e*x^6+48*a^2*b^2*
f*x^4-54*a*b^3*e*x^4+63*b^4*d*x^4-64*a^3*b*f*x^2+72*a^2*b^2*e*x^2-84*a*b^3*
d*x^2+105*b^4*c*x^2+128*a^4*f-144*a^3*b*e+168*a^2*b^2*d-210*a*b^3*c)/b^5
```

maxima [A] time = 1.45, size = 263, normalized size = 1.57

$$\frac{\sqrt{bx^2+a}\,fx^8}{9\,b} + \frac{\sqrt{bx^2+a}\,ex^6}{7\,b} - \frac{8\,\sqrt{bx^2+a}\,afx^6}{63\,b^2} + \frac{\sqrt{bx^2+a}\,dx^4}{5\,b} - \frac{6\,\sqrt{bx^2+a}\,aex^4}{35\,b^2} + \frac{16\,\sqrt{bx^2+a}\,a^2fx^4}{105\,b^3} + \frac{\sqrt{bx^2+a}\,c}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(b*x^2 + a)*f*x^8/b + 1/7*sqrt(b*x^2 + a)*e*x^6/b - 8/63*sqrt(b*x^2
 + a)*a*f*x^6/b^2 + 1/5*sqrt(b*x^2 + a)*d*x^4/b - 6/35*sqrt(b*x^2 + a)*a*e*
x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*f*x^4/b^3 + 1/3*sqrt(b*x^2 + a)*c*x^2/
```

```
b - 4/15*sqrt(b*x^2 + a)*a*d*x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*e*x^2/b^3 -
 64/315*sqrt(b*x^2 + a)*a^3*f*x^2/b^4 - 2/3*sqrt(b*x^2 + a)*a*c/b^2 + 8/15*
sqrt(b*x^2 + a)*a^2*d/b^3 - 16/35*sqrt(b*x^2 + a)*a^3*e/b^4 + 128/315*sqrt(
b*x^2 + a)*a^4*f/b^5
```

mupad [B] time = 1.11, size = 146, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{128\,f\,a^4-144\,e\,a^3\,b+168\,d\,a^2\,b^2-210\,c\,a\,b^3}{315\,b^5}+\frac{x^4\,\left(48\,f\,a^2\,b^2-54\,e\,a\,b^3+63\,d\,b^4\right)}{315\,b^5}+\frac{f\,x^8}{9\,b}+\frac{x^6\,\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((128*a^4*f + 168*a^2*b^2*d - 210*a*b^3*c - 144*a^3*b*e)/
(315*b^5) + (x^4*(63*b^4*d + 48*a^2*b^2*f - 54*a*b^3*e))/(315*b^5) + (f*x^8
)/(9*b) + (x^6*(45*b^4*e - 40*a*b^3*f))/(315*b^5) + (x^2*(105*b^4*c + 72*a^
2*b^2*e - 84*a*b^3*d - 64*a^3*b*f))/(315*b^5))
```

sympy [A] time = 2.90, size = 340, normalized size = 2.04

$$\begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5}-\frac{16a^3e\sqrt{a+bx^2}}{35b^4}-\frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4}+\frac{8a^2d\sqrt{a+bx^2}}{15b^3}+\frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3}+\frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3}-\frac{2ac\sqrt{a+bx^2}}{3b^2}-\frac{4adx^2\sqrt{a\cdots}}{15b^2} \\ \frac{\frac{cx^4}{4}+\frac{dx^6}{6}+\frac{ex^8}{8}+\frac{fx^{10}}{10}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x*
*2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*
f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*
d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) -
 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*
x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*sqrt(a
 + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x**10/10)
/sqrt(a), True))
```

3.145 $\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=121

$$\frac{(a+bx^2)^{3/2}(3a^2f-2abe+b^2d)}{3b^4}+\frac{\sqrt{a+bx^2}\,(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}+\frac{(a+bx^2)^{5/2}(be-3af)}{5b^4}+\frac{f(a+bx^2)^{7}}{7b^4}$$

```
[Out] 1/3*(3*a^2*f-2*a*b*e+b^2*d)*(b*x^2+a)^(3/2)/b^4+1/5*(-3*a*f+b*e)*(b*x^2+a)^
(5/2)/b^4+1/7*f*(b*x^2+a)^(7/2)/b^4+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a
)^(1/2)/b^4
```

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1799, 1850}

$$\frac{\sqrt{a+bx^2}\,(a^2be+a^3(-f)-ab^2d+b^3c)}{b^4}+\frac{(a+bx^2)^{3/2}(3a^2f-2abe+b^2d)}{3b^4}+\frac{(a+bx^2)^{5/2}(be-3af)}{5b^4}+\frac{f(a+bx^2)^{7}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}}+\frac{(b^2d-2abe+3a^2f)\sqrt{a+bx}}{b^3}+\frac{(be-3\right.\right.$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^4}+\frac{(b^2d-2abe+3a^2f)(a+bx^2)^{3/2}}{3b^4}+\frac{(be-}{}$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-48a^3f+8a^2b(7e+3fx^2)-2ab^2(35d+14ex^2+9fx^4)+b^3(105c+35dx^2+21ex^4+15fx^6)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14*
e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4)
```

fricas [A] time = 0.64, size = 94, normalized size = 0.78

$$\frac{\left(15\,b^{3}fx^{6}+3\left(7\,b^{3}e-6\,ab^{2}f\right)x^{4}+105\,b^{3}c-70\,ab^{2}d+56\,a^{2}be-48\,a^{3}f+\left(35\,b^{3}d-28\,ab^{2}e+24\,a^{2}bf\right)x^{2}\right)\sqrt{b\ldots}}{105\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 70*a*b^2*d
+ 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x^2)*sqrt(b*
x^2 + a)/b^4
```

giac [A] time = 0.41, size = 130, normalized size = 1.07

$$\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\sqrt{bx^{2}+a}}{b^{4}}+\frac{35\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}d+15\left(bx^{2}+a\right)^{\frac{7}{2}}f-63\left(bx^{2}+a\right)^{\frac{5}{2}}af+105\left(bx^{2}+a\right)^{\frac{3}{2}}a\ldots}{105\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*sqrt(b*x^2 + a)/b^4 + 1/105*(35*(b*x^2
+ a)^(3/2)*b^2*d + 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*
(b*x^2 + a)^(3/2)*a^2*f + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a
*b*e)/b^4
```

maple [A] time = 0.00, size = 99, normalized size = 0.82

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(-15f\,x^{6}b^{3}+18a\,b^{2}f\,x^{4}-21b^{3}e\,x^{4}-24a^{2}bf\,x^{2}+28a\,b^{2}e\,x^{2}-35b^{3}d\,x^{2}+48a^{3}f-56a^{2}be+70a\,b\ldots\right.}{105b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(-15*b^3*f*x^6+18*a*b^2*f*x^4-21*b^3*e*x^4-24*a^2*b*
f*x^2+28*a*b^2*e*x^2-35*b^3*d*x^2+48*a^3*f-56*a^2*b*e+70*a*b^2*d-105*b^3*c)
/b^4
```

maxima [A] time = 1.35, size = 180, normalized size = 1.49

$$\frac{\sqrt{bx^{2}+a}\,fx^{6}}{7\,b}+\frac{\sqrt{bx^{2}+a}\,ex^{4}}{5\,b}-\frac{6\,\sqrt{bx^{2}+a}\,afx^{4}}{35\,b^{2}}+\frac{\sqrt{bx^{2}+a}\,dx^{2}}{3\,b}-\frac{4\,\sqrt{bx^{2}+a}\,aex^{2}}{15\,b^{2}}+\frac{8\,\sqrt{bx^{2}+a}\,a^{2}fx^{2}}{35\,b^{3}}+\frac{\sqrt{bx^{2}+\ldots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*f*x^6/b + 1/5*sqrt(b*x^2 + a)*e*x^4/b - 6/35*sqrt(b*x^2
 + a)*a*f*x^4/b^2 + 1/3*sqrt(b*x^2 + a)*d*x^2/b - 4/15*sqrt(b*x^2 + a)*a*e*
x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*f*x^2/b^3 + sqrt(b*x^2 + a)*c/b - 2/3*sq
rt(b*x^2 + a)*a*d/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*e/b^3 - 16/35*sqrt(b*x^2 +
 a)*a^3*f/b^4
```

mupad [B] time = 1.06, size = 103, normalized size = 0.85

$$\sqrt{b\,x^{2}+a}\,\left(\frac{-48\,f\,a^{3}+56\,e\,a^{2}\,b-70\,d\,a\,b^{2}+105\,c\,b^{3}}{105\,b^{4}}+\frac{f\,x^{6}}{7\,b}+\frac{x^{2}\,\left(24\,f\,a^{2}\,b-28\,e\,a\,b^{2}+35\,d\,b^{3}\right)}{105\,b^{4}}+\frac{x^{4}\,\left(21\,b^{3}\ldots\right.}{1\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((105*b^3*c - 48*a^3*f - 70*a*b^2*d + 56*a^2*b*e)/(105*b^
4) + (f*x^6)/(7*b) + (x^2*(35*b^3*d - 28*a*b^2*e + 24*a^2*b*f))/(105*b^4) +
 (x^4*(21*b^3*e - 18*a*b^2*f))/(105*b^4))
```

sympy [A] time = 2.19, size = 238, normalized size = 1.97

$$\begin{cases} -\frac{16a^3f\sqrt{a+bx^2}}{35b^4} + \frac{8a^2e\sqrt{a+bx^2}}{15b^3} + \frac{8a^2fx^2\sqrt{a+bx^2}}{35b^3} - \frac{2ad\sqrt{a+bx^2}}{3b^2} - \frac{4aex^2\sqrt{a+bx^2}}{15b^2} - \frac{6afx^4\sqrt{a+bx^2}}{35b^2} + \frac{c\sqrt{a+bx^2}}{b} + \frac{dx^2\sqrt{a+bx^2}}{3b} + \frac{ex^4\sqrt{\cdots}}{5\cdots} \\ \frac{\frac{cx^2}{2}+\frac{dx^4}{4}+\frac{ex^6}{6}+\frac{fx^8}{8}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**2
)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b*x
**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(a +
 b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) +
 e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)),
((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

3.146 $\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{a+bx^2}\left(a^2f-abe+b^2d\right)}{b^3}+\frac{\left(a+bx^2\right)^{3/2}(be-2af)}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] 1/3*(-2*a*f+b*e)*(b*x^2+a)^(3/2)/b^3+1/5*f*(b*x^2+a)^(5/2)/b^3-c*arctanh((b
*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+(a^2*f-a*b*e+b^2*d)*(b*x^2+a)^(1/2)/b^3
```

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1799, 1620, 63, 208}

$$\frac{\sqrt{a+bx^2}\left(a^2f-abe+b^2d\right)}{b^3}+\frac{\left(a+bx^2\right)^{3/2}(be-2af)}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*Sqrt[a + b*x^2])/b^3 + ((b*e - 2*a*f)*(a + b*x^2)^
(3/2))/(3*b^3) + (f*(a + b*x^2)^(5/2))/(5*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^2d-abe+a^2f}{b^2\sqrt{a+bx}}+\frac{c}{x\sqrt{a+bx}}+\frac{(be-2af)\sqrt{a+bx}}{b^2}+\frac{f(a+bx)^{3/2}}{b^2}\right) d$$

$$= \frac{\left(b^2d-abe+a^2f\right)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{1}{2}c\,\text{Subst}$$

$$= \frac{\left(b^2d-abe+a^2f\right)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{c\,\text{Subst}\left(\right.}{}$$

$$= \frac{\left(b^2d-abe+a^2f\right)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\right.}{\sqrt{a}}$$

Mathematica [A] time = 0.13, size = 86, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\left(8a^2f-2ab\left(5e+2fx^2\right)+b^2\left(15d+5ex^2+3fx^4\right)\right)}{15b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*f - 2*a*b*(5*e + 2*f*x^2) + b^2*(15*d + 5*e*x^2 + 3
*f*x^4)))/(15*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 0.71, size = 205, normalized size = 1.99

$$\left[\frac{15\sqrt{a}\,b^3c\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(3ab^2fx^4+15ab^2d-10a^2be+8a^3f+\left(5ab^2e-4a^2bf\right)x^2\right)\sqrt{bx^2+a}}{30ab^3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*b^3*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(3*a*b^2*f*x^4 + 15*a*b^2*d - 10*a^2*b*e + 8*a^3*f + (5*a*b^2*e - 4*a^
2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3), 1/15*(15*sqrt(-a)*b^3*c*arctan(sqrt(-
a)/sqrt(b*x^2 + a)) + (3*a*b^2*f*x^4 + 15*a*b^2*d - 10*a^2*b*e + 8*a^3*f +
(5*a*b^2*e - 4*a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3)]
```

giac [A] time = 0.40, size = 127, normalized size = 1.23

$$\frac{c\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{15\sqrt{bx^2+a}\,b^{14}d+3\left(bx^2+a\right)^{\frac{5}{2}}b^{12}f-10\left(bx^2+a\right)^{\frac{3}{2}}ab^{12}f+15\sqrt{bx^2+a}\,a^2b^{12}f+5\left(bx^2+a\right.}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] c*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(15*sqrt(b*x^2 + a)*b^14
*d + 3*(b*x^2 + a)^(5/2)*b^12*f - 10*(b*x^2 + a)^(3/2)*a*b^12*f + 15*sqrt(b
```

```
*x^2 + a)*a^2*b^12*f + 5*(b*x^2 + a)^(3/2)*b^13*e - 15*sqrt(b*x^2 + a)*a*b^
13*e)/b^15
```

maple [A] time = 0.01, size = 134, normalized size = 1.30

$$\frac{\sqrt{b\,x^2+a}\;f\,x^4}{5b}-\frac{4\sqrt{b\,x^2+a}\;af\,x^2}{15b^2}+\frac{\sqrt{b\,x^2+a}\;e\,x^2}{3b}-\frac{c\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{8\sqrt{b\,x^2+a}\;a^2f}{15b^3}-\frac{2\sqrt{b\,x^2+a}\;ae}{3b^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*f*x^4/b*(b*x^2+a)^(1/2)-4/15*f*a/b^2*x^2*(b*x^2+a)^(1/2)+8/15*f*a^2/b^3
*(b*x^2+a)^(1/2)+1/3*e*x^2/b*(b*x^2+a)^(1/2)-2/3*e*a/b^2*(b*x^2+a)^(1/2)+d/
b*(b*x^2+a)^(1/2)-c/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.35, size = 122, normalized size = 1.18

$$\frac{\sqrt{bx^2+a}\,fx^4}{5\,b}+\frac{\sqrt{bx^2+a}\,ex^2}{3\,b}-\frac{4\,\sqrt{bx^2+a}\,afx^2}{15\,b^2}-\frac{c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{\sqrt{bx^2+a}\,d}{b}-\frac{2\,\sqrt{bx^2+a}\,ae}{3\,b^2}+\frac{8\,\sqrt{bx^2+a}\,a^2\cdots}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(b*x^2 + a)*f*x^4/b + 1/3*sqrt(b*x^2 + a)*e*x^2/b - 4/15*sqrt(b*x^2
 + a)*a*f*x^2/b^2 - c*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 +
a)*d/b - 2/3*sqrt(b*x^2 + a)*a*e/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*f/b^3
```

mupad [B] time = 1.81, size = 99, normalized size = 0.96

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,a^2\,f}{15\,b^3}+\frac{f\,x^4}{5\,b}-\frac{4\,a\,f\,x^2}{15\,b^2}\right)+\frac{d\,\sqrt{b\,x^2+a}}{b}-\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{e\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^2*f)/(15*b^3) + (f*x^4)/(5*b) - (4*a*f*x^2)/(15*b^2
)) + (d*(a + b*x^2)^(1/2))/b - (c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2)
 - (e*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

sympy [A] time = 37.87, size = 102, normalized size = 0.99

$$\frac{f\left(a+bx^2\right)^{\frac{5}{2}}}{5b^3}-\frac{\left(a+bx^2\right)^{\frac{3}{2}}\left(2af-be\right)}{3b^3}+\frac{\sqrt{a+bx^2}\,\left(a^2f-abe+b^2d\right)}{b^3}+\frac{c\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\,\sqrt{a+bx^2}}\right)}{a\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] f*(a + b*x**2)**(5/2)/(5*b**3) - (a + b*x**2)**(3/2)*(2*a*f - b*e)/(3*b**3)
 + sqrt(a + b*x**2)*(a**2*f - a*b*e + b**2*d)/b**3 + c*atan(1/(sqrt(-1/a)*s
qrt(a + b*x**2)))/(a*sqrt(-1/a))
```

3.147 $\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}\,(be-af)}{b^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}$$

```
[Out] 1/3*f*(b*x^2+a)^(3/2)/b^2+1/2*(-2*a*d+b*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))
/a^(3/2)+(-a*f+b*e)*(b*x^2+a)^(1/2)/b^2-1/2*c*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A] time = 0.20, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1799, 1621, 897, 1153, 208}

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}\,(be-af)}{b^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b*e - a*f)*Sqrt[a + b*x^2])/b^2 - (c*Sqrt[a + b*x^2])/(2*a*x^2) + (f*(a +
 b*x^2)^(3/2))/(3*b^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(
2*a^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx=\frac{1}{2}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)$$
$$=-\frac{c\sqrt{a+bx^2}}{2ax^2}-\frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc-2ad)-aex-afx^2}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}$$
$$=-\frac{c\sqrt{a+bx^2}}{2ax^2}-\frac{\operatorname{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(bc-2ad)+a^2be-a^3f}{b^2}-\frac{(abe-2a^2f)x^2}{b^2}-\frac{afx^4}{b^2}}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}$$
$$=-\frac{c\sqrt{a+bx^2}}{2ax^2}-\frac{\operatorname{Subst}\left(\int \left(-a\left(e-\frac{af}{b}\right)-\frac{afx^2}{b}+\frac{bc-2ad}{2\left(-\frac{a}{b}+\frac{x^2}{b}\right)}\right)dx,x,\sqrt{a+bx^2}\right)}{ab}$$
$$=\frac{(be-af)\sqrt{a+bx^2}}{b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{(bc-2ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,d\right.}{2ab}$$
$$=\frac{(be-af)\sqrt{a+bx^2}}{b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.41, size = 131, normalized size = 1.31

$$\frac{3b^3cx^2\sqrt{\frac{bx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)-\left(a+bx^2\right)\left(4a^2fx^2-2abx^2\left(3e+fx^2\right)+3b^2c\right)}{6ab^2x^2\sqrt{a+bx^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -((d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]) + (-((a + b*x^2)*(3*b^2*c +
 4*a^2*f*x^2 - 2*a*b*x^2*(3*e + f*x^2))) + 3*b^3*c*x^2*Sqrt[1 + (b*x^2)/a]*
ArcTanh[Sqrt[1 + (b*x^2)/a]])/(6*a*b^2*x^2*Sqrt[a + b*x^2])
```

fricas [A] time = 0.69, size = 210, normalized size = 2.10

$$\left[-\frac{3\left(b^3c-2\,ab^2d\right)\sqrt{a}\,x^2\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(2\,a^2bfx^4-3\,ab^2c+2\left(3\,a^2be-2\,a^3f\right)x^2\right)\sqrt{bx^2+a}}{12\,a^2b^2x^2},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^3*c - 2*a*b^2*d)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - 2*(2*a^2*b*f*x^4 - 3*a*b^2*c + 2*(3*a^2*b*e - 2*a^3*f)
```

```
*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2), -1/6*(3*(b^3*c - 2*a*b^2*d)*sqrt(-a)*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*a^2*b*f*x^4 - 3*a*b^2*c + 2*(3*a^
2*b*e - 2*a^3*f)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2)]
```

giac [A] time = 0.55, size = 114, normalized size = 1.14

$$-\frac{\frac{3\left(b^{2}c-2\,abd\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{3\,\sqrt{bx^{2}+a}\,bc}{ax^{2}}-\frac{2\left(\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}f-3\,\sqrt{bx^{2}+a}\,ab^{2}f+3\,\sqrt{bx^{2}+a}\,b^{3}e\right)}{b^{3}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*(b^2*c - 2*a*b*d)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 3
*sqrt(b*x^2 + a)*b*c/(a*x^2) - 2*((b*x^2 + a)^(3/2)*b^2*f - 3*sqrt(b*x^2 +
a)*a*b^2*f + 3*sqrt(b*x^2 + a)*b^3*e)/b^3)/b
```

maple [A] time = 0.01, size = 127, normalized size = 1.27

$$\frac{\sqrt{b\,x^{2}+a}\;f\,x^{2}}{3b}-\frac{d\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{bc\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{2\sqrt{b\,x^{2}+a}\;af}{3b^{2}}+\frac{\sqrt{b\,x^{2}+a}\;e}{b}-\frac{\sqrt{b\,x^{2}+a}\;c}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*f*x^2/b*(b*x^2+a)^(1/2)-2/3*f*a/b^2*(b*x^2+a)^(1/2)+e/b*(b*x^2+a)^(1/2)
-1/2*c*(b*x^2+a)^(1/2)/a/x^2+1/2*c*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)-d/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.33, size = 104, normalized size = 1.04

$$\frac{\sqrt{bx^{2}+a}\,fx^{2}}{3\,b}+\frac{bc\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{\sqrt{bx^{2}+a}\,e}{b}-\frac{2\,\sqrt{bx^{2}+a}\,af}{3\,b^{2}}-\frac{\sqrt{bx^{2}+a}\,c}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*f*x^2/b + 1/2*b*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2)
 - d*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 + a)*e/b - 2/3*sqrt
(b*x^2 + a)*a*f/b^2 - 1/2*sqrt(b*x^2 + a)*c/(a*x^2)
```

mupad [B] time = 1.95, size = 99, normalized size = 0.99

$$\frac{e\,\sqrt{b\,x^{2}+a}}{b}-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{c\,\sqrt{b\,x^{2}+a}}{2\,a\,x^{2}}+\frac{b\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{f\,\sqrt{b\,x^{2}+a}\,\left(2\,a-b\,x^{2}\right)}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (e*(a + b*x^2)^(1/2))/b - (d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (c
*(a + b*x^2)^(1/2))/(2*a*x^2) + (b*c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a
^(3/2)) - (f*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

sympy [**A**] time = 127.43, size = 138, normalized size = 1.38

$$e\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)+f\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)-\frac{\sqrt{b}\,c\sqrt{\frac{a}{bx^2}+1}}{2ax}-\frac{d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}+\frac{bc\operatorname{asin}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, True)) + f*P
iecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne(
b, 0)), (x**4/(4*sqrt(a)), True)) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(2*a*x)
- d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + b*c*asinh(sqrt(a)/(sqrt(b)*x))/(2*
a**(3/2))
```

3.148 $\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{a+bx^2}\,(3bc-4ad)}{8a^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2e-4abd+3b^2c\right)}{8a^{5/2}}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{f\sqrt{a+bx^2}}{b}$$

```
[Out] -1/8*(8*a^2*e-4*a*b*d+3*b^2*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+f*(
b*x^2+a)^(1/2)/b-1/4*c*(b*x^2+a)^(1/2)/a/x^4+1/8*(-4*a*d+3*b*c)*(b*x^2+a)^(
1/2)/a^2/x^2
```

Rubi [A] time = 0.23, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1799, 1621, 897, 1157, 388, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2e-4abd+3b^2c\right)}{8a^{5/2}}+\frac{\sqrt{a+bx^2}\,(3bc-4ad)}{8a^2x^2}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{f\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x^2])/b - (c*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*b*c - 4*a*d)*Sq
rt[a + b*x^2])/(8*a^2*x^2) - ((3*b^2*c - 4*a*b*d + 8*a^2*e)*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(3bc-4ad)-2aex-2afx^2}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} - \frac{\operatorname{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(3bc-4ad)+2a^2be-2a^3f}{b^2}-\frac{(2abe-4a^2f)x^2}{b^2}-\frac{2afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^2}\,dx, x, \sqrt{a+bx^2}\right)}{2ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}\left(-3bc+4ad-\frac{8a^2e}{b}+\frac{8a^3f}{b^2}\right)-\frac{4a^2fx^2}{b^2}}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right.}{4a^2}$$

$$= \frac{f\sqrt{a+bx^2}}{b} - \frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} + \frac{\left(3bc-4ad+\frac{8a^2e}{b}\right)\operatorname{Subst}\left(\int\right.}{8a^2}$$

$$= \frac{f\sqrt{a+bx^2}}{b} - \frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} - \frac{\left(3b^2c-4abd+8a^2e\right)\tanh^{-1}}{8a^{5/2}}$$

Mathematica [C] time = 0.37, size = 141, normalized size = 1.24

$$-\frac{b^2c\sqrt{a+bx^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^3}-\frac{bd\sqrt{a+bx^2}\left(\frac{a}{bx^2}-\frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{2a^2}-\frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{f\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x^2])/b - (e*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a] - (b*d
*Sqrt[a + b*x^2]*(a/(b*x^2) - ArcTanh[Sqrt[1 + (b*x^2)/a]]/Sqrt[1 + (b*x^2)
/a]))/(2*a^2) - (b^2*c*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (
b*x^2)/a])/a^3
```

fricas [A] time = 0.87, size = 221, normalized size = 1.94

$$\left[\frac{\left(3\,b^{3}c-4\,ab^{2}d+8\,a^{2}be\right)\sqrt{a}\,x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(8\,a^{3}fx^{4}-2\,a^{2}bc+\left(3\,ab^{2}c-4\,a^{2}bd\right)x^{2}\right)\sqrt{bx^{2}+a}}{16\,a^{3}bx^{4}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*a^3*f*x^4 - 2*a^2*b*c + (3*a*b^2*c - 4
*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4), 1/8*((3*b^3*c - 4*a*b^2*d + 8*
a^2*b*e)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*a^3*f*x^4 - 2*a
^2*b*c + (3*a*b^2*c - 4*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4)]
```

giac [A] time = 0.40, size = 141, normalized size = 1.24

$$\frac{8\,\sqrt{bx^{2}+a}\,f+\frac{\left(3\,b^{3}c-4\,ab^{2}d+8\,a^{2}be\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{3\,\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}c-5\,\sqrt{bx^{2}+a}\,ab^{3}c-4\,\left(bx^{2}+a\right)^{\frac{3}{2}}ab^{2}d+4\,\sqrt{bx^{2}+a}\,a^{2}b^{2}d}{a^{2}b^{2}x^{4}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(b*x^2 + a)*f + (3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*arctan(sqrt(b*
x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)^(3/2)*b^3*c - 5*sqrt(b*x
^2 + a)*a*b^3*c - 4*(b*x^2 + a)^(3/2)*a*b^2*d + 4*sqrt(b*x^2 + a)*a^2*b^2*d
)/(a^2*b^2*x^4))/b
```

maple [A] time = 0.01, size = 162, normalized size = 1.42

$$-\frac{e\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{bd\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^{2}c\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}+\frac{\sqrt{b\,x^{2}+a}\,f}{b}-\frac{\sqrt{b\,x^{2}+a}\,d}{2a\,x^{2}}+\frac{3\sqrt{b\,x^{2}+a}}{8a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] f*(b*x^2+a)^(1/2)/b-1/2*d/a/x^2*(b*x^2+a)^(1/2)+1/2*d*b/a^(3/2)*ln((2*a+2*(
b*x^2+a)^(1/2)*a^(1/2))/x)-1/4*c*(b*x^2+a)^(1/2)/a/x^4+3/8*c/a^2*b/x^2*(b*x
^2+a)^(1/2)-3/8*c/a^(5/2)*b^2*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-e/a^(1/
2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.38, size = 128, normalized size = 1.12

$$-\frac{3\,b^{2}c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{bd\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{e\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{\sqrt{bx^{2}+a}\,f}{b}+\frac{3\,\sqrt{bx^{2}+a}\,bc}{8\,a^{2}x^{2}}-\frac{\sqrt{bx^{2}+a}\,d}{2\,ax^{2}}-\frac{\sqrt{bx^{2}+}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*b^2*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/2*b*d*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(3/2) - e*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^
2 + a)*f/b + 3/8*sqrt(b*x^2 + a)*b*c/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*d/(a*x
^2) - 1/4*sqrt(b*x^2 + a)*c/(a*x^4)
```

mupad [B] time = 2.19, size = 133, normalized size = 1.17

$$\frac{f\sqrt{b\,x^2+a}}{b}-\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{5\,c\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,c\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{d\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{3\,b^2\,c\,\mathrm{ata}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^5*(a + b*x^2)^(1/2)),x)
```

```
[Out] (f*(a + b*x^2)^(1/2))/b - (e*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (5
*c*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*c*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (d*
(a + b*x^2)^(1/2))/(2*a*x^2) + (b*d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^
(3/2)) - (3*b^2*c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

3.149 $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{a+bx^2}\,(5bc-6ad)}{24a^2x^4}-\frac{\sqrt{a+bx^2}\,\left(8a^2e-6abd+5b^2c\right)}{16a^3x^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-16a^3f+8a^2be-6ab^2d+5b^3c\right)}{16a^{7/2}}-\frac{c\sqrt{a}}{6}$$

```
[Out] 1/16*(-16*a^3*f+8*a^2*b*e-6*a*b^2*d+5*b^3*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2
))/a^(7/2)-1/6*c*(b*x^2+a)^(1/2)/a/x^6+1/24*(-6*a*d+5*b*c)*(b*x^2+a)^(1/2)/
a^2/x^4-1/16*(8*a^2*e-6*a*b*d+5*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^2
```

Rubi [A] time = 0.28, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1799, 1621, 897, 1157, 385, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2be-16a^3f-6ab^2d+5b^3c\right)}{16a^{7/2}}-\frac{\sqrt{a+bx^2}\,\left(8a^2e-6abd+5b^2c\right)}{16a^3x^2}+\frac{\sqrt{a+bx^2}\,(5bc-6ad)}{24a^2x^4}-\frac{c\sqrt{a}}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(6*a*x^6) + ((5*b*c - 6*a*d)*Sqrt[a + b*x^2])/(24*a^2*
x^4) - ((5*b^2*c - 6*a*b*d + 8*a^2*e)*Sqrt[a + b*x^2])/(16*a^3*x^2) + ((5*b
^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(16*a^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
```

```
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc-6ad)-3aex-3afx^2}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(5bc-6ad)+3a^2be-3a^3f}{b^2} - \frac{(3abe-6a^2f)x^2}{b^2} - \frac{3afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^3}\,dx, x, \sqrt{a+bx^2}\right)}{3ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}\left(5bc-6ad+\frac{8a^2e}{b}-\frac{8a^3f}{b^2}\right)-\frac{12a^2fx^2}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^2}\,dx, \ldots}{12a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\left(5b^2c-6abd+8a^2e\right)\sqrt{a+bx^2}}{16a^3x^2} + \frac{\left(b^2\left(12\ldots\right.\right.}{\ldots}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\left(5b^2c-6abd+8a^2e\right)\sqrt{a+bx^2}}{16a^3x^2} + \frac{\left(5b^3c\ldots\right.}{\ldots}$$

Mathematica [C] time = 1.02, size = 162, normalized size = 1.11

$$\frac{b^3c\sqrt{a+bx^2}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^4} - \frac{b^2d\sqrt{a+bx^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^3} - \frac{be\sqrt{a+bx^2}\left(\frac{a}{bx^2} - \frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{2a^2} - \frac{f\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -((f*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]) - (b*e*Sqrt[a + b*x^2]*(a/(
b*x^2) - ArcTanh[Sqrt[1 + (b*x^2)/a]]/Sqrt[1 + (b*x^2)/a]))/(2*a^2) - (b^2*
```

```
d*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x^2)/a])/a^3 + (b^3
*c*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x^2)/a])/a^4
```

fricas [A] time = 0.58, size = 261, normalized size = 1.79

$$\left[-\frac{3\left(5\,b^3c-6\,ab^2d+8\,a^2be-16\,a^3f\right)\sqrt{a}\,x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(3\left(5\,ab^2c-6\,a^2bd+8\,a^3e\right)x^4+8\,a^3c-\cdots\right.}{96\,a^4x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*sqrt(a)*x^6*log(-(b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(5*a*b^2*c - 6*a^2*b*d +
 8*a^3*e)*x^4 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2 + a))/(a^
4*x^6), -1/48*(3*(5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*sqrt(-a)*x^6*
arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(5*a*b^2*c - 6*a^2*b*d + 8*a^3*e)*x^4
 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6)]
```

giac [A] time = 0.39, size = 232, normalized size = 1.59

$$-\frac{\frac{3\left(5\,b^4c-6\,ab^3d-16\,a^3bf+8\,a^2b^2e\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{15\left(bx^2+a\right)^{\frac{5}{2}}b^4c-40\left(bx^2+a\right)^{\frac{3}{2}}ab^4c+33\,\sqrt{bx^2+a}\,a^2b^4c-18\left(bx^2+a\right)^{\frac{5}{2}}ab^3d+48\left(bx^2+a\right)^{\frac{3}{2}}a^2b^3d\cdots}{a^3b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(3*(5*b^4*c - 6*a*b^3*d - 16*a^3*b*f + 8*a^2*b^2*e)*arctan(sqrt(b*x^2
 + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x^2 + a)^(5/2)*b^4*c - 40*(b*x^2 +
a)^(3/2)*a*b^4*c + 33*sqrt(b*x^2 + a)*a^2*b^4*c - 18*(b*x^2 + a)^(5/2)*a*b^
3*d + 48*(b*x^2 + a)^(3/2)*a^2*b^3*d - 30*sqrt(b*x^2 + a)*a^3*b^3*d + 24*(b
*x^2 + a)^(5/2)*a^2*b^2*e - 48*(b*x^2 + a)^(3/2)*a^3*b^2*e + 24*sqrt(b*x^2
+ a)*a^4*b^2*e)/(a^3*b^3*x^6))/b
```

maple [A] time = 0.01, size = 238, normalized size = 1.63

$$-\frac{f\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{be\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2d\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}+\frac{5b^3c\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{7}{2}}}-\frac{\sqrt{b\,x^2+a}\,e}{2a\,x^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*e/a/x^2*(b*x^2+a)^(1/2)+1/2*e*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)-1/6*c*(b*x^2+a)^(1/2)/a/x^6+5/24*c/a^2*b/x^4*(b*x^2+a)^(1/2)-5/16*c
/a^3*b^2/x^2*(b*x^2+a)^(1/2)+5/16*c/a^(7/2)*b^3*ln((2*a+2*(b*x^2+a)^(1/2)*a
^(1/2))/x)-1/4*d/a/x^4*(b*x^2+a)^(1/2)+3/8*d/a^2*b/x^2*(b*x^2+a)^(1/2)-3/8*
d/a^(5/2)*b^2*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-f/a^(1/2)*ln((2*a+2*(b*
x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.39, size = 193, normalized size = 1.32

$$\frac{5\,b^3c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{7}{2}}}-\frac{3\,b^2d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{be\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{f\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}-\frac{5\,\sqrt{bx^2+a}\,b^2c}{16\,a^3x^2}+\frac{3\,\sqrt{bx^2+a}\,\cdots}{8\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] 5/16*b^3*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 3/8*b^2*d*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/2*b*e*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - f*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - 5/16*sqrt(b*x^2 + a)*b^2*c/(a^3*x^2) + 3/8*sqrt(b*x^2 + a)*b*d/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*e/(a*x^2) + 5/24*sqrt(b*x^2 + a)*b*c/(a^2*x^4) - 1/4*sqrt(b*x^2 + a)*d/(a*x^4) - 1/6*sqrt(b*x^2 + a)*c/(a*x^6)

mupad [B] time = 2.54, size = 199, normalized size = 1.36

$$\frac{5\,c\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a^2\,x^6}-\frac{11\,c\,\sqrt{b\,x^2+a}}{16\,a\,x^6}-\frac{f\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{5\,c\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^3\,x^6}-\frac{5\,d\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,d\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{e\,\sqrt{b\,x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^7*(a + b*x^2)^(1/2)),x)

[Out] (5*c*(a + b*x^2)^(3/2))/(6*a^2*x^6) - (11*c*(a + b*x^2)^(1/2))/(16*a*x^6) - (f*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (5*c*(a + b*x^2)^(5/2))/(16*a^3*x^6) - (5*d*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*d*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (e*(a + b*x^2)^(1/2))/(2*a*x^2) + (b*e*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (b^3*c*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(7/2)) - (3*b^2*d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**7/(b*x**2+a)**(1/2),x)

[Out] Timed out

3.150 $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{a+bx^2}\,(7bc-8ad)}{48a^2x^6}-\frac{\sqrt{a+bx^2}\,\left(48a^2e-40abd+35b^2c\right)}{192a^3x^4}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-64a^3f+48a^2be-40ab^2d+35b\right.}{128a^{9/2}}$$

```
[Out] -1/128*b*(-64*a^3*f+48*a^2*b*e-40*a*b^2*d+35*b^3*c)*arctanh((b*x^2+a)^(1/2)
/a^(1/2))/a^(9/2)-1/8*c*(b*x^2+a)^(1/2)/a/x^8+1/48*(-8*a*d+7*b*c)*(b*x^2+a)
^(1/2)/a^2/x^6-1/192*(48*a^2*e-40*a*b*d+35*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^4+1
/128*(-64*a^3*f+48*a^2*b*e-40*a*b^2*d+35*b^3*c)*(b*x^2+a)^(1/2)/a^4/x^2
```

Rubi [A] time = 0.35, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1799, 1621, 897, 1157, 385, 199, 208}

$$\frac{\sqrt{a+bx^2}\,\left(48a^2be-64a^3f-40ab^2d+35b^3c\right)}{128a^4x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(48a^2be-64a^3f-40ab^2d+35b^3c\right)}{128a^{9/2}}-\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(8*a*x^8) + ((7*b*c - 8*a*d)*Sqrt[a + b*x^2])/(48*a^2*
x^6) - ((35*b^2*c - 40*a*b*d + 48*a^2*e)*Sqrt[a + b*x^2])/(192*a^3*x^4) + (
(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*Sqrt[a + b*x^2])/(128*a^4*x
^2) - (b*(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(128*a^(9/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx}}\,dx,x,x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(7bc-8ad)-4aex-4afx^2}{x^4\sqrt{a+bx}}\,dx,x,x^2\right)}{8a}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} - \frac{\operatorname{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(7bc-8ad)+4a^2be-4a^3f}{b^2}-\frac{\left(4abe-8a^2f\right)x^2}{b^2}-\frac{4afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^4}\,dx,x,\sqrt{a+bx^2}\right)}{4ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}\left(-35bc+40ad-\frac{48a^2e}{b}+\frac{48a^3f}{b^2}\right)-\frac{24a^2fx^2}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^3}\right.}{24a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} + \frac{\left(b^2\right.}{}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} + \frac{(35}{}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} + \frac{(35}{}$$

Mathematica [C] time = 0.34, size = 140, normalized size = 0.72

$$\frac{b\sqrt{a+bx^2}\left(-\frac{a^4f}{bx^2}+\frac{a^3f\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}-2a^2be\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)-2b^3c\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};\frac{bx^2}{a}+1\right)+2ab^2d\,{}_2F_1\left(\frac{1}{2},4;\right.\right.}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*(-((a^4*f)/(b*x^2)) + (a^3*f*ArcTanh[Sqrt[1 + (b*x^2)/a]
])/Sqrt[1 + (b*x^2)/a] - 2*a^2*b*e*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x^
2)/a] + 2*a*b^2*d*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x^2)/a] - 2*b^3*c*H
ypergeometric2F1[1/2, 5, 3/2, 1 + (b*x^2)/a]))/(2*a^5)
```

fricas [A] time = 1.00, size = 341, normalized size = 1.75

$$\left[-\frac{3\left(35\,b^4c-40\,ab^3d+48\,a^2b^2e-64\,a^3bf\right)\sqrt{a}\,x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(3\left(35\,ab^3c-40\,a^2b^2d+48\,a^3be\right.\right.}{768\,a^5x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(35*b^4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*sqrt(a)*x^8*
log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(3*(35*a*b^3*c - 40
*a^2*b^2*d + 48*a^3*b*e - 64*a^4*f)*x^6 - 48*a^4*c - 2*(35*a^2*b^2*c - 40*a
^3*b*d + 48*a^4*e)*x^4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2)*sqrt(b*x^2 + a))/(a^5
*x^8), 1/384*(3*(35*b^4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*sqrt(-a
)*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(35*a*b^3*c - 40*a^2*b^2*d + 48
*a^3*b*e - 64*a^4*f)*x^6 - 48*a^4*c - 2*(35*a^2*b^2*c - 40*a^3*b*d + 48*a^4
*e)*x^4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2)*sqrt(b*x^2 + a))/(a^5*x^8)]
```

giac [B] time = 0.40, size = 361, normalized size = 1.85

$$\frac{\frac{3\left(35\,b^5c-40\,ab^4d-64\,a^3b^2f+48\,a^2b^3e\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}+\frac{105\left(bx^2+a\right)^{\frac{7}{2}}b^5c-385\left(bx^2+a\right)^{\frac{5}{2}}ab^5c+511\left(bx^2+a\right)^{\frac{3}{2}}a^2b^5c-279\,\sqrt{bx^2+a}\,a^3b^5c-120\left(bx^2+a\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(3*(35*b^5*c - 40*a*b^4*d - 64*a^3*b^2*f + 48*a^2*b^3*e)*arctan(sqrt(
b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + (105*(b*x^2 + a)^(7/2)*b^5*c - 385*(b
*x^2 + a)^(5/2)*a*b^5*c + 511*(b*x^2 + a)^(3/2)*a^2*b^5*c - 279*sqrt(b*x^2
+ a)*a^3*b^5*c - 120*(b*x^2 + a)^(7/2)*a*b^4*d + 440*(b*x^2 + a)^(5/2)*a^2*
b^4*d - 584*(b*x^2 + a)^(3/2)*a^3*b^4*d + 264*sqrt(b*x^2 + a)*a^4*b^4*d - 1
92*(b*x^2 + a)^(7/2)*a^3*b^2*f + 576*(b*x^2 + a)^(5/2)*a^4*b^2*f - 576*(b*x
^2 + a)^(3/2)*a^5*b^2*f + 192*sqrt(b*x^2 + a)*a^6*b^2*f + 144*(b*x^2 + a)^(
7/2)*a^2*b^3*e - 528*(b*x^2 + a)^(5/2)*a^3*b^3*e + 624*(b*x^2 + a)^(3/2)*a^
4*b^3*e - 240*sqrt(b*x^2 + a)*a^5*b^3*e)/(a^4*b^4*x^8))/b
```

maple [A] time = 0.02, size = 320, normalized size = 1.64

$$\frac{bf\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2e\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}+\frac{5b^3d\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{7}{2}}}-\frac{35b^4c\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{128a^{\frac{9}{2}}}-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/8*c*(b*x^2+a)^(1/2)/a/x^8+7/48*c/a^2*b/x^6*(b*x^2+a)^(1/2)-35/192*c/a^3*
b^2/x^4*(b*x^2+a)^(1/2)+35/128*c/a^4*b^3/x^2*(b*x^2+a)^(1/2)-35/128*c/a^(9/
2)*b^4*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/2*f/a/x^2*(b*x^2+a)^(1/2)+1/
2*f*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/6*d/a/x^6*(b*x^2+a)^(
1/2)+5/24*d/a^2*b/x^4*(b*x^2+a)^(1/2)-5/16*d/a^3*b^2/x^2*(b*x^2+a)^(1/2)+5/
16*d/a^(7/2)*b^3*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/4*e/a/x^4*(b*x^2+a
)^(1/2)+3/8*e/a^2*b/x^2*(b*x^2+a)^(1/2)-3/8*e/a^(5/2)*b^2*ln((2*a+2*(b*x^2+
a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.35, size = 275, normalized size = 1.41

$$-\frac{35\,b^4c\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{9}{2}}}+\frac{5\,b^3d\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{7}{2}}}-\frac{3\,b^2e\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{bf\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}+\frac{35\,\sqrt{bx^2+a}\,b^3c}{128\,a^4x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -35/128*b^4*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 5/16*b^3*d*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(7/2) - 3/8*b^2*e*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/
2) + 1/2*b*f*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 35/128*sqrt(b*x^2 + a)
*b^3*c/(a^4*x^2) - 5/16*sqrt(b*x^2 + a)*b^2*d/(a^3*x^2) + 3/8*sqrt(b*x^2 +
a)*b*e/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*f/(a*x^2) - 35/192*sqrt(b*x^2 + a)*b
^2*c/(a^3*x^4) + 5/24*sqrt(b*x^2 + a)*b*d/(a^2*x^4) - 1/4*sqrt(b*x^2 + a)*e
/(a*x^4) + 7/48*sqrt(b*x^2 + a)*b*c/(a^2*x^6) - 1/6*sqrt(b*x^2 + a)*d/(a*x^
6) - 1/8*sqrt(b*x^2 + a)*c/(a*x^8)
```

mupad [B] time = 2.91, size = 277, normalized size = 1.42

$$\frac{511\,c\,\left(b\,x^2+a\right)^{3/2}}{384\,a^2\,x^8}-\frac{93\,c\,\sqrt{b\,x^2+a}}{128\,a\,x^8}-\frac{385\,c\,\left(b\,x^2+a\right)^{5/2}}{384\,a^3\,x^8}+\frac{35\,c\,\left(b\,x^2+a\right)^{7/2}}{128\,a^4\,x^8}-\frac{11\,d\,\sqrt{b\,x^2+a}}{16\,a\,x^6}+\frac{5\,d\,\left(b\,x^2+a\right)^{3/2}}{6\,a^2\,x^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^9*(a + b*x^2)^(1/2)),x)
```

```
[Out] (511*c*(a + b*x^2)^(3/2))/(384*a^2*x^8) - (93*c*(a + b*x^2)^(1/2))/(128*a*x
^8) - (385*c*(a + b*x^2)^(5/2))/(384*a^3*x^8) + (35*c*(a + b*x^2)^(7/2))/(1
28*a^4*x^8) - (11*d*(a + b*x^2)^(1/2))/(16*a*x^6) + (5*d*(a + b*x^2)^(3/2))
/(6*a^2*x^6) - (5*d*(a + b*x^2)^(5/2))/(16*a^3*x^6) - (5*e*(a + b*x^2)^(1/2
))/(8*a*x^4) + (3*e*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (f*(a + b*x^2)^(1/2))/
(2*a*x^2) + (b*f*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) + (b^4*c*ata
n(((a + b*x^2)^(1/2)*1i)/a^(1/2))*35i)/(128*a^(9/2)) - (b^3*d*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(7/2)) - (3*b^2*e*atanh((a + b*x^2)^(1/2)
/a^(1/2)))/(8*a^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**9/(b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

3.151 $\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=245

$$\frac{x^5\sqrt{a+bx^2}\,\left(63a^2f-70abe+80b^2d\right)}{480b^3}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(-63a^3f+70a^2be-80ab^2d+96b^3c\right)}{256b^{11/2}}-\frac{ax\sqrt{a+bx^2}\,\left(\cdots\right.}{\cdots}$$

```
[Out] 1/256*a^2*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*arctanh(x*b^(1/2)/(b*x
^2+a)^(1/2))/b^(11/2)-1/256*a*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*x*
(b*x^2+a)^(1/2)/b^5+1/384*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*x^3*(b
*x^2+a)^(1/2)/b^4+1/480*(63*a^2*f-70*a*b*e+80*b^2*d)*x^5*(b*x^2+a)^(1/2)/b^
3+1/80*(-9*a*f+10*b*e)*x^7*(b*x^2+a)^(1/2)/b^2+1/10*f*x^9*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.26, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1809, 1267, 459, 321, 217, 206}

$$\frac{x^3\sqrt{a+bx^2}\,\left(70a^2be-63a^3f-80ab^2d+96b^3c\right)}{384b^4}-\frac{ax\sqrt{a+bx^2}\,\left(70a^2be-63a^3f-80ab^2d+96b^3c\right)}{256b^5}+\frac{a^2\tanh^{-1}\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -(a*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x*Sqrt[a + b*x^2])/(256
*b^5) + ((96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x^3*Sqrt[a + b*x^2
])/(384*b^4) + ((80*b^2*d - 70*a*b*e + 63*a^2*f)*x^5*Sqrt[a + b*x^2])/(480*
b^3) + ((10*b*e - 9*a*f)*x^7*Sqrt[a + b*x^2])/(80*b^2) + (f*x^9*Sqrt[a + b*
x^2])/(10*b) + (a^2*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx = \frac{fx^9\sqrt{a+bx^2}}{10b} + \frac{\int \frac{x^4\left(10bc+10bdx^2+(10be-9af)x^4\right)}{\sqrt{a+bx^2}}\,dx}{10b}$$

$$= \frac{(10be-9af)x^7\sqrt{a+bx^2}}{80b^2} + \frac{fx^9\sqrt{a+bx^2}}{10b} + \frac{\int \frac{x^4\left(80b^2c+\left(80b^2d-70abe+63a^2f\right)x^2\right)}{\sqrt{a+bx^2}}\,d}{80b^2}$$

$$= \frac{\left(80b^2d-70abe+63a^2f\right)x^5\sqrt{a+bx^2}}{480b^3} + \frac{(10be-9af)x^7\sqrt{a+bx^2}}{80b^2} + \frac{fx^9\sqrt{a}}{10}$$

$$= \frac{\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x^3\sqrt{a+bx^2}}{384b^4} + \frac{\left(80b^2d-70abe+63a^2f\right.}{480b^3}$$

$$= -\frac{a\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x\sqrt{a+bx^2}}{256b^5} + \frac{\left(96b^3c-80ab^2d+70a\right.}{3}$$

$$= -\frac{a\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x\sqrt{a+bx^2}}{256b^5} + \frac{\left(96b^3c-80ab^2d+70a\right.}{3}$$

$$= -\frac{a\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x\sqrt{a+bx^2}}{256b^5} + \frac{\left(96b^3c-80ab^2d+70a\right.}{3}$$

Mathematica [A] time = 0.24, size = 184, normalized size = 0.75

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\,\left(945a^4f-210a^3b\left(5e+3fx^2\right)+4a^2b^2\left(300d+175ex^2+126fx^4\right)-16ab^3\left(90c+50dx^2+35ex\right.\right.}{38}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(945*a^4*f - 210*a^3*b*(5*e + 3*f*x^2) + 4*a^2*b
^2*(300*d + 175*e*x^2 + 126*f*x^4) + 32*b^4*x^2*(30*c + 20*d*x^2 + 15*e*x^4
 + 12*f*x^6) - 16*a*b^3*(90*c + 50*d*x^2 + 35*e*x^4 + 27*f*x^6)) - 15*a^2*(
-96*b^3*c + 80*a*b^2*d - 70*a^2*b*e + 63*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(3840*b^(11/2))
```

fricas [A] time = 0.99, size = 414, normalized size = 1.69

$$\left[-\frac{15\left(96\,a^2b^3c-80\,a^3b^2d+70\,a^4be-63\,a^5f\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(384\,b^5fx^9+48\left(10\,b^5e\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(96*a^2*b^3*c - 80*a^3*b^2*d + 70*a^4*b*e - 63*a^5*f)*sqrt(b)*
log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(384*b^5*f*x^9 + 48*(10
*b^5*e - 9*a*b^4*f)*x^7 + 8*(80*b^5*d - 70*a*b^4*e + 63*a^2*b^3*f)*x^5 + 10
*(96*b^5*c - 80*a*b^4*d + 70*a^2*b^3*e - 63*a^3*b^2*f)*x^3 - 15*(96*a*b^4*c
 - 80*a^2*b^3*d + 70*a^3*b^2*e - 63*a^4*b*f)*x)*sqrt(b*x^2 + a))/b^6, -1/38
40*(15*(96*a^2*b^3*c - 80*a^3*b^2*d + 70*a^4*b*e - 63*a^5*f)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*b^5*f*x^9 + 48*(10*b^5*e - 9*a*b^4*f)*
x^7 + 8*(80*b^5*d - 70*a*b^4*e + 63*a^2*b^3*f)*x^5 + 10*(96*b^5*c - 80*a*b^
4*d + 70*a^2*b^3*e - 63*a^3*b^2*f)*x^3 - 15*(96*a*b^4*c - 80*a^2*b^3*d + 70
*a^3*b^2*e - 63*a^4*b*f)*x)*sqrt(b*x^2 + a))/b^6]
```

giac [A] time = 0.53, size = 224, normalized size = 0.91

$$\frac{1}{3840}\left(2\left(4\left(6\left(\frac{8\,fx^2}{b}-\frac{9\,ab^7f-10\,b^8e}{b^9}\right)x^2+\frac{80\,b^8d+63\,a^2b^6f-70\,ab^7e}{b^9}\right)x^2+\frac{5\left(96\,b^8c-80\,ab^7d-63\,a^3b^5f+\right.}{b^9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*f*x^2/b - (9*a*b^7*f - 10*b^8*e)/b^9)*x^2 + (80*b^8*d +
63*a^2*b^6*f - 70*a*b^7*e)/b^9)*x^2 + 5*(96*b^8*c - 80*a*b^7*d - 63*a^3*b^5
*f + 70*a^2*b^6*e)/b^9)*x^2 - 15*(96*a*b^7*c - 80*a^2*b^6*d - 63*a^4*b^4*f
+ 70*a^3*b^5*e)/b^9)*sqrt(b*x^2 + a)*x - 1/256*(96*a^2*b^3*c - 80*a^3*b^2*d
 - 63*a^5*f + 70*a^4*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

maple [A] time = 0.02, size = 368, normalized size = 1.50

$$\frac{\sqrt{b\,x^2+a}\;f\,x^9}{10b}-\frac{9\sqrt{b\,x^2+a}\;af\,x^7}{80b^2}+\frac{\sqrt{b\,x^2+a}\;e\,x^7}{8b}+\frac{21\sqrt{b\,x^2+a}\;a^2f\,x^5}{160b^3}-\frac{7\sqrt{b\,x^2+a}\;ae\,x^5}{48b^2}+\frac{\sqrt{b\,x^2+a}\;d\,x^5}{6b}-\frac{21\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/10*f*x^9*(b*x^2+a)^(1/2)/b-9/80*f*a/b^2*x^7*(b*x^2+a)^(1/2)+21/160*f*a^2/
b^3*x^5*(b*x^2+a)^(1/2)-21/128*f*a^3/b^4*x^3*(b*x^2+a)^(1/2)+63/256*f*a^4/b
^5*x*(b*x^2+a)^(1/2)-63/256*f*a^5/b^(11/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/
8*e*x^7/b*(b*x^2+a)^(1/2)-7/48*e*a/b^2*x^5*(b*x^2+a)^(1/2)+35/192*e*a^2/b^3
*x^3*(b*x^2+a)^(1/2)-35/128*e*a^3/b^4*x*(b*x^2+a)^(1/2)+35/128*e*a^4/b^(9/2
)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/6*d*x^5/b*(b*x^2+a)^(1/2)-5/24*d*a/b^2*x^
3*(b*x^2+a)^(1/2)+5/16*d*a^2/b^3*x*(b*x^2+a)^(1/2)-5/16*d*a^3/b^(7/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))+1/4*c*x^3/b*(b*x^2+a)^(1/2)-3/8*c*a/b^2*x*(b*x^2+a
)^(1/2)+3/8*c*a^2/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.46, size = 339, normalized size = 1.38

$$\frac{\sqrt{bx^2+a}\,fx^9}{10\,b}+\frac{\sqrt{bx^2+a}\,ex^7}{8\,b}-\frac{9\,\sqrt{bx^2+a}\,afx^7}{80\,b^2}+\frac{\sqrt{bx^2+a}\,dx^5}{6\,b}-\frac{7\,\sqrt{bx^2+a}\,aex^5}{48\,b^2}+\frac{21\,\sqrt{bx^2+a}\,a^2fx^5}{160\,b^3}+\frac{\sqrt{bx^2+a}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(b*x^2 + a)*f*x^9/b + 1/8*sqrt(b*x^2 + a)*e*x^7/b - 9/80*sqrt(b*x^
2 + a)*a*f*x^7/b^2 + 1/6*sqrt(b*x^2 + a)*d*x^5/b - 7/48*sqrt(b*x^2 + a)*a*e
*x^5/b^2 + 21/160*sqrt(b*x^2 + a)*a^2*f*x^5/b^3 + 1/4*sqrt(b*x^2 + a)*c*x^3
/b - 5/24*sqrt(b*x^2 + a)*a*d*x^3/b^2 + 35/192*sqrt(b*x^2 + a)*a^2*e*x^3/b^
3 - 21/128*sqrt(b*x^2 + a)*a^3*f*x^3/b^4 - 3/8*sqrt(b*x^2 + a)*a*c*x/b^2 +
5/16*sqrt(b*x^2 + a)*a^2*d*x/b^3 - 35/128*sqrt(b*x^2 + a)*a^3*e*x/b^4 + 63/
256*sqrt(b*x^2 + a)*a^4*f*x/b^5 + 3/8*a^2*c*arcsinh(b*x/sqrt(a*b))/b^(5/2)
- 5/16*a^3*d*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 35/128*a^4*e*arcsinh(b*x/sqrt
(a*b))/b^(9/2) - 63/256*a^5*f*arcsinh(b*x/sqrt(a*b))/b^(11/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(f\,x^6+e\,x^4+d\,x^2+c\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2), x)
```

sympy [B] time = 42.12, size = 586, normalized size = 2.39

$$\frac{63a^{\frac{9}{2}}fx}{256b^5\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{7}{2}}ex}{128b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{21a^{\frac{7}{2}}fx^3}{256b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{5}{2}}dx}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{5}{2}}ex^3}{384b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{21a^{\frac{5}{2}}fx^5}{640b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a}{8b^2\sqrt{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] 63*a**(9/2)*f*x/(256*b**5*sqrt(1 + b*x**2/a)) - 35*a**(7/2)*e*x/(128*b**4*s
qrt(1 + b*x**2/a)) + 21*a**(7/2)*f*x**3/(256*b**4*sqrt(1 + b*x**2/a)) + 5*a
**(5/2)*d*x/(16*b**3*sqrt(1 + b*x**2/a)) - 35*a**(5/2)*e*x**3/(384*b**3*sqr
t(1 + b*x**2/a)) - 21*a**(5/2)*f*x**5/(640*b**3*sqrt(1 + b*x**2/a)) - 3*a**
(3/2)*c*x/(8*b**2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*d*x**3/(48*b**2*sqrt(1 +
 b*x**2/a)) + 7*a**(3/2)*e*x**5/(192*b**2*sqrt(1 + b*x**2/a)) + 3*a**(3/2)*
f*x**7/(160*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*c*x**3/(8*b*sqrt(1 + b*x**2/
a)) - sqrt(a)*d*x**5/(24*b*sqrt(1 + b*x**2/a)) - sqrt(a)*e*x**7/(48*b*sqrt(
1 + b*x**2/a)) - sqrt(a)*f*x**9/(80*b*sqrt(1 + b*x**2/a)) - 63*a**5*f*asinh
(sqrt(b)*x/sqrt(a))/(256*b**(11/2)) + 35*a**4*e*asinh(sqrt(b)*x/sqrt(a))/(1
28*b**(9/2)) - 5*a**3*d*asinh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + 3*a**2*c*a
sinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + c*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)
) + d*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + e*x**9/(8*sqrt(a)*sqrt(1 + b*x*
*2/a)) + f*x**11/(10*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.152 $\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{x^3\sqrt{a+bx^2}\left(35a^2f-40abe+48b^2d\right)}{192b^3}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(-35a^3f+40a^2be-48ab^2d+64b^3c\right)}{128b^{9/2}}+\frac{x\sqrt{a+bx^2}\,\left(-3\right.}{}$$

```
[Out] -1/128*a*(-35*a^3*f+40*a^2*b*e-48*a*b^2*d+64*b^3*c)*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))/b^(9/2)+1/128*(-35*a^3*f+40*a^2*b*e-48*a*b^2*d+64*b^3*c)*x*(b*x
^2+a)^(1/2)/b^4+1/192*(35*a^2*f-40*a*b*e+48*b^2*d)*x^3*(b*x^2+a)^(1/2)/b^3+
1/48*(-7*a*f+8*b*e)*x^5*(b*x^2+a)^(1/2)/b^2+1/8*f*x^7*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1809, 1267, 459, 321, 217, 206}

$$\frac{x\sqrt{a+bx^2}\left(40a^2be-35a^3f-48ab^2d+64b^3c\right)}{128b^4}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(40a^2be-35a^3f-48ab^2d+64b^3c\right)}{128b^{9/2}}+\frac{x^3\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] ((64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*x*Sqrt[a + b*x^2])/(128*b^
4) + ((48*b^2*d - 40*a*b*e + 35*a^2*f)*x^3*Sqrt[a + b*x^2])/(192*b^3) + ((8
*b*e - 7*a*f)*x^5*Sqrt[a + b*x^2])/(48*b^2) + (f*x^7*Sqrt[a + b*x^2])/(8*b)
 - (a*(64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(128*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx = \frac{fx^7\sqrt{a+bx^2}}{8b}+\frac{\int \frac{x^2\left(8bc+8bdx^2+(8be-7af)x^4\right)}{\sqrt{a+bx^2}}\,dx}{8b}$$

$$= \frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2}+\frac{fx^7\sqrt{a+bx^2}}{8b}+\frac{\int \frac{x^2\left(48b^2c+\left(48b^2d-40abe+35a^2f\right)x^2\right)}{\sqrt{a+bx^2}}\,dx}{48b^2}$$

$$= \frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+bx^2}}{192b^3}+\frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2}+\frac{fx^7\sqrt{a+}}{8b}$$

$$= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b}+\frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+b}}{192b^3}$$

$$= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b}+\frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+b}}{192b^3}$$

$$= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b}+\frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+b}}{192b^3}$$

Mathematica [A] time = 0.17, size = 149, normalized size = 0.77

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(35a^3f-40a^2be+48ab^2d-64b^3c\right)+\sqrt{b}\,x\sqrt{a+bx^2}\,\left(-105a^3f+10a^2b\left(12e+7fx^2\right)-8a\right.}{384b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^3*f + 10*a^2*b*(12*e + 7*f*x^2) - 8*a*b^
2*(18*d + 10*e*x^2 + 7*f*x^4) + 16*b^3*(12*c + 6*d*x^2 + 4*e*x^4 + 3*f*x^6)
) + 3*a*(-64*b^3*c + 48*a*b^2*d - 40*a^2*b*e + 35*a^3*f)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(384*b^(9/2))
```

fricas [A] time = 0.79, size = 329, normalized size = 1.70

$$\left[-\frac{3\left(64\,ab^3c-48\,a^2b^2d+40\,a^3be-35\,a^4f\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(48\,b^4fx^7+8\left(8\,b^4e-7\,a\right.\right.}{768\,b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*e - 35*a^4*f)*sqrt(b)*log(
-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*b^4*f*x^7 + 8*(8*b^4*e
- 7*a*b^3*f)*x^5 + 2*(48*b^4*d - 40*a*b^3*e + 35*a^2*b^2*f)*x^3 + 3*(64*b^4
*c - 48*a*b^3*d + 40*a^2*b^2*e - 35*a^3*b*f)*x)*sqrt(b*x^2 + a))/b^5, 1/384
*(3*(64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*e - 35*a^4*f)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) + (48*b^4*f*x^7 + 8*(8*b^4*e - 7*a*b^3*f)*x^5 + 2*
(48*b^4*d - 40*a*b^3*e + 35*a^2*b^2*f)*x^3 + 3*(64*b^4*c - 48*a*b^3*d + 40*
a^2*b^2*e - 35*a^3*b*f)*x)*sqrt(b*x^2 + a))/b^5]
```

giac [A] time = 0.54, size = 175, normalized size = 0.90

$$\frac{1}{384}\left(2\left(4\left(\frac{6\,fx^2}{b}-\frac{7\,ab^5f-8\,b^6e}{b^7}\right)x^2+\frac{48\,b^6d+35\,a^2b^4f-40\,ab^5e}{b^7}\right)x^2+\frac{3\left(64\,b^6c-48\,ab^5d-35\,a^3b^3f+40\,a^2\right.}{b^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*f*x^2/b - (7*a*b^5*f - 8*b^6*e)/b^7)*x^2 + (48*b^6*d + 35*a^
2*b^4*f - 40*a*b^5*e)/b^7)*x^2 + 3*(64*b^6*c - 48*a*b^5*d - 35*a^3*b^3*f +
40*a^2*b^4*e)/b^7)*sqrt(b*x^2 + a)*x + 1/128*(64*a*b^3*c - 48*a^2*b^2*d - 3
5*a^4*f + 40*a^3*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

maple [A] time = 0.01, size = 284, normalized size = 1.46

$$\frac{\sqrt{b\,x^2+a}\;f\,x^7}{8b}-\frac{7\sqrt{b\,x^2+a}\;af\,x^5}{48b^2}+\frac{\sqrt{b\,x^2+a}\;e\,x^5}{6b}+\frac{35\sqrt{b\,x^2+a}\;a^2f\,x^3}{192b^3}-\frac{5\sqrt{b\,x^2+a}\;ae\,x^3}{24b^2}+\frac{\sqrt{b\,x^2+a}\;d\,x^3}{4b}+\frac{35a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*f*x^7*(b*x^2+a)^(1/2)/b-7/48*f*a/b^2*x^5*(b*x^2+a)^(1/2)+35/192*f*a^2/b
^3*x^3*(b*x^2+a)^(1/2)-35/128*f*a^3/b^4*x*(b*x^2+a)^(1/2)+35/128*f*a^4/b^(9
/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/6*e*x^5/b*(b*x^2+a)^(1/2)-5/24*e*a/b^2*
x^3*(b*x^2+a)^(1/2)+5/16*e*a^2/b^3*x*(b*x^2+a)^(1/2)-5/16*e*a^3/b^(7/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*d*x^3/b*(b*x^2+a)^(1/2)-3/8*d*a/b^2*x*(b*x^2
+a)^(1/2)+3/8*d*a^2/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*c*x/b*(b*x^2+
a)^(1/2)-1/2*c*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.39, size = 255, normalized size = 1.31

$$\frac{\sqrt{bx^2+a}\,fx^7}{8\,b}+\frac{\sqrt{bx^2+a}\,ex^5}{6\,b}-\frac{7\,\sqrt{bx^2+a}\,afx^5}{48\,b^2}+\frac{\sqrt{bx^2+a}\,dx^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}\,aex^3}{24\,b^2}+\frac{35\,\sqrt{bx^2+a}\,a^2fx^3}{192\,b^3}+\frac{\sqrt{bx^2+a}\,}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*f*x^7/b + 1/6*sqrt(b*x^2 + a)*e*x^5/b - 7/48*sqrt(b*x^2
 + a)*a*f*x^5/b^2 + 1/4*sqrt(b*x^2 + a)*d*x^3/b - 5/24*sqrt(b*x^2 + a)*a*e*
x^3/b^2 + 35/192*sqrt(b*x^2 + a)*a^2*f*x^3/b^3 + 1/2*sqrt(b*x^2 + a)*c*x/b
- 3/8*sqrt(b*x^2 + a)*a*d*x/b^2 + 5/16*sqrt(b*x^2 + a)*a^2*e*x/b^3 - 35/128
*sqrt(b*x^2 + a)*a^3*f*x/b^4 - 1/2*a*c*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8
*a^2*d*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/16*a^3*e*arcsinh(b*x/sqrt(a*b))/b
^(7/2) + 35/128*a^4*f*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(f\,x^6+e\,x^4+d\,x^2+c\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2), x)
```

sympy [B] time = 32.56, size = 444, normalized size = 2.29

$$-\frac{35a^{\frac{7}{2}}fx}{128b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{5}{2}}ex}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{5}{2}}fx^3}{384b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}dx}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}ex^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{7a^{\frac{3}{2}}fx^5}{192b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,cx\sqrt{\cdots}}{2b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] -35*a**(7/2)*f*x/(128*b**4*sqrt(1 + b*x**2/a)) + 5*a**(5/2)*e*x/(16*b**3*sq
rt(1 + b*x**2/a)) - 35*a**(5/2)*f*x**3/(384*b**3*sqrt(1 + b*x**2/a)) - 3*a*
*(3/2)*d*x/(8*b**2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*e*x**3/(48*b**2*sqrt(1
+ b*x**2/a)) + 7*a**(3/2)*f*x**5/(192*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*c*
x*sqrt(1 + b*x**2/a)/(2*b) - sqrt(a)*d*x**3/(8*b*sqrt(1 + b*x**2/a)) - sqrt
(a)*e*x**5/(24*b*sqrt(1 + b*x**2/a)) - sqrt(a)*f*x**7/(48*b*sqrt(1 + b*x**2
/a)) + 35*a**4*f*asinh(sqrt(b)*x/sqrt(a))/(128*b**(9/2)) - 5*a**3*e*asinh(s
qrt(b)*x/sqrt(a))/(16*b**(7/2)) + 3*a**2*d*asinh(sqrt(b)*x/sqrt(a))/(8*b**(
5/2)) - a*c*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) + d*x**5/(4*sqrt(a)*sqrt(
1 + b*x**2/a)) + e*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + f*x**9/(8*sqrt(a)*
sqrt(1 + b*x**2/a))
```

3.153 $\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=145

$$\frac{x\sqrt{a+bx^2}\left(5a^2f-6abe+8b^2d\right)}{16b^3}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(-5a^3f+6a^2be-8ab^2d+16b^3c\right)}{16b^{7/2}}+\frac{x^3\sqrt{a+bx^2}\,(6be-5af)}{24b^2}+$$

```
[Out] 1/16*(-5*a^3*f+6*a^2*b*e-8*a*b^2*d+16*b^3*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))/b^(7/2)+1/16*(5*a^2*f-6*a*b*e+8*b^2*d)*x*(b*x^2+a)^(1/2)/b^3+1/24*(-5*
a*f+6*b*e)*x^3*(b*x^2+a)^(1/2)/b^2+1/6*f*x^5*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1815, 1159, 388, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(6a^2be-5a^3f-8ab^2d+16b^3c\right)}{16b^{7/2}}+\frac{x\sqrt{a+bx^2}\left(5a^2f-6abe+8b^2d\right)}{16b^3}+\frac{x^3\sqrt{a+bx^2}\,(6be-5af)}{24b^2}+\frac{f}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] ((8*b^2*d - 6*a*b*e + 5*a^2*f)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*b*e - 5*a*
f)*x^3*Sqrt[a + b*x^2])/(24*b^2) + (f*x^5*Sqrt[a + b*x^2])/(6*b) + ((16*b^3
*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(16*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
```

```
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx = \frac{fx^5\sqrt{a+bx^2}}{6b} + \frac{\int \frac{6bc+6bdx^2+(6be-5af)x^4}{\sqrt{a+bx^2}}\,dx}{6b}$$

$$= \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} + \frac{\int \frac{24b^2c+3\left(8b^2d-6abe+5a^2f\right)x^2}{\sqrt{a+bx^2}}\,dx}{24b^2}$$

$$= \frac{\left(8b^2d-6abe+5a^2f\right)x\sqrt{a+bx^2}}{16b^3} + \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} - \frac{1}{1}$$

$$= \frac{\left(8b^2d-6abe+5a^2f\right)x\sqrt{a+bx^2}}{16b^3} + \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} - \frac{1}{1}$$

$$= \frac{\left(8b^2d-6abe+5a^2f\right)x\sqrt{a+bx^2}}{16b^3} + \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} + $$

Mathematica [A] time = 0.11, size = 118, normalized size = 0.81

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\left(15a^2f-2ab\left(9e+5fx^2\right)+4b^2\left(6d+3ex^2+2fx^4\right)\right)+3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(-5a^3f+6a^2be-8a\right.}{48b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^2*f - 2*a*b*(9*e + 5*f*x^2) + 4*b^2*(6*d +
 3*e*x^2 + 2*f*x^4)) + 3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(48*b^(7/2))
```

fricas [A] time = 0.70, size = 250, normalized size = 1.72

$$\left[-\frac{3\left(16\,b^3c-8\,ab^2d+6\,a^2be-5\,a^3f\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(8\,b^3fx^5+2\left(6\,b^3e-5\,ab^2f\right)\right.}{96\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*sqrt(b)*log(-2*b*x^2
 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*b^3*f*x^5 + 2*(6*b^3*e - 5*a*b^2
*f)*x^3 + 3*(8*b^3*d - 6*a*b^2*e + 5*a^2*b*f)*x)*sqrt(b*x^2 + a))/b^4, -1/4
8*(3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*sqrt(-b)*arctan(sqrt(-b)*
x/sqrt(b*x^2 + a)) - (8*b^3*f*x^5 + 2*(6*b^3*e - 5*a*b^2*f)*x^3 + 3*(8*b^3*
d - 6*a*b^2*e + 5*a^2*b*f)*x)*sqrt(b*x^2 + a))/b^4]
```

giac [A] time = 0.56, size = 129, normalized size = 0.89

$$\frac{1}{48}\left(2\left(\frac{4\,fx^2}{b}-\frac{5\,ab^3f-6\,b^4e}{b^5}\right)x^2+\frac{3\left(8\,b^4d+5\,a^2b^2f-6\,ab^3e\right)}{b^5}\right)\sqrt{bx^2+a}\,x-\frac{\left(16\,b^3c-8\,ab^2d-5\,a^3f+6\,a^2b\right.}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*f*x^2/b - (5*a*b^3*f - 6*b^4*e)/b^5)*x^2 + 3*(8*b^4*d + 5*a^2*b^
2*f - 6*a*b^3*e)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3*c - 8*a*b^2*d - 5*a^
3*f + 6*a^2*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.01, size = 203, normalized size = 1.40

$$\frac{\sqrt{b\,x^2+a}\;f\,x^5}{6b}-\frac{5\sqrt{b\,x^2+a}\;af\,x^3}{24b^2}+\frac{\sqrt{b\,x^2+a}\;e\,x^3}{4b}-\frac{5a^3f\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{7}{2}}}+\frac{3a^2e\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*f*x^5*(b*x^2+a)^(1/2)/b-5/24*f*a/b^2*x^3*(b*x^2+a)^(1/2)+5/16*f*a^2/b^3
*x*(b*x^2+a)^(1/2)-5/16*f*a^3/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*e*x
^3/b*(b*x^2+a)^(1/2)-3/8*e*a/b^2*x*(b*x^2+a)^(1/2)+3/8*e*a^2/b^(5/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))+1/2*d*x/b*(b*x^2+a)^(1/2)-1/2*d*a/b^(3/2)*ln(b^(1/2
)*x+(b*x^2+a)^(1/2))+c*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.35, size = 174, normalized size = 1.20

$$\frac{\sqrt{bx^2+a}\,fx^5}{6\,b}+\frac{\sqrt{bx^2+a}\,ex^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}\,afx^3}{24\,b^2}+\frac{\sqrt{bx^2+a}\,dx}{2\,b}-\frac{3\,\sqrt{bx^2+a}\,aex}{8\,b^2}+\frac{5\,\sqrt{bx^2+a}\,a^2fx}{16\,b^3}+\frac{c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*f*x^5/b + 1/4*sqrt(b*x^2 + a)*e*x^3/b - 5/24*sqrt(b*x^2
 + a)*a*f*x^3/b^2 + 1/2*sqrt(b*x^2 + a)*d*x/b - 3/8*sqrt(b*x^2 + a)*a*e*x/b
^2 + 5/16*sqrt(b*x^2 + a)*a^2*f*x/b^3 + c*arcsinh(b*x/sqrt(a*b))/sqrt(b) -
1/2*a*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*a^2*e*arcsinh(b*x/sqrt(a*b))/b
^(5/2) - 5/16*a^3*f*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^6+e\,x^4+d\,x^2+c}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^(1/2), x)
```

sympy [A] time = 13.71, size = 362, normalized size = 2.50

$$\frac{5a^{\frac{5}{2}}fx}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}ex}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}fx^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,dx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}\,ex^3}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,fx^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^3f\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*f*x/(16*b**3*sqrt(1 + b*x**2/a)) - 3*a**(3/2)*e*x/(8*b**2*sqrt(1
 + b*x**2/a)) + 5*a**(3/2)*f*x**3/(48*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*d*
x*sqrt(1 + b*x**2/a)/(2*b) - sqrt(a)*e*x**3/(8*b*sqrt(1 + b*x**2/a)) - sqrt
(a)*f*x**5/(24*b*sqrt(1 + b*x**2/a)) - 5*a**3*f*asinh(sqrt(b)*x/sqrt(a))/(1
6*b**(7/2)) + 3*a**2*e*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) - a*d*asinh(sq
rt(b)*x/sqrt(a))/(2*b**(3/2)) + c*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/
sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0)
 & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0)))
+ e*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + f*x**7/(6*sqrt(a)*sqrt(1 + b*x**2
/a))
```

3.154 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=117

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}\,(4be-3af)}{8b^2}-\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}$$

```
[Out] 1/8*(3*a^2*f-4*a*b*e+8*b^2*d)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-c*
(b*x^2+a)^(1/2)/a/x+1/8*(-3*a*f+4*b*e)*x*(b*x^2+a)^(1/2)/b^2+1/4*f*x^3*(b*x
^2+a)^(1/2)/b
```

Rubi [A] time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1807, 1585, 1159, 388, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}\,(4be-3af)}{8b^2}-\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((c*Sqrt[a + b*x^2])/(a*x)) + ((4*b*e - 3*a*f)*x*Sqrt[a + b*x^2])/(8*b^2)
+ (f*x^3*Sqrt[a + b*x^2])/(4*b) + ((8*b^2*d - 4*a*b*e + 3*a^2*f)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
```

```
Q[r - p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx &= -\frac{c\sqrt{a+bx^2}}{ax} - \frac{\int \frac{-adx-aex^3-afx^5}{x\sqrt{a+bx^2}}\,dx}{a} \\
&= -\frac{c\sqrt{a+bx^2}}{ax} - \frac{\int \frac{-ad-aex^2-afx^4}{\sqrt{a+bx^2}}\,dx}{a} \\
&= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{fx^3\sqrt{a+bx^2}}{4b} - \frac{\int \frac{-4abd-a(4be-3af)x^2}{\sqrt{a+bx^2}}\,dx}{4ab} \\
&= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2} + \frac{fx^3\sqrt{a+bx^2}}{4b} + \frac{\left(8ab^2d-a^2(4be-3af\right.}{8ab^2} \\
&= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2} + \frac{fx^3\sqrt{a+bx^2}}{4b} + \frac{\left(8ab^2d-a^2(4be-3af\right.}{} \\
&= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2} + \frac{fx^3\sqrt{a+bx^2}}{4b} + \frac{\left(8b^2d-4abe+3a^2f\right)\,\mathrm{t}}{8b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 103, normalized size = 0.88

$$\frac{\frac{\sqrt{b}\sqrt{a+bx^2}\left(-3a^2fx^2+2abx^2\left(2e+fx^2\right)-8b^2c\right)}{ax}+\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] ((Sqrt[b]*Sqrt[a + b*x^2]*(-8*b^2*c - 3*a^2*f*x^2 + 2*a*b*x^2*(2*e + f*x^2)
))/(a*x) + (8*b^2*d - 4*a*b*e + 3*a^2*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(8*b^(5/2))
```

fricas [A] time = 0.61, size = 216, normalized size = 1.85

$$\left[\frac{\left(8\,ab^2d-4\,a^2be+3\,a^3f\right)\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,ab^2fx^4-8\,b^3c+\left(4\,ab^2e-3\,a^2bf\right)x^2\right.}{16\,ab^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*a*b^2*d - 4*a^2*b*e + 3*a^3*f)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) + 2*(2*a*b^2*f*x^4 - 8*b^3*c + (4*a*b^2*e - 3*a^2*b
```

```
*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x), -1/8*((8*a*b^2*d - 4*a^2*b*e + 3*a^3*f
)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*a*b^2*f*x^4 - 8*b^3*c
+ (4*a*b^2*e - 3*a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x)]
```

giac [A] time = 0.52, size = 121, normalized size = 1.03

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,fx^2}{b}-\frac{3\,abf-4\,b^2e}{b^3}\right)x+\frac{2\,\sqrt{b}\,c}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}-\frac{\left(8\,b^{\frac{5}{2}}d+3\,a^2\sqrt{b}\,f-4\,ab^{\frac{3}{2}}e\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2\right.\right.}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*f*x^2/b - (3*a*b*f - 4*b^2*e)/b^3)*x + 2*sqrt(b)*c/(
(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) - 1/16*(8*b^(5/2)*d + 3*a^2*sqrt(b)*f
- 4*a*b^(3/2)*e)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^3
```

maple [A] time = 0.01, size = 140, normalized size = 1.20

$$\frac{\sqrt{b\,x^2+a}\;f\,x^3}{4b}+\frac{3a^2f\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}-\frac{ae\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{d\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{3\sqrt{b\,x^2+a}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*f*x^3*(b*x^2+a)^(1/2)/b-3/8*f*a/b^2*x*(b*x^2+a)^(1/2)+3/8*f*a^2/b^(5/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*e*x/b*(b*x^2+a)^(1/2)-1/2*e*a/b^(3/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))+d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-c*(b*x^
2+a)^(1/2)/a/x
```

maxima [A] time = 1.26, size = 118, normalized size = 1.01

$$\frac{\sqrt{bx^2+a}\,fx^3}{4\,b}+\frac{\sqrt{bx^2+a}\,ex}{2\,b}-\frac{3\,\sqrt{bx^2+a}\,afx}{8\,b^2}+\frac{d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{ae\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{3\,a^2f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*f*x^3/b + 1/2*sqrt(b*x^2 + a)*e*x/b - 3/8*sqrt(b*x^2 +
a)*a*f*x/b^2 + d*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*e*arcsinh(b*x/sqrt(
a*b))/b^(3/2) + 3/8*a^2*f*arcsinh(b*x/sqrt(a*b))/b^(5/2) - sqrt(b*x^2 + a)*
c/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^6+e\,x^4+d\,x^2+c}{x^2\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^(1/2)), x)
```

sympy [A] time = 9.05, size = 250, normalized size = 2.14

$$-\frac{3a^{\frac{3}{2}}fx}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,ex\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}\,fx^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2f\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{ae\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+d\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\,\mathrm{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{fo}\\ \frac{\sqrt{\frac{a}{b}}\,\mathrm{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{fo}\\ \frac{\sqrt{-\frac{a}{b}}\,\mathrm{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{fo}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*f*x/(8*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*e*x*sqrt(1 + b*x**2/a
)/(2*b) - sqrt(a)*f*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*a**2*f*asinh(sqrt(b)*
x/sqrt(a))/(8*b**(5/2)) - a*e*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) + d*Pie
cewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/
b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt
(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/a +
f*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.155 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{a+bx^2}\,(2bc-3ad)}{3a^2x}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{fx\sqrt{a+bx^2}}{2b}$$

```
[Out] 1/2*(-a*f+2*b*e)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/3*c*(b*x^2+a)
^(1/2)/a/x^3+1/3*(-3*a*d+2*b*c)*(b*x^2+a)^(1/2)/a^2/x+1/2*f*x*(b*x^2+a)^(1/
2)/b
```

Rubi [A] time = 0.13, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1807, 1585, 1265, 388, 217, 206}

$$\frac{\sqrt{a+bx^2}\,(2bc-3ad)}{3a^2x}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{fx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(3*a*x^3) + ((2*b*c - 3*a*d)*Sqrt[a + b*x^2])/(3*a^2*x
) + (f*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*e - a*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
```

```
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx &= -\frac{c\sqrt{a+bx^2}}{3ax^3}-\frac{\int \frac{(2bc-3ad)x-3aex^3-3afx^5}{x^3\sqrt{a+bx^2}}\,dx}{3a}\\
&= -\frac{c\sqrt{a+bx^2}}{3ax^3}-\frac{\int \frac{2bc-3ad-3aex^2-3afx^4}{x^2\sqrt{a+bx^2}}\,dx}{3a}\\
&= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{\int \frac{3a^2e+3a^2fx^2}{\sqrt{a+bx^2}}\,dx}{3a^2}\\
&= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\
&= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\right.}{2b}\\
&= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx}}\right.}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 93, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\left(3a^2fx^4-2ab\left(c+3dx^2\right)+4b^2cx^2\right)}{6a^2bx^3}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(4*b^2*c*x^2 + 3*a^2*f*x^4 - 2*a*b*(c + 3*d*x^2)))/(6*a^2*
b*x^3) + ((2*b*e - a*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

fricas [A] time = 0.79, size = 210, normalized size = 1.91

$$\left[-\frac{3\left(2a^2be-a^3f\right)\sqrt{b}\,x^3\log\left(-2bx^2+2\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(3a^2bfx^4-2ab^2c+2\left(2b^3c-3ab^2d\right)x^2\right)\sqrt{bx}}{12a^2b^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2*b*e - a^3*f)*sqrt(b)*x^3*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) - 2*(3*a^2*b*f*x^4 - 2*a*b^2*c + 2*(2*b^3*c - 3*a*b^2*d)*x^2
```

```
)*sqrt(b*x^2 + a))/(a^2*b^2*x^3), -1/6*(3*(2*a^2*b*e - a^3*f)*sqrt(-b)*x^3*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*a^2*b*f*x^4 - 2*a*b^2*c + 2*(2*b^3*
c - 3*a*b^2*d)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^3)]
```

giac [A] time = 0.56, size = 176, normalized size = 1.60

$$\frac{\sqrt{bx^2+a}\,fx}{2\,b}+\frac{\left(a\sqrt{b}\,f-2\,b^{\frac{3}{2}}e\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{4\,b^2}+\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4\sqrt{b}\,d+6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)\right.}{3\left(\left(\sqrt{b}\,x-\sqrt{\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*f*x/b + 1/4*(a*sqrt(b)*f - 2*b^(3/2)*e)*log((sqrt(b)*x
- sqrt(b*x^2 + a))^2)/b^2 + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*sqrt(b)*
d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c - 6*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a*sqrt(b)*d - 2*a*b^(3/2)*c + 3*a^2*sqrt(b)*d)/((sqrt(b)*x - sqrt(
b*x^2 + a))^2 - a)^3
```

maple [A] time = 0.01, size = 117, normalized size = 1.06

$$-\frac{af\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{e\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\;fx}{2b}-\frac{\sqrt{b\,x^2+a}\;d}{ax}+\frac{2\sqrt{b\,x^2+a}\;bc}{3a^2x}-\frac{\sqrt{b\,x^2+a}\;c}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(b*x^2+a)^(1/2)/b-1/2*f*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+e*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-1/3*c*(b*x^2+a)^(1/2)/a/x^3+2/3*c*b/a^
2/x*(b*x^2+a)^(1/2)-d/a/x*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 102, normalized size = 0.93

$$\frac{\sqrt{bx^2+a}\,fx}{2\,b}+\frac{e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{af\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{2\,\sqrt{bx^2+a}\,bc}{3\,a^2x}-\frac{\sqrt{bx^2+a}\,d}{ax}-\frac{\sqrt{bx^2+a}\,c}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*f*x/b + e*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*f*arcs
inh(b*x/sqrt(a*b))/b^(3/2) + 2/3*sqrt(b*x^2 + a)*b*c/(a^2*x) - sqrt(b*x^2 +
 a)*d/(a*x) - 1/3*sqrt(b*x^2 + a)*c/(a*x^3)
```

mupad [B] time = 2.20, size = 143, normalized size = 1.30

$$\begin{cases} -\frac{-f\,x^6-3\,e\,x^4+3\,d\,x^2+c}{3\,\sqrt{a}\,x^3} & \text{if } b=0 \\ \frac{e\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{d\,\sqrt{b\,x^2+a}}{a\,x}-\frac{a\,f\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{f\,x\,\sqrt{b\,x^2+a}}{2\,b}-\frac{c\,\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^(1/2)),x)
```

```
[Out] piecewise(b == 0, -(c + 3*d*x^2 - 3*e*x^4 - f*x^6)/(3*a^(1/2)*x^3), b ~= 0,
 (e*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (d*(a + b*x^2)^(1/2))/(a*
x) - (a*f*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (f*x*(a + b
*x^2)^(1/2))/(2*b) - (c*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3))
```

sympy [A] time = 4.73, size = 197, normalized size = 1.79

$$\frac{\sqrt{a}fx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{af\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+e\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{3ax^2}-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x*sqrt(1 + b*x**2/a)/(2*b) - a*f*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) + e*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b <
0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)
*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*c*sqrt(a/(b*x*
*2) + 1)/(3*a*x**2) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*c*sqrt(
a/(b*x**2) + 1)/(3*a**2)
```

3.156 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=118

$$\frac{\sqrt{a+bx^2}\,(4bc-5ad)}{15a^2x^3}-\frac{\sqrt{a+bx^2}\,\left(15a^2e-10abd+8b^2c\right)}{15a^3x}-\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] f*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)-1/5*c*(b*x^2+a)^(1/2)/a/x^5+1/
15*(-5*a*d+4*b*c)*(b*x^2+a)^(1/2)/a^2/x^3-1/15*(15*a^2*e-10*a*b*d+8*b^2*c)*
(b*x^2+a)^(1/2)/a^3/x
```

Rubi [A] time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1807, 1585, 1265, 451, 217, 206}

$$-\frac{\sqrt{a+bx^2}\,\left(15a^2e-10abd+8b^2c\right)}{15a^3x}+\frac{\sqrt{a+bx^2}\,(4bc-5ad)}{15a^2x^3}-\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(5*a*x^5) + ((4*b*c - 5*a*d)*Sqrt[a + b*x^2])/(15*a^2*
x^3) - ((8*b^2*c - 10*a*b*d + 15*a^2*e)*Sqrt[a + b*x^2])/(15*a^3*x) + (f*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{5ax^5} - \frac{\int \frac{(4bc-5ad)x-5aex^3-5afx^5}{x^5\sqrt{a+bx^2}}\,dx}{5a}$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} - \frac{\int \frac{4bc-5ad-5aex^2-5afx^4}{x^4\sqrt{a+bx^2}}\,dx}{5a}$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} + \frac{\int \frac{8b^2c-10abd+15a^2e+15a^2fx^2}{x^2\sqrt{a+bx^2}}\,dx}{15a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{\left(8b^2c-10abd+15a^2e\right)\sqrt{a+bx^2}}{15a^3x} + f\int$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{\left(8b^2c-10abd+15a^2e\right)\sqrt{a+bx^2}}{15a^3x} + f\,\mathrm{Su}$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{\left(8b^2c-10abd+15a^2e\right)\sqrt{a+bx^2}}{15a^3x} + \frac{f\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.11, size = 95, normalized size = 0.81

$$\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{\sqrt{a+bx^2}\left(a^2\left(3c+5dx^2+15ex^4\right)-2abx^2\left(2c+5dx^2\right)+8b^2cx^4\right)}{15a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/15*(Sqrt[a + b*x^2]*(8*b^2*c*x^4 - 2*a*b*x^2*(2*c + 5*d*x^2) + a^2*(3*c
+ 5*d*x^2 + 15*e*x^4)))/(a^3*x^5) + (f*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/Sqrt[b]
```

fricas [A] time = 0.79, size = 221, normalized size = 1.87

$$\left[\frac{15\,a^3\sqrt{b}\,fx^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(\left(8\,b^3c-10\,ab^2d+15\,a^2be\right)x^4+3\,a^2bc-\left(4\,ab^2c-5\,a^2bc\right.\right.}{30\,a^3bx^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^3*sqrt(b)*f*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 - 2*((8*b^3*c - 10*a*b^2*d + 15*a^2*b*e)*x^4 + 3*a^2*b*c - (4*a*b^2*c - 5*
a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5), -1/15*(15*a^3*sqrt(-b)*f*x^5*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((8*b^3*c - 10*a*b^2*d + 15*a^2*b*e)*x^4
 + 3*a^2*b*c - (4*a*b^2*c - 5*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5)]
```

giac [B] time = 0.60, size = 324, normalized size = 2.75

$$-\frac{f\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\sqrt{b}}+\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8\sqrt{b}e+30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 b^{\frac{3}{2}}d-60\left(\sqrt{b}x-\sqrt{bx^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*f*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2/15*(15*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*sqrt(b)*e + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(3/2)*
d - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*sqrt(b)*e + 80*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*b^(5/2)*c - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(3/2)*d + 9
0*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*sqrt(b)*e - 40*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a*b^(5/2)*c + 50*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2)*d -
60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*e + 8*a^2*b^(5/2)*c - 10*a^3
*b^(3/2)*d + 15*a^4*sqrt(b)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

maple [A] time = 0.01, size = 136, normalized size = 1.15

$$\frac{f\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{\sqrt{b\,x^2+a}\;e}{ax}+\frac{2\sqrt{b\,x^2+a}\;bd}{3a^2x}-\frac{8\sqrt{b\,x^2+a}\;b^2c}{15a^3x}-\frac{\sqrt{b\,x^2+a}\;d}{3a\,x^3}+\frac{4\sqrt{b\,x^2+a}\;bc}{15a^2x^3}-\frac{\sqrt{b\,x^2+a}}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x)
```

```
[Out] f*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-1/5*c*(b*x^2+a)^(1/2)/a/x^5+4/15*c/
a^2*b/x^3*(b*x^2+a)^(1/2)-8/15*c/a^3*b^2/x*(b*x^2+a)^(1/2)-1/3*d/a/x^3*(b*x
^2+a)^(1/2)+2/3*d*b/a^2/x*(b*x^2+a)^(1/2)-e/a/x*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 128, normalized size = 1.08

$$\frac{f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{8\sqrt{bx^2+a}\,b^2c}{15\,a^3x}+\frac{2\sqrt{bx^2+a}\,bd}{3\,a^2x}-\frac{\sqrt{bx^2+a}\,e}{ax}+\frac{4\sqrt{bx^2+a}\,bc}{15\,a^2x^3}-\frac{\sqrt{bx^2+a}\,d}{3\,ax^3}-\frac{\sqrt{bx^2+a}\,c}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] f*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 8/15*sqrt(b*x^2 + a)*b^2*c/(a^3*x) + 2/3
*sqrt(b*x^2 + a)*b*d/(a^2*x) - sqrt(b*x^2 + a)*e/(a*x) + 4/15*sqrt(b*x^2 +
a)*b*c/(a^2*x^3) - 1/3*sqrt(b*x^2 + a)*d/(a*x^3) - 1/5*sqrt(b*x^2 + a)*c/(a
*x^5)
```

mupad [B] time = 1.72, size = 105, normalized size = 0.89

$$\frac{f\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{e\,\sqrt{b\,x^2+a}}{a\,x}-\frac{d\,\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3}-\frac{c\,\sqrt{b\,x^2+a}\,\left(3\,a^2-4\,a\,b\,x^2+8\,b^2\,x^4\right)}{15\,a^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^(1/2)),x)
```

```
[Out] (f*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (e*(a + b*x^2)^(1/2))/(a*x
) - (d*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3) - (c*(a + b*x^2)^(1/2)*
(3*a^2 + 8*b^2*x^4 - 4*a*b*x^2))/(15*a^3*x^5)
```

sympy [A] time = 6.21, size = 456, normalized size = 3.86

$$-\frac{3a^4b^{\frac{9}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{2a^3b^{\frac{11}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{3a^2b^{\frac{13}{2}}cx^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{\ }{15a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**4*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x
**6 + 15*a**3*b**6*x**8) - 2*a**3*b**(11/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(15
*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*a**2*b**(13/2)
*c*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a*
*3*b**6*x**8) - 12*a*b**(15/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x*
*4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*b**(17/2)*c*x**8*sqrt(a/(b*
x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + f*
Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt
(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*s
qrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(
3*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*d*sqrt(a/(b*x**2)
 + 1)/(3*a**2)
```

3.157 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{a+bx^2}\,(6bc-7ad)}{35a^2x^5}-\frac{\sqrt{a+bx^2}\,\left(35a^2e-28abd+24b^2c\right)}{105a^3x^3}+\frac{\sqrt{a+bx^2}\,\left(-105a^3f+70a^2be-56ab^2d+48b^3c\right)}{105a^4x}-c\ \cdots$$

```
[Out] -1/7*c*(b*x^2+a)^(1/2)/a/x^7+1/35*(-7*a*d+6*b*c)*(b*x^2+a)^(1/2)/a^2/x^5-1/
105*(35*a^2*e-28*a*b*d+24*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^3+1/105*(-105*a^3*f+
70*a^2*b*e-56*a*b^2*d+48*b^3*c)*(b*x^2+a)^(1/2)/a^4/x
```

Rubi [A] time = 0.18, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1803, 12, 264}

$$\frac{\sqrt{a+bx^2}\,\left(70a^2be-105a^3f-56ab^2d+48b^3c\right)}{105a^4x}-\frac{\sqrt{a+bx^2}\,\left(35a^2e-28abd+24b^2c\right)}{105a^3x^3}+\frac{\sqrt{a+bx^2}\,(6bc-7ad)}{35a^2x^5}-c\sqrt{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*b*c - 7*a*d)*Sqrt[a + b*x^2])/(35*a^2*
x^5) - ((24*b^2*c - 28*a*b*d + 35*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^3) + (
(48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(105*a^4*
x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{7ax^7} - \frac{\int \frac{6bc-7a\left(d+ex^2+fx^4\right)}{x^6\sqrt{a+bx^2}}\,dx}{7a}$$

$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} + \frac{\int \frac{4b(6bc-7ad)-5a\left(-7ae-7afx^2\right)}{x^4\sqrt{a+bx^2}}\,dx}{35a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3} - \int$$

$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3} - (48$$

$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3} + (48$$

Mathematica [A] time = 0.08, size = 103, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-a^3\left(15c+21dx^2+35x^4\left(e+3fx^2\right)\right)+2a^2bx^2\left(9c+14dx^2+35ex^4\right)-8ab^2x^4\left(3c+7dx^2\right)+48b^3c\right.}{105a^4x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*b^3*c*x^6 - 8*a*b^2*x^4*(3*c + 7*d*x^2) + 2*a^2*b*x^2*
(9*c + 14*d*x^2 + 35*e*x^4) - a^3*(15*c + 21*d*x^2 + 35*x^4*(e + 3*f*x^2)))
)/(105*a^4*x^7)
```

fricas [A] time = 0.81, size = 100, normalized size = 0.71

$$\frac{\left(\left(48\,b^3c-56\,ab^2d+70\,a^2be-105\,a^3f\right)x^6-\left(24\,ab^2c-28\,a^2bd+35\,a^3e\right)x^4-15\,a^3c+3\left(6\,a^2bc-7\,a^3d\right)x^2\right)\sqrt{b}}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*((48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*x^6 - (24*a*b^2*c -
 28*a^2*b*d + 35*a^3*e)*x^4 - 15*a^3*c + 3*(6*a^2*b*c - 7*a^3*d)*x^2)*sqrt(
b*x^2 + a)/(a^4*x^7)
```

giac [B] time = 0.57, size = 554, normalized size = 3.96

$$2\left(105\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}\sqrt{b}\,f-630\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}a\sqrt{b}\,f+210\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}b^{\frac{3}{2}}e+560\left(\sqrt{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*sqrt(b)*f - 630*(sqrt(b)*x - sq
rt(b*x^2 + a))^10*a*sqrt(b)*f + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(3/2
)*e + 560*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2)*d + 1575*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a^2*sqrt(b)*f - 910*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(3/
2)*e + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(7/2)*c - 1400*(sqrt(b)*x - s
```

```
qrt(b*x^2 + a))^6*a*b^(5/2)*d - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*sq
rt(b)*f + 1540*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(3/2)*e - 1008*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c + 1176*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^2*b^(5/2)*d + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*sqrt(b)*f - 1260*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(3/2)*e + 336*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^2*b^(7/2)*c - 392*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/2)*d
- 630*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*sqrt(b)*f + 490*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a^4*b^(3/2)*e - 48*a^3*b^(7/2)*c + 56*a^4*b^(5/2)*d + 105*a^
6*sqrt(b)*f - 70*a^5*b^(3/2)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

maple [A] time = 0.01, size = 111, normalized size = 0.79

$$-\frac{\sqrt{b\,x^2+a}\,\left(105a^3 f\,x^6-70a^2be\,x^6+56a\,b^2d\,x^6-48b^3c\,x^6+35a^3e\,x^4-28a^2bd\,x^4+24a\,b^2c\,x^4+21a^3d\,x^2-18a\right.}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(105*a^3*f*x^6-70*a^2*b*e*x^6+56*a*b^2*d*x^6-48*b^3*
c*x^6+35*a^3*e*x^4-28*a^2*b*d*x^4+24*a*b^2*c*x^4+21*a^3*d*x^2-18*a^2*b*c*x^
2+15*a^3*c)/x^7/a^4
```

maxima [A] time = 1.40, size = 193, normalized size = 1.38

$$\frac{16\,\sqrt{bx^2+a}\,b^3c}{35\,a^4x}-\frac{8\,\sqrt{bx^2+a}\,b^2d}{15\,a^3x}+\frac{2\,\sqrt{bx^2+a}\,be}{3\,a^2x}-\frac{\sqrt{bx^2+a}\,f}{ax}-\frac{8\,\sqrt{bx^2+a}\,b^2c}{35\,a^3x^3}+\frac{4\,\sqrt{bx^2+a}\,bd}{15\,a^2x^3}-\frac{\sqrt{bx^2+a}\,e}{3\,ax^3}+\frac{6\,\sqrt{b}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 16/35*sqrt(b*x^2 + a)*b^3*c/(a^4*x) - 8/15*sqrt(b*x^2 + a)*b^2*d/(a^3*x) +
2/3*sqrt(b*x^2 + a)*b*e/(a^2*x) - sqrt(b*x^2 + a)*f/(a*x) - 8/35*sqrt(b*x^2
 + a)*b^2*c/(a^3*x^3) + 4/15*sqrt(b*x^2 + a)*b*d/(a^2*x^3) - 1/3*sqrt(b*x^2
 + a)*e/(a*x^3) + 6/35*sqrt(b*x^2 + a)*b*c/(a^2*x^5) - 1/5*sqrt(b*x^2 + a)*
d/(a*x^5) - 1/7*sqrt(b*x^2 + a)*c/(a*x^7)
```

mupad [B] time = 1.28, size = 124, normalized size = 0.89

$$\frac{\sqrt{b\,x^2+a}\,\left(-105\,f\,a^3+70\,e\,a^2\,b-56\,d\,a\,b^2+48\,c\,b^3\right)}{105\,a^4\,x}-\frac{\sqrt{b\,x^2+a}\,\left(7\,a\,d-6\,b\,c\right)}{35\,a^2\,x^5}-\frac{\sqrt{b\,x^2+a}\,\left(35\,e\,a^2-28\,d\,a\,b+\right.}{105\,a^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*(48*b^3*c - 105*a^3*f - 56*a*b^2*d + 70*a^2*b*e))/(105*a
^4*x) - ((a + b*x^2)^(1/2)*(7*a*d - 6*b*c))/(35*a^2*x^5) - ((a + b*x^2)^(1/
2)*(24*b^2*c + 35*a^2*e - 28*a*b*d))/(105*a^3*x^3) - (c*(a + b*x^2)^(1/2))/
(7*a*x^7)
```

sympy [B] time = 6.71, size = 891, normalized size = 6.36

$$-\frac{5a^6b^{\frac{19}{2}}c\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{9a^5b^{\frac{21}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{}{35a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -5*a**6*b**(19/2)*c*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**1
0*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 9*a**5*b**(21/2)*c*x
**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**
5*b**11*x**10 + 35*a**4*b**12*x**12) - 5*a**4*b**(23/2)*c*x**4*sqrt(a/(b*x*
*2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 +
35*a**4*b**12*x**12) - 3*a**4*b**(9/2)*d*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4
*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 5*a**3*b**(25/2)*c*x**6*sq
rt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**1
1*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*d*x**2*sqrt(a/(b*x**2) +
1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 30*a**2*b*
*(27/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**
8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 3*a**2*b**(13/2)*d*x**4*s
qrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x
**8) + 40*a*b**(29/2)*c*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105
*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 12*a*b**(1
5/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 1
5*a**3*b**6*x**8) + 16*b**(31/2)*c*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9
*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) -
 8*b**(17/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - sqr
t(b)*f*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*e*sqrt(a/(b*x**2) + 1)/(3*a**2)
```

3.158 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{a+bx^2}\,(8bc-9ad)}{63a^2x^7}-\frac{\sqrt{a+bx^2}\,\left(21a^2e-18abd+16b^2c\right)}{105a^3x^5}-\frac{2b\sqrt{a+bx^2}\,\left(-105a^3f+84a^2be-72ab^2d+64b^3c\right)}{315a^5x}+$$

```
[Out] -1/9*c*(b*x^2+a)^(1/2)/a/x^9+1/63*(-9*a*d+8*b*c)*(b*x^2+a)^(1/2)/a^2/x^7-1/
105*(21*a^2*e-18*a*b*d+16*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^5+1/315*(-105*a^3*f+
84*a^2*b*e-72*a*b^2*d+64*b^3*c)*(b*x^2+a)^(1/2)/a^4/x^3-2/315*b*(-105*a^3*f
+84*a^2*b*e-72*a*b^2*d+64*b^3*c)*(b*x^2+a)^(1/2)/a^5/x
```

Rubi [A] time = 0.25, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1803, 12, 271, 264}

$$-\frac{2b\sqrt{a+bx^2}\,\left(84a^2be-105a^3f-72ab^2d+64b^3c\right)}{315a^5x}+\frac{\sqrt{a+bx^2}\,\left(84a^2be-105a^3f-72ab^2d+64b^3c\right)}{315a^4x^3}-\frac{\sqrt{a+bx^2}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(9*a*x^9) + ((8*b*c - 9*a*d)*Sqrt[a + b*x^2])/(63*a^2*
x^7) - ((16*b^2*c - 18*a*b*d + 21*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^5) + (
(64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(315*a^4*
x^3) - (2*b*(64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2
])/(315*a^5*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{9ax^9} - \frac{\int \frac{8bc-9a\left(d+ex^2+fx^4\right)}{x^8\sqrt{a+bx^2}}dx}{9a}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} + \frac{\int \frac{6b(8bc-9ad)-7a\left(-9ae-9afx^2\right)}{x^6\sqrt{a+bx^2}}dx}{63a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} - \frac{\int \frac{4}{}}{}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} - \frac{(64}{}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} + \frac{(64}{}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} + \frac{(64}{}$$

Mathematica [A] time = 0.10, size = 134, normalized size = 0.71

$$-\frac{\sqrt{a+bx^2}\left(a^4\left(35c+45dx^2+63ex^4+105fx^6\right)-2a^3bx^2\left(20c+27dx^2+42ex^4+105fx^6\right)+24a^2b^2x^4\left(2c+\right.\right.}{315a^5x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/315*(Sqrt[a + b*x^2]*(128*b^4*c*x^8 - 16*a*b^3*x^6*(4*c + 9*d*x^2) + 24*
a^2*b^2*x^4*(2*c + 3*d*x^2 + 7*e*x^4) - 2*a^3*b*x^2*(20*c + 27*d*x^2 + 42*e
*x^4 + 105*f*x^6) + a^4*(35*c + 45*d*x^2 + 63*e*x^4 + 105*f*x^6)))/(a^5*x^9
)
```

fricas [A] time = 1.18, size = 141, normalized size = 0.75

$$-\frac{\left(2\left(64\,b^4c-72\,ab^3d+84\,a^2b^2e-105\,a^3bf\right)x^8-\left(64\,ab^3c-72\,a^2b^2d+84\,a^3be-105\,a^4f\right)x^6+35\,a^4c+3\left(16\right.\right.}{315\,a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/315*(2*(64*b^4*c - 72*a*b^3*d + 84*a^2*b^2*e - 105*a^3*b*f)*x^8 - (64*a*
b^3*c - 72*a^2*b^2*d + 84*a^3*b*e - 105*a^4*f)*x^6 + 35*a^4*c + 3*(16*a^2*b
^2*c - 18*a^3*b*d + 21*a^4*e)*x^4 - 5*(8*a^3*b*c - 9*a^4*d)*x^2)*sqrt(b*x^2
 + a)/(a^5*x^9)
```

giac [B] time = 0.60, size = 667, normalized size = 3.53

$$4\left(315\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{14}b^{\frac{3}{2}}f-1995\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{3}{2}}f+840\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}b^{\frac{5}{2}}e+2520\left(\sqrt{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(3/2)*f - 1995*(sqrt(b)*x - s
qrt(b*x^2 + a))^12*a*b^(3/2)*f + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(5/
2)*e + 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*d + 5355*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*a^2*b^(3/2)*f - 3780*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a
*b^(5/2)*e + 8064*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c - 6552*(sqrt(b)
*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d - 7875*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
a^3*b^(3/2)*f + 6804*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*e - 5376*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c + 6048*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^2*b^(7/2)*d + 6825*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*f -
 6216*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/2)*e + 2304*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*a^2*b^(9/2)*c - 2592*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^
(7/2)*d - 3465*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*f + 3024*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*e - 576*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^3*b^(9/2)*c + 648*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7/2)*d + 945*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*f - 756*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^5*b^(5/2)*e + 64*a^4*b^(9/2)*c - 72*a^5*b^(7/2)*d - 105*a^7*b^(3/
2)*f + 84*a^6*b^(5/2)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

maple [A] time = 0.01, size = 157, normalized size = 0.83

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(-210a^{3}bf\,x^{8}+168a^{2}b^{2}e\,x^{8}-144a\,b^{3}d\,x^{8}+128b^{4}c\,x^{8}+105a^{4}f\,x^{6}-84a^{3}be\,x^{6}+72a^{2}b^{2}d\,x^{6}-64a\,b\right.}{315a^{5}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/315*(b*x^2+a)^(1/2)*(-210*a^3*b*f*x^8+168*a^2*b^2*e*x^8-144*a*b^3*d*x^8+
128*b^4*c*x^8+105*a^4*f*x^6-84*a^3*b*e*x^6+72*a^2*b^2*d*x^6-64*a*b^3*c*x^6+
63*a^4*e*x^4-54*a^3*b*d*x^4+48*a^2*b^2*c*x^4+45*a^4*d*x^2-40*a^3*b*c*x^2+35
*a^4*c)/x^9/a^5
```

maxima [A] time = 1.39, size = 275, normalized size = 1.46

$$-\frac{128\sqrt{bx^{2}+a}\,b^{4}c}{315\,a^{5}x}+\frac{16\sqrt{bx^{2}+a}\,b^{3}d}{35\,a^{4}x}-\frac{8\sqrt{bx^{2}+a}\,b^{2}e}{15\,a^{3}x}+\frac{2\sqrt{bx^{2}+a}\,bf}{3\,a^{2}x}+\frac{64\sqrt{bx^{2}+a}\,b^{3}c}{315\,a^{4}x^{3}}-\frac{8\sqrt{bx^{2}+a}\,b^{2}d}{35\,a^{3}x^{3}}+\frac{4\sqrt{bx^{2}}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -128/315*sqrt(b*x^2 + a)*b^4*c/(a^5*x) + 16/35*sqrt(b*x^2 + a)*b^3*d/(a^4*x
) - 8/15*sqrt(b*x^2 + a)*b^2*e/(a^3*x) + 2/3*sqrt(b*x^2 + a)*b*f/(a^2*x) +
64/315*sqrt(b*x^2 + a)*b^3*c/(a^4*x^3) - 8/35*sqrt(b*x^2 + a)*b^2*d/(a^3*x^
3) + 4/15*sqrt(b*x^2 + a)*b*e/(a^2*x^3) - 1/3*sqrt(b*x^2 + a)*f/(a*x^3) - 1
6/105*sqrt(b*x^2 + a)*b^2*c/(a^3*x^5) + 6/35*sqrt(b*x^2 + a)*b*d/(a^2*x^5)
- 1/5*sqrt(b*x^2 + a)*e/(a*x^5) + 8/63*sqrt(b*x^2 + a)*b*c/(a^2*x^7) - 1/7*
sqrt(b*x^2 + a)*d/(a*x^7) - 1/9*sqrt(b*x^2 + a)*c/(a*x^9)
```

mupad [B] time = 1.28, size = 171, normalized size = 0.90

$$\frac{\sqrt{b\,x^{2}+a}\,\left(-105\,f\,a^{3}+84\,e\,a^{2}\,b-72\,d\,a\,b^{2}+64\,c\,b^{3}\right)}{315\,a^{4}\,x^{3}}-\frac{\sqrt{b\,x^{2}+a}\,\left(9\,a\,d-8\,b\,c\right)}{63\,a^{2}\,x^{7}}-\frac{\sqrt{b\,x^{2}+a}\,\left(21\,e\,a^{2}-18\,d\,a\,b+\right.}{105\,a^{3}\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*(64*b^3*c - 105*a^3*f - 72*a*b^2*d + 84*a^2*b*e))/(315*a
^4*x^3) - ((a + b*x^2)^(1/2)*(9*a*d - 8*b*c))/(63*a^2*x^7) - ((a + b*x^2)^(
1/2)*(16*b^2*c + 21*a^2*e - 18*a*b*d))/(105*a^3*x^5) - ((a + b*x^2)^(1/2)*(
```

```
128*b^4*c + 168*a^2*b^2*e - 144*a*b^3*d - 210*a^3*b*f))/(315*a^5*x) - (c*(a
 + b*x^2)^(1/2))/(9*a*x^9)
```

sympy [B] time = 7.69, size = 1642, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**(1/2),x)
```

```
[Out] -35*a**8*b**(33/2)*c*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*
b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**2
0*x**16) - 100*a**7*b**(35/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x
**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14
 + 315*a**5*b**20*x**16) - 98*a**6*b**(37/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(3
15*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a
**6*b**19*x**14 + 315*a**5*b**20*x**16) - 5*a**6*b**(19/2)*d*sqrt(a/(b*x**2
) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35
*a**4*b**12*x**12) - 28*a**5*b**(39/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(315*a**
9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b*
*19*x**14 + 315*a**5*b**20*x**16) - 9*a**5*b**(21/2)*d*x**2*sqrt(a/(b*x**2)
 + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*
a**4*b**12*x**12) - 35*a**4*b**(41/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(315*a**9
*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**
19*x**14 + 315*a**5*b**20*x**16) - 5*a**4*b**(23/2)*d*x**4*sqrt(a/(b*x**2)
+ 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a
**4*b**12*x**12) - 3*a**4*b**(9/2)*e*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**
4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 280*a**3*b**(43/2)*c*x**10*sqr
t(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*
b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) + 5*a**3*b**(25
/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 +
105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*e*x**2*sqrt(
a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8)
 - 560*a**2*b**(45/2)*c*x**12*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1
260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*
a**5*b**20*x**16) + 30*a**2*b**(27/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*
b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**1
2) - 3*a**2*b**(13/2)*e*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a
**4*b**5*x**6 + 15*a**3*b**6*x**8) - 448*a*b**(47/2)*c*x**14*sqrt(a/(b*x**2
) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12
 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) + 40*a*b**(29/2)*d*x**10*s
qrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**
11*x**10 + 35*a**4*b**12*x**12) - 12*a*b**(15/2)*e*x**6*sqrt(a/(b*x**2) + 1
)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 128*b**(49/
2)*c*x**16*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**1
0 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) +
 16*b**(31/2)*d*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b*
*10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 8*b**(17/2)*e*x**8
*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6
*x**8) - sqrt(b)*f*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*f*sqrt(a/(b
*x**2) + 1)/(3*a**2)
```

3.159 $\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=381

$$-\frac{x^9\left(2Ab^3-a\left(23a^2D-16abC+9b^2B\right)\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x\sqrt{a+bx^2}\left(16Ab^3-3a\left(143a^2D-66abC+\right.\right.}{16ab^7}$$

```
[Out] 1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^9/a/(b*x^2+a)^(7/2)-1/35*(2*A*b^3-a*(9*
B*b^2-16*C*a*b+23*D*a^2))*x^9/a^2/b^3/(b*x^2+a)^(5/2)-1/210*(16*A*b^3-3*a*(
24*B*b^2-66*C*a*b+143*D*a^2))*x^7/a^2/b^4/(b*x^2+a)^(3/2)+1/6*D*x^9/b^3/(b*
x^2+a)^(3/2)+1/16*(16*A*b^3-72*B*a*b^2+198*C*a^2*b-429*D*a^3)*arctanh(x*b^(
1/2)/(b*x^2+a)^(1/2))/b^(15/2)-1/30*(16*A*b^3-3*a*(24*B*b^2-66*C*a*b+143*D*
a^2))*x^5/a^2/b^5/(b*x^2+a)^(1/2)-1/16*(16*A*b^3-3*a*(24*B*b^2-66*C*a*b+143
*D*a^2))*x*(b*x^2+a)^(1/2)/a/b^7+1/24*(16*A*b^3-3*a*(24*B*b^2-66*C*a*b+143*
D*a^2))*x^3*(b*x^2+a)^(1/2)/a^2/b^6
```

Rubi [A] time = 0.66, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {1804, 1585, 1263, 1584, 459, 288, 321, 217, 206}

$$-\frac{x^9\left(2Ab^3-a\left(23a^2D-16abC+9b^2B\right)\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x^7\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{210a^2b^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^9)/(7*a*(a + b*x^2)^(7/2)) - ((2*A
*b^3 - a*(9*b^2*B - 16*a*b*C + 23*a^2*D))*x^9)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) - ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^7)/(210*a^2*b^4*
(a + b*x^2)^(3/2)) + (D*x^9)/(6*b^3*(a + b*x^2)^(3/2)) - ((16*A*b^3 - 3*a*(
24*b^2*B - 66*a*b*C + 143*a^2*D))*x^5)/(30*a^2*b^5*Sqrt[a + b*x^2]) - ((16*
A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x*Sqrt[a + b*x^2])/(16*a*b^7
) + ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^3*Sqrt[a + b*x^2]
)/(24*a^2*b^6) + ((16*A*b^3 - 72*a*b^2*B + 198*a^2*b*C - 429*a^3*D)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^7\left(\left(2Ab-\frac{9a\left(b^2B-abC+a^2D\right)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^8\left(2Ab-\frac{9a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^7\left(\left(8\cdots\right.\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^8\left(8A\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{D\cdots}{6b^3\left(a+\cdots\right.}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\cdots\right.}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\cdots\right.}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\cdots\right.}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\cdots\right.}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\cdots\right.}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.58, size = 273, normalized size = 0.72

$$\frac{\sqrt{a+bx^2}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{16\sqrt{a}\,b^{15/2}\sqrt{\frac{bx^2}{a}+1}}+\frac{x\left(45045a^6D-2310a^5b\left(9C-65Dx^2\right)+42a\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(45045*a^6*D - 2310*a^5*b*(9*C - 65*D*x^2) + 42*a^4*b^2*(180*B - 1650*C*
x^2 + 4147*D*x^4) - 12*a^3*b^3*(140*A - 2100*B*x^2 + 6699*C*x^4 - 6292*D*x^
6) - 2*a*b^5*x^4*(3248*A - 6336*B*x^2 + 1155*C*x^4 + 455*D*x^6) + a^2*b^4*x
^2*(-5600*A + 29232*B*x^2 - 34848*C*x^4 + 5005*D*x^6) + 4*b^6*x^6*(-704*A +
 35*(6*B*x^2 + 3*C*x^4 + 2*D*x^6))))/(1680*b^7*(a + b*x^2)^(7/2)) + ((16*A*
b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[
b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(15/2)*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 1.13, size = 987, normalized size = 2.59

$$\left[\frac{105\left(\left(429\,Da^3b^4 - 198\,Ca^2b^5 + 72\,Bab^6 - 16\,Ab^7\right)x^8 + 429\,Da^7 - 198\,Ca^6b + 72\,Ba^5b^2 - 16\,Aa^4b^3 + 4\left(429\,\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*((429*D*a^3*b^4 - 198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^8 +
 429*D*a^7 - 198*C*a^6*b + 72*B*a^5*b^2 - 16*A*a^4*b^3 + 4*(429*D*a^4*b^3 -
 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6)*x^6 + 6*(429*D*a^5*b^2 - 198*C*
a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^4 + 4*(429*D*a^6*b - 198*C*a^5*b^2
 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) + 2*(280*D*b^7*x^13 - 70*(13*D*a*b^6 - 6*C*b^7)*x^11 + 35
*(143*D*a^2*b^5 - 66*C*a*b^6 + 24*B*b^7)*x^9 + 176*(429*D*a^3*b^4 - 198*C*a
^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^7 + 406*(429*D*a^4*b^3 - 198*C*a^3*b^4 +
72*B*a^2*b^5 - 16*A*a*b^6)*x^5 + 350*(429*D*a^5*b^2 - 198*C*a^4*b^3 + 72*B*
a^3*b^4 - 16*A*a^2*b^5)*x^3 + 105*(429*D*a^6*b - 198*C*a^5*b^2 + 72*B*a^4*b
^3 - 16*A*a^3*b^4)*x)*sqrt(b*x^2 + a))/(b^12*x^8 + 4*a*b^11*x^6 + 6*a^2*b^1
0*x^4 + 4*a^3*b^9*x^2 + a^4*b^8), 1/1680*(105*((429*D*a^3*b^4 - 198*C*a^2*b
^5 + 72*B*a*b^6 - 16*A*b^7)*x^8 + 429*D*a^7 - 198*C*a^6*b + 72*B*a^5*b^2 -
16*A*a^4*b^3 + 4*(429*D*a^4*b^3 - 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6
)*x^6 + 6*(429*D*a^5*b^2 - 198*C*a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^4
 + 4*(429*D*a^6*b - 198*C*a^5*b^2 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x^2)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (280*D*b^7*x^13 - 70*(13*D*a*b^6 -
 6*C*b^7)*x^11 + 35*(143*D*a^2*b^5 - 66*C*a*b^6 + 24*B*b^7)*x^9 + 176*(429*
D*a^3*b^4 - 198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^7 + 406*(429*D*a^4*b^3
 - 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6)*x^5 + 350*(429*D*a^5*b^2 - 19
8*C*a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^3 + 105*(429*D*a^6*b - 198*C*a
^5*b^2 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x)*sqrt(b*x^2 + a))/(b^12*x^8 + 4*a*b
^11*x^6 + 6*a^2*b^10*x^4 + 4*a^3*b^9*x^2 + a^4*b^8)]
```

giac [A] time = 0.64, size = 342, normalized size = 0.90

$$\frac{\left(\left(\left(\left(35\left(2\left(\frac{4\,Dx^2}{b} - \frac{13\,Da^4b^{11} - 6\,Ca^3b^{12}}{a^3b^{13}}\right)x^2 + \frac{143\,Da^5b^{10} - 66\,Ca^4b^{11} + 24\,Ba^3b^{12}}{a^3b^{13}}\right)x^2 + \frac{176\left(429\,Da^6b^9 - 198\,Ca^5b^{10} + 72\,Ba^4b^{11} - 16\,Aa^3b\right.}{a^3b^{13}}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/1680*((((35*(2*(4*D*x^2/b - (13*D*a^4*b^11 - 6*C*a^3*b^12)/(a^3*b^13))*x^
2 + (143*D*a^5*b^10 - 66*C*a^4*b^11 + 24*B*a^3*b^12)/(a^3*b^13))*x^2 + 176*
(429*D*a^6*b^9 - 198*C*a^5*b^10 + 72*B*a^4*b^11 - 16*A*a^3*b^12)/(a^3*b^13)
)*x^2 + 406*(429*D*a^7*b^8 - 198*C*a^6*b^9 + 72*B*a^5*b^10 - 16*A*a^4*b^11)
/(a^3*b^13))*x^2 + 350*(429*D*a^8*b^7 - 198*C*a^7*b^8 + 72*B*a^6*b^9 - 16*A
*a^5*b^10)/(a^3*b^13))*x^2 + 105*(429*D*a^9*b^6 - 198*C*a^8*b^7 + 72*B*a^7*
```

b^8 - 16*A*a^6*b^9)/(a^3*b^13))*x/(b*x^2 + a)^(7/2) + 1/16*(429*D*a^3 - 198
*C*a^2*b + 72*B*a*b^2 - 16*A*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^
(15/2)

maple [A] time = 0.30, size = 517, normalized size = 1.36

$$\frac{Dx^{13}}{6\left(bx^2+a\right)^{\frac{7}{2}} b}+\frac{Cx^{11}}{4\left(bx^2+a\right)^{\frac{7}{2}} b}-\frac{13Da\,x^{11}}{24\left(bx^2+a\right)^{\frac{7}{2}} b^2}+\frac{Bx^9}{2\left(bx^2+a\right)^{\frac{7}{2}} b}-\frac{11Ca\,x^9}{8\left(bx^2+a\right)^{\frac{7}{2}} b^2}+\frac{143Da^2x^9}{48\left(bx^2+a\right)^{\frac{7}{2}} b^3}-\frac{Ax}{7\left(bx^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/4*C*x^11/b/(b*x^2+a)^(7/2)+99/8*C*a^2/b^(13/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/
2))+1/2*B*x^9/b/(b*x^2+a)^(7/2)-9/2*B*a/b^(11/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/
2))+1/6*D*x^13/b/(b*x^2+a)^(7/2)-429/16*D*a^3/b^(15/2)*ln(b^(1/2)*x+(b*x^2+
a)^(1/2))-1/7*A*x^7/b/(b*x^2+a)^(7/2)-1/5*A/b^2*x^5/(b*x^2+a)^(5/2)-1/3*A/b
^3*x^3/(b*x^2+a)^(3/2)-A/b^4*x/(b*x^2+a)^(1/2)+9/10*B*a/b^3*x^5/(b*x^2+a)^(
5/2)+3/2*B*a/b^4*x^3/(b*x^2+a)^(3/2)+9/2*B*a/b^5*x/(b*x^2+a)^(1/2)-13/24*D*
a/b^2*x^11/(b*x^2+a)^(7/2)+143/48*D*a^2/b^3*x^9/(b*x^2+a)^(7/2)+429/112*D*a
^3/b^4*x^7/(b*x^2+a)^(7/2)+429/80*D*a^3/b^5*x^5/(b*x^2+a)^(5/2)+143/16*D*a^
3/b^6*x^3/(b*x^2+a)^(3/2)+429/16*D*a^3/b^7*x/(b*x^2+a)^(1/2)-11/8*C*a/b^2*x
^9/(b*x^2+a)^(7/2)-99/56*C*a^2/b^3*x^7/(b*x^2+a)^(7/2)-99/40*C*a^2/b^4*x^5/
(b*x^2+a)^(5/2)-33/8*C*a^2/b^5*x^3/(b*x^2+a)^(3/2)+A/b^(9/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2))-99/8*C*a^2/b^6*x/(b*x^2+a)^(1/2)+9/14*B*a/b^2*x^7/(b*x^2+a)
^(7/2)
```

maxima [B] time = 1.89, size = 1221, normalized size = 3.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/6*D*x^13/((b*x^2 + a)^(7/2)*b) - 13/24*D*a*x^11/((b*x^2 + a)^(7/2)*b^2) +
 1/4*C*x^11/((b*x^2 + a)^(7/2)*b) + 143/48*D*a^2*x^9/((b*x^2 + a)^(7/2)*b^3
) - 11/8*C*a*x^9/((b*x^2 + a)^(7/2)*b^2) + 1/2*B*x^9/((b*x^2 + a)^(7/2)*b)
- 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*A*x + 4
29/560*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*D*a^3*x
/b^3 - 99/280*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b
^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*
C*a^2*x/b^2 + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7
/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b
^4))*B*a*x/b + 143/80*D*a^3*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*
x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^4 - 33/40*C*a^2*x*(1
5*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*
x^2 + a)^(5/2)*b^3))/b^3 + 3/10*B*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*
x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/15*A*x
*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/(
(b*x^2 + a)^(5/2)*b^3))/b + 143/16*D*a^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2
*a/((b*x^2 + a)^(3/2)*b^2))/b^5 - 33/8*C*a^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b)
 + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 3/2*B*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b
) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*A*x*(3*x^2/((b*x^2 + a)^(3/2)*b)
 + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 429/16*D*a^4*x^3/((b*x^2 + a)^(5/2)*b
^6) - 99/8*C*a^3*x^3/((b*x^2 + a)^(5/2)*b^5) + 9/2*B*a^2*x^3/((b*x^2 + a)^(
5/2)*b^4) - A*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 19877/560*D*a^3*x/(sqrt(b*x^2
 + a)*b^7) - 2431/560*D*a^4*x/((b*x^2 + a)^(3/2)*b^7) + 12441/560*D*a^5*x/(
```

```
(b*x^2 + a)^(5/2)*b^7) + 4587/280*C*a^2*x/(sqrt(b*x^2 + a)*b^6) + 561/280*C
*a^3*x/((b*x^2 + a)^(3/2)*b^6) - 2871/280*C*a^4*x/((b*x^2 + a)^(5/2)*b^6) -
 417/70*B*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*B*a^2*x/((b*x^2 + a)^(3/2)*b^5)
 + 261/70*B*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*A*x/(sqrt(b*x^2 + a)*b^
4) + 17/105*A*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*A*a^2*x/((b*x^2 + a)^(5/2
)*b^4) - 429/16*D*a^3*arcsinh(b*x/sqrt(a*b))/b^(15/2) + 99/8*C*a^2*arcsinh(
b*x/sqrt(a*b))/b^(13/2) - 9/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(11/2) + A*arcsi
nh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.160 $\int \frac{x^6\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=279

$$\frac{x^7\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(99a^2D-36abC+8b^2B\right)}{8b^{13/2}}-\frac{x\sqrt{a+bx^2}\left(99a^2D-36abC+8b^2B\right)}{8ab^6}+\frac{x^3\left(\right.}{}$$

```
[Out] 1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^7/a/(b*x^2+a)^(7/2)+1/5*(B*b^2-2*C*a*b+
3*D*a^2)*x^7/a/b^3/(b*x^2+a)^(5/2)+1/60*(8*B*b^2-36*C*a*b+99*D*a^2)*x^5/a/b
^4/(b*x^2+a)^(3/2)+1/4*D*x^7/b^3/(b*x^2+a)^(3/2)+1/8*(8*B*b^2-36*C*a*b+99*D
*a^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(13/2)+1/12*(8*B*b^2-36*C*a*b+99
*D*a^2)*x^3/a/b^5/(b*x^2+a)^(1/2)-1/8*(8*B*b^2-36*C*a*b+99*D*a^2)*x*(b*x^2+
a)^(1/2)/a/b^6
```

Rubi [A] time = 0.45, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1804, 1585, 1263, 1584, 459, 288, 321, 217, 206}

$$\frac{x^7\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(3a^2D-2abC+b^2B\right)}{5ab^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(99a^2D-36abC+8b^2B\right)}{60ab^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(99a^2D-36abC+8b^2B\right)}{12ab^5\sqrt{a+bx^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^7)/(7*a*(a + b*x^2)^(7/2)) + ((b^2
*B - 2*a*b*C + 3*a^2*D)*x^7)/(5*a*b^3*(a + b*x^2)^(5/2)) + ((8*b^2*B - 36*a
*b*C + 99*a^2*D)*x^5)/(60*a*b^4*(a + b*x^2)^(3/2)) + (D*x^7)/(4*b^3*(a + b*
x^2)^(3/2)) + ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x^3)/(12*a*b^5*Sqrt[a + b*x^
2]) - ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x*Sqrt[a + b*x^2])/(8*a*b^6) + ((8*b
^2*B - 36*a*b*C + 99*a^2*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(13/
2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^5\left(-7a\left(B-\frac{a(bC-aD)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^6\left(-7a\left(B-\frac{a(bC-aD)}{b^2}\right)-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^5\left(-7a\left(2B-\frac{a(9bC-16aD)}{b^2}\right)x \cdots\right.}{\left(a+bx^2\right)^{5/2}}}{35a^2b}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^6\left(-7a\left(2B-\frac{a(9bC-16aD)}{b^2}\right) \cdots\right.}{\left(a+bx^2\right)^{5/2}}}{35a^2b}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{Dx^7}{4b^3\left(a+bx^2\right)^{3/2}} - \frac{(8b \cdots}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2 \cdots\right.}{60ab^4\left(a+bx^2\right)^{3/ \cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2 \cdots\right.}{60ab^4\left(a+bx^2\right)^{3/ \cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2 \cdots\right.}{60ab^4\left(a+bx^2\right)^{3/ \cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2 \cdots\right.}{60ab^4\left(a+bx^2\right)^{3/ \cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2 \cdots\right.}{60ab^4\left(a+bx^2\right)^{3/ \cdots}}$$

Mathematica [A] time = 0.48, size = 229, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(99a^2D-36abC+8b^2B\right)}{8\sqrt{a}\,b^{13/2}\sqrt{\frac{bx^2}{a}+1}} - \frac{x\left(10395a^6D-630a^5b\left(6C-55Dx^2\right)+42a^4b^2\left(20B-300Cx \cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/840*(x*(10395*a^6*D - 120*A*b^6*x^6 - 630*a^5*b*(6*C - 55*D*x^2) + 42*a^
4*b^2*(20*B - 300*C*x^2 + 957*D*x^4) + a^2*b^4*x^4*(3248*B - 6336*C*x^2 + 1
```

```
155*D*x^4) + 8*a^3*b^3*x^2*(350*B - 1827*C*x^2 + 2178*D*x^4) + 2*a*b^5*x^6*
(704*B - 105*(2*C*x^2 + D*x^4))))/(a*b^6*(a + b*x^2)^(7/2)) + ((8*b^2*B - 3
6*a*b*C + 99*a^2*D)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a
]*b^(13/2)*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 0.94, size = 816, normalized size = 2.92

$$\left[\frac{105\left(\left(99\,Da^3b^4-36\,Ca^2b^5+8\,Bab^6\right)x^8+99\,Da^7-36\,Ca^6b+8\,Ba^5b^2+4\left(99\,Da^4b^3-36\,Ca^3b^4+8\,Ba^2b^5\right)x\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/1680*(105*((99*D*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^8 + 99*D*a^7 - 36
*C*a^6*b + 8*B*a^5*b^2 + 4*(99*D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^6
+ 6*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*(99*D*a^6*b - 36*C*
a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) + 2*(210*D*a*b^6*x^11 - 105*(11*D*a^2*b^5 - 4*C*a*b^6)*x^9 - 8*(21
78*D*a^3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(99*D*a^4*
b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*D*a^5*b^2 - 36*C*a^4*b^3 +
8*B*a^3*b^4)*x^3 - 105*(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x)*sqrt(b*
x^2 + a))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b^8*x^2 + a^
5*b^7), -1/840*(105*((99*D*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^8 + 99*D*a
^7 - 36*C*a^6*b + 8*B*a^5*b^2 + 4*(99*D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^
5)*x^6 + 6*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*(99*D*a^6*b
- 36*C*a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) - (210*D*a*b^6*x^11 - 105*(11*D*a^2*b^5 - 4*C*a*b^6)*x^9 - 8*(2178*D*a^
3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(99*D*a^4*b^3 - 3
6*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3
*b^4)*x^3 - 105*(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x)*sqrt(b*x^2 + a
))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b^8*x^2 + a^5*b^7)]
```

giac [A] time = 0.59, size = 265, normalized size = 0.95

$$\frac{\left(\left(\left(\left(105\left(\frac{2\,Dx^2}{b}-\frac{11\,Da^4b^9-4\,Ca^3b^{10}}{a^3b^{11}}\right)x^2-\frac{8\left(2178\,Da^5b^8-792\,Ca^4b^9+176\,Ba^3b^{10}-15\,Aa^2b^{11}\right)}{a^3b^{11}}\right)x^2-\frac{406\left(99\,Da^6b^7-36\,Ca^5b^8+8\,Ba^4b^9\right)}{a^3b^{11}}\cdots}{840\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/840*(((((105*(2*D*x^2/b - (11*D*a^4*b^9 - 4*C*a^3*b^10)/(a^3*b^11))*x^2 -
8*(2178*D*a^5*b^8 - 792*C*a^4*b^9 + 176*B*a^3*b^10 - 15*A*a^2*b^11)/(a^3*b^
11))*x^2 - 406*(99*D*a^6*b^7 - 36*C*a^5*b^8 + 8*B*a^4*b^9)/(a^3*b^11))*x^2
- 350*(99*D*a^7*b^6 - 36*C*a^6*b^7 + 8*B*a^5*b^8)/(a^3*b^11))*x^2 - 105*(99
*D*a^8*b^5 - 36*C*a^7*b^6 + 8*B*a^6*b^7)/(a^3*b^11))*x/(b*x^2 + a)^(7/2) -
1/8*(99*D*a^2 - 36*C*a*b + 8*B*b^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/
b^(13/2)
```

maple [A] time = 0.01, size = 460, normalized size = 1.65

$$\frac{Dx^{11}}{4\left(b\,x^2+a\right)^{\frac{7}{2}}b}+\frac{C\,x^9}{2\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{11Da\,x^9}{8\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}-\frac{B\,x^7}{7\left(b\,x^2+a\right)^{\frac{7}{2}}b}+\frac{9Ca\,x^7}{14\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}-\frac{99Da^2x^7}{56\left(b\,x^2+a\right)^{\frac{7}{2}}b^3}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/4*D*x^11/b/(b*x^2+a)^(7/2)-11/8*D*a/b^2*x^9/(b*x^2+a)^(7/2)-99/56*D*a^2/b
^3*x^7/(b*x^2+a)^(7/2)-99/40*D*a^2/b^4*x^5/(b*x^2+a)^(5/2)-33/8*D*a^2/b^5*x
^3/(b*x^2+a)^(3/2)-99/8*D*a^2/b^6*x/(b*x^2+a)^(1/2)+99/8*D*a^2/b^(13/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*C*x^9/b/(b*x^2+a)^(7/2)+9/14*C*a/b^2*x^7/(b*
x^2+a)^(7/2)+9/10*C*a/b^3*x^5/(b*x^2+a)^(5/2)+3/2*C*a/b^4*x^3/(b*x^2+a)^(3/
2)+9/2*C*a/b^5*x/(b*x^2+a)^(1/2)-9/2*C*a/b^(11/2)*ln(b^(1/2)*x+(b*x^2+a)^(1
/2))-1/7/(b*x^2+a)^(7/2)*B/b*x^7-1/5/(b*x^2+a)^(5/2)*B/b^2*x^5-1/3/(b*x^2+a
)^(3/2)*B/b^3*x^3-1/(b*x^2+a)^(1/2)*B/b^4*x+B/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2))-1/2/(b*x^2+a)^(7/2)*A/b*x^5-5/8/(b*x^2+a)^(7/2)*A*a/b^2*x^3-15/56/
(b*x^2+a)^(7/2)*A*a^2/b^3*x+3/56/(b*x^2+a)^(5/2)*A*a/b^3*x+1/14/(b*x^2+a)^(
3/2)*A/b^3*x+1/7/(b*x^2+a)^(1/2)*A/a/b^3*x
```

maxima [B] time = 1.79, size = 986, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/4*D*x^11/((b*x^2 + a)^(7/2)*b) - 11/8*D*a*x^9/((b*x^2 + a)^(7/2)*b^2) + 1
/2*C*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*
x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/(
(b*x^2 + a)^(7/2)*b^4))*B*x - 99/280*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x
^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((
b*x^2 + a)^(7/2)*b^4))*D*a^2*x/b^2 + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 7
0*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a
^3/((b*x^2 + a)^(7/2)*b^4))*C*a*x/b - 33/40*D*a^2*x*(15*x^4/((b*x^2 + a)^(5
/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/
b^3 + 3/10*C*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2
)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/15*B*x*(15*x^4/((b*x^2 + a)
^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3
))/b - 1/2*A*x^5/((b*x^2 + a)^(7/2)*b) - 33/8*D*a^2*x*(3*x^2/((b*x^2 + a)^(
3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 3/2*C*a*x*(3*x^2/((b*x^2 + a)^
(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*B*x*(3*x^2/((b*x^2 + a)^(
3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - 99/8*D*a^3*x^3/((b*x^2 + a)^(5
/2)*b^5) + 9/2*C*a^2*x^3/((b*x^2 + a)^(5/2)*b^4) - B*a*x^3/((b*x^2 + a)^(5/
2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) + 4587/280*D*a^2*x/(sqrt(b*x^
2 + a)*b^6) + 561/280*D*a^3*x/((b*x^2 + a)^(3/2)*b^6) - 2871/280*D*a^4*x/((
b*x^2 + a)^(5/2)*b^6) - 417/70*C*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*C*a^2*x/
((b*x^2 + a)^(3/2)*b^5) + 261/70*C*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*
B*x/(sqrt(b*x^2 + a)*b^4) + 17/105*B*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*B*
a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*A*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*A*x/
(sqrt(b*x^2 + a)*a*b^3) + 3/56*A*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*A*a^2*
x/((b*x^2 + a)^(7/2)*b^3) + 99/8*D*a^2*arcsinh(b*x/sqrt(a*b))/b^(13/2) - 9/
2*C*a*arcsinh(b*x/sqrt(a*b))/b^(11/2) + B*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.161 $\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=210

$$\frac{x^5\left(a\left(19a^2D-12abC+5b^2B\right)+2Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{(2bC-9aD)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x(4bC-15aD)}{3b^5\sqrt{a+bx^2}}$$

```
[Out] 1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^5/a/(b*x^2+a)^(7/2)+1/35*(2*A*b^3+a*(5*
B*b^2-12*C*a*b+19*D*a^2))*x^5/a^2/b^3/(b*x^2+a)^(5/2)+1/3*a*(C*b-3*D*a)*x/b
^5/(b*x^2+a)^(3/2)+1/2*(2*C*b-9*D*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
11/2)-1/3*(4*C*b-15*D*a)*x/b^5/(b*x^2+a)^(1/2)+1/2*D*x*(b*x^2+a)^(1/2)/b^5
```

Rubi [A] time = 0.39, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1804, 1585, 1263, 1584, 455, 1157, 388, 217, 206}

$$\frac{x^5\left(a\left(19a^2D-12abC+5b^2B\right)+2Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x(4bC-15aD)}{3b^5\sqrt{a+bx^2}}+\frac{ax(bC-3aD)}{3b^5\left(a+bx^2\right)^{3/2}}+\frac{(2bC-9a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^5)/(7*a*(a + b*x^2)^(7/2)) + ((2*A
*b^3 + a*(5*b^2*B - 12*a*b*C + 19*a^2*D))*x^5)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) + (a*(b*C - 3*a*D)*x)/(3*b^5*(a + b*x^2)^(3/2)) - ((4*b*C - 15*a*D)*x)/(
3*b^5*Sqrt[a + b*x^2]) + (D*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*C - 9*a*D)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^3\left(-\left(2Ab+\frac{5a\left(b^2B-abC+a^2D\right)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^4\left(-2Ab-\frac{5a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^3\left(35\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^4\left(35\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC\cdots}{3b^5\left(a\cdots\right.}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC\cdots}{3b^5\left(a\cdots\right.}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC\cdots}{3b^5\left(a\cdots\right.}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC\cdots}{3b^5\left(a\cdots\right.}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC\cdots}{3b^5\left(a\cdots\right.}$$

Mathematica [A] time = 0.50, size = 194, normalized size = 0.92

$$\frac{105a^{5/2}\sqrt{\frac{bx^2}{a}+1}\left(a+bx^2\right)^3(2bC-9aD)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x\left(945a^6D-210a^5b\left(C-15Dx^2\right)+14a^4b^2x^2\left(261D\cdots\right.\right.}{210a^2b^{11/2}\left(a+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[b]*x*(945*a^6*D + 12*A*b^6*x^6 + 6*a*b^5*x^4*(7*A + 5*B*x^2) - 210*a^
5*b*(C - 15*D*x^2) + a^2*b^4*x^6*(-352*C + 105*D*x^2) + 14*a^4*b^2*x^2*(-50
*C + 261*D*x^2) + 4*a^3*b^3*x^4*(-203*C + 396*D*x^2)) + 105*a^(5/2)*(2*b*C
- 9*a*D)*(a + b*x^2)^3*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2
10*a^2*b^(11/2)*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.96, size = 653, normalized size = 3.11

$$\left[\frac{105\left(\left(9\,Da^3b^4-2\,Ca^2b^5\right)x^8+9\,Da^7-2\,Ca^6b+4\left(9\,Da^4b^3-2\,Ca^3b^4\right)x^6+6\left(9\,Da^5b^2-2\,Ca^4b^3\right)x^4+4\left(9\,D\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(105*((9*D*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*a^6*b + 4*(9*D
*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(9*D*a^
6*b - 2*C*a^5*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) + 2*(105*D*a^2*b^5*x^9 + 2*(792*D*a^3*b^4 - 176*C*a^2*b^5 + 15*B*a*b^6
 + 6*A*b^7)*x^7 + 14*(261*D*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 + 350*(
9*D*a^5*b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*D*a^6*b - 2*C*a^5*b^2)*x)*sqrt(b*x^
2 + a))/(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x^2 + a^6
*b^6), 1/210*(105*((9*D*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*a^6*b +
4*(9*D*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(
9*D*a^6*b - 2*C*a^5*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) +
 (105*D*a^2*b^5*x^9 + 2*(792*D*a^3*b^4 - 176*C*a^2*b^5 + 15*B*a*b^6 + 6*A*b
^7)*x^7 + 14*(261*D*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 + 350*(9*D*a^5*
b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*D*a^6*b - 2*C*a^5*b^2)*x)*sqrt(b*x^2 + a))/
(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x^2 + a^6*b^6)]
```

giac [A] time = 0.60, size = 203, normalized size = 0.97

$$\frac{\left(\left(\left(\left(\frac{105\,Dx^2}{b}+\frac{2\left(792\,Da^4b^7-176\,Ca^3b^8+15\,Ba^2b^9+6\,Aab^{10}\right)}{a^3b^9}\right)x^2+\frac{14\left(261\,Da^5b^6-58\,Ca^4b^7+3\,Aa^2b^9\right)}{a^3b^9}\right)x^2+\frac{350\left(9\,Da^6b^5-2\,Ca^5b^6\right)}{a^3b^9}\right)x^2\right.}{210\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/210*((((105*D*x^2/b + 2*(792*D*a^4*b^7 - 176*C*a^3*b^8 + 15*B*a^2*b^9 + 6
*A*a*b^10)/(a^3*b^9))*x^2 + 14*(261*D*a^5*b^6 - 58*C*a^4*b^7 + 3*A*a^2*b^9)
/(a^3*b^9))*x^2 + 350*(9*D*a^6*b^5 - 2*C*a^5*b^6)/(a^3*b^9))*x^2 + 105*(9*D
*a^7*b^4 - 2*C*a^6*b^5)/(a^3*b^9))*x/(b*x^2 + a)^(7/2) + 1/2*(9*D*a - 2*C*b
)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

maple [B] time = 0.01, size = 405, normalized size = 1.93

$$\frac{Dx^9}{2\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{C\,x^7}{7\left(b\,x^2+a\right)^{\frac{7}{2}}b}+\frac{9Da\,x^7}{14\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}-\frac{B\,x^5}{2\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{C\,x^5}{5\left(b\,x^2+a\right)^{\frac{5}{2}}b^2}+\frac{9Da\,x^5}{10\left(b\,x^2+a\right)^{\frac{5}{2}}b^3}-\frac{A}{4\left(b\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/2*D*x^9/b/(b*x^2+a)^(7/2)+9/14*D*a/b^2*x^7/(b*x^2+a)^(7/2)+9/10*D*a/b^3*x
^5/(b*x^2+a)^(5/2)+3/2*D*a/b^4*x^3/(b*x^2+a)^(3/2)+9/2*D*a/b^5*x/(b*x^2+a)^
(1/2)-9/2*D*a/b^(11/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-1/7/(b*x^2+a)^(7/2)*C/
b*x^7-1/5/(b*x^2+a)^(5/2)*C/b^2*x^5-1/3/(b*x^2+a)^(3/2)*C/b^3*x^3-1/(b*x^2+
a)^(1/2)*C/b^4*x+C/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-1/2*B*x^5/b/(b*x^2
+a)^(7/2)-5/8*B*a/b^2*x^3/(b*x^2+a)^(7/2)-15/56*B*a^2/b^3*x/(b*x^2+a)^(7/2)
+3/56*B*a/b^3*x/(b*x^2+a)^(5/2)+1/14*B/b^3*x/(b*x^2+a)^(3/2)+1/7*B/a/b^3*x/
(b*x^2+a)^(1/2)-1/4*A*x^3/b/(b*x^2+a)^(7/2)-3/28*A*a/b^2*x/(b*x^2+a)^(7/2)+
```

```
3/140*A/b^2*x/(b*x^2+a)^(5/2)+1/35*A/a/b^2*x/(b*x^2+a)^(3/2)+2/35*A/a^2/b^2
*x/(b*x^2+a)^(1/2)
```

maxima [B] time = 1.69, size = 753, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/2*D*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/
((b*x^2 + a)^(7/2)*b^4))*C*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^
4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b
*x^2 + a)^(7/2)*b^4))*D*a*x/b + 3/10*D*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) +
20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/1
5*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*
a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) + 3/2*D*a*
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*C*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 9/2*D*a^
2*x^3/((b*x^2 + a)^(5/2)*b^4) - C*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*B*a*x
^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) - 417/70*D*a*x
/(sqrt(b*x^2 + a)*b^5) - 51/70*D*a^2*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*D*a
^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*C*x/(sqrt(b*x^2 + a)*b^4) + 17/105*C
*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14
*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*B*a*x
/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*A*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*A*x/(
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b*x^2 + a)^(7/2)*b^2) - 9/2*D*a*arc
sinh(b*x/sqrt(a*b))/b^(11/2) + C*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.162 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=179

$$\frac{x^3\left(Ab^3-10a^3D\right)}{3ab^3\left(a+bx^2\right)^{7/2}}-\frac{a^3Dx}{b^4\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(-176a^3D+15a^2bC+6ab^2B+8Ab^3\right)}{105a^3b\left(a+bx^2\right)^{7/2}}+\frac{x^5\left(-58a^3D+3ab^2B+4Ab^3\right)}{15a^2b^2\left(a+bx^2\right)^{7/2}}$$

```
[Out] -a^3*D*x/b^4/(b*x^2+a)^(7/2)+1/3*(A*b^3-10*D*a^3)*x^3/a/b^3/(b*x^2+a)^(7/2)
+1/15*(4*A*b^3+3*B*a*b^2-58*D*a^3)*x^5/a^2/b^2/(b*x^2+a)^(7/2)+1/105*(8*A*b
^3+6*B*a*b^2+15*C*a^2*b-176*D*a^3)*x^7/a^3/b/(b*x^2+a)^(7/2)+D*arctanh(x*b^
(1/2)/(b*x^2+a)^(1/2))/b^(9/2)
```

Rubi [A] time = 0.31, antiderivative size = 192, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1804, 1585, 1263, 1584, 452, 288, 217, 206}

$$\frac{x^3\left(a\left(-71a^2D+15abC+6b^2B\right)+8Ab^3\right)}{105a^3b^3\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(17a^2D-10abC+3b^2B\right)+4Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^3)/(7*a*(a + b*x^2)^(7/2)) + ((4*A
*b^3 + a*(3*b^2*B - 10*a*b*C + 17*a^2*D))*x^3)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) + ((8*A*b^3 + a*(6*b^2*B + 15*a*b*C - 71*a^2*D))*x^3)/(105*a^3*b^3*(a +
b*x^2)^(3/2)) - (D*x)/(b^4*Sqrt[a + b*x^2]) + (D*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x\left(-\left(4Ab+\frac{3a\left(b^2B-abC+a^2D\right)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(-4Ab-\frac{3a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x(\cdots}{\cdots}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^2\cdots}{\cdots}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{(8A\cdots}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{(8A\cdots}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{(8A\cdots}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{(8A\cdots}{\cdots}$$

Mathematica [**A**] time = 0.46, size = 168, normalized size = 0.94

$$\frac{-105a^6Dx-350a^5bDx^3-406a^4b^2Dx^5-176a^3b^3Dx^7+a^2b^4x^3\left(35A+21Bx^2+15Cx^4\right)+2ab^5x^5\left(14A+3B\cdots\right.}{105a^3b^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-105*a^6*D*x - 350*a^5*b*D*x^3 - 406*a^4*b^2*D*x^5 + 8*A*b^6*x^7 - 176*a^3
*b^3*D*x^7 + 2*a*b^5*x^5*(14*A + 3*B*x^2) + a^2*b^4*x^3*(35*A + 21*B*x^2 +
15*C*x^4))/(105*a^3*b^4*(a + b*x^2)^(7/2)) + (Sqrt[a]*D*Sqrt[1 + (b*x^2)/a]
*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(b^(9/2)*Sqrt[a + b*x^2])
```

fricas [**A**] time = 0.76, size = 491, normalized size = 2.74

$$\left[\frac{105\left(Da^3b^4x^8+4\,Da^4b^3x^6+6\,Da^5b^2x^4+4\,Da^6bx^2+Da^7\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(105\,\cdots\right.}{210\left(a^3b^9x^8+4\,\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 + 6*D*a^5*b^2*x^4 + 4*D*a^6*b*
x^2 + D*a^7)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1
05*D*a^6*b*x + (176*D*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 - 8*A*b^7)*x^7 + 7
*(58*D*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*D*a^5*b^2 - A*a^2*b^
5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 + 6*a^5*b^7*x^4 + 4*a
^6*b^6*x^2 + a^7*b^5), -1/105*(105*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 + 6*D*a
^5*b^2*x^4 + 4*D*a^6*b*x^2 + D*a^7)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) + (105*D*a^6*b*x + (176*D*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 - 8*A*b^7
)*x^7 + 7*(58*D*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*D*a^5*b^2 -
 A*a^2*b^5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 + 6*a^5*b^7*
x^4 + 4*a^6*b^6*x^2 + a^7*b^5)]
```

giac [A] time = 0.55, size = 160, normalized size = 0.89

$$-\frac{\left(\left(x^2\left(\frac{\left(176\,Da^3b^6-15\,Ca^2b^7-6\,Bab^8-8\,Ab^9\right)x^2}{a^3b^7}+\frac{7\left(58\,Da^4b^5-3\,Ba^2b^7-4\,Aab^8\right)}{a^3b^7}\right)+\frac{35\left(10\,Da^5b^4-Aa^2b^7\right)}{a^3b^7}\right)x^2+\frac{105\,Da^3}{b^4}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{D\log\left(\left|-\sqrt{\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((176*D*a^3*b^6 - 15*C*a^2*b^7 - 6*B*a*b^8 - 8*A*b^9)*x^2/(a^3
*b^7) + 7*(58*D*a^4*b^5 - 3*B*a^2*b^7 - 4*A*a*b^8)/(a^3*b^7)) + 35*(10*D*a^
5*b^4 - A*a^2*b^7)/(a^3*b^7))*x^2 + 105*D*a^3/b^4)*x/(b*x^2 + a)^(7/2) - D*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

maple [B] time = 0.01, size = 363, normalized size = 2.03

$$-\frac{Dx^7}{7\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Cx^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Dx^5}{5\left(bx^2+a\right)^{\frac{5}{2}}b^2}-\frac{Bx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5Ca\,x^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Ax}{7\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{3Bax}{28\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*D*x^7/b/(b*x^2+a)^(7/2)-1/5*D/b^2*x^5/(b*x^2+a)^(5/2)-1/3*D/b^3*x^3/(b
*x^2+a)^(3/2)-D*x/b^4/(b*x^2+a)^(1/2)+D/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2
))-1/2*C*x^5/b/(b*x^2+a)^(7/2)-5/8*C*a/b^2*x^3/(b*x^2+a)^(7/2)-15/56*C*a^2/
b^3*x/(b*x^2+a)^(7/2)+3/56*C*a/b^3*x/(b*x^2+a)^(5/2)+1/14*C/b^3*x/(b*x^2+a)
^(3/2)+1/7*C/a/b^3*x/(b*x^2+a)^(1/2)-1/4*B*x^3/b/(b*x^2+a)^(7/2)-3/28*B*a/b
^2*x/(b*x^2+a)^(7/2)+3/140*B/b^2*x/(b*x^2+a)^(5/2)+1/35*B/a/b^2*x/(b*x^2+a)
^(3/2)+2/35*B*x/a^2/b^2/(b*x^2+a)^(1/2)-1/7*A/b*x/(b*x^2+a)^(7/2)+1/35*A/a/
b*x/(b*x^2+a)^(5/2)+4/105*A/a^2/b*x/(b*x^2+a)^(3/2)+8/105*A/a^3/b*x/(b*x^2+
a)^(1/2)
```

maxima [B] time = 1.66, size = 533, normalized size = 2.98

$$-\frac{1}{35}\left(\frac{35\,x^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{70\,ax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{\left(bx^2+a\right)^{\frac{7}{2}}b^4}\right)Dx-\frac{Dx\left(\frac{15\,x^4}{\left(bx^2+a\right)^{\frac{5}{2}}b}+\frac{20\,ax^2}{\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{\left(bx^2+a\right)^{\frac{5}{2}}b^3}\right)}{15\,b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*D*x - 1/
15*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8
*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*D*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - D*a*x^3/
((b*x^2 + a)^(5/2)*b^3) - 5/8*C*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B*x^3/(
(b*x^2 + a)^(7/2)*b) + 139/105*D*x/(sqrt(b*x^2 + a)*b^4) + 17/105*D*a*x/((b
*x^2 + a)^(3/2)*b^4) - 29/35*D*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*C*x/((b
*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2
 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*B*x/((b*x^
2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/((b*x^2 +
 a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/((b*x^2 + a
)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^2 + a)^(3/
2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) + D*arcsinh(b*x/sqrt(a*b))/b^(
9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.163 $\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{9/2}} dx$

Optimal. Leaf size=134

$$\frac{x^5\left(a(3aC+4bB)+24Ab^2\right)}{15a^3\left(a+bx^2\right)^{7/2}}+\frac{x^3(aB+6Ab)}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)}{105a^4\left(a+bx^2\right)^{7/2}}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

```
[Out] A*x/a/(b*x^2+a)^(7/2)+1/3*(6*A*b+B*a)*x^3/a^2/(b*x^2+a)^(7/2)+1/15*(24*A*b^
2+a*(4*B*b+3*C*a))*x^5/a^3/(b*x^2+a)^(7/2)+1/105*(48*A*b^3+a*(8*B*b^2+6*C*a
*b+15*D*a^2))*x^7/a^4/(b*x^2+a)^(7/2)
```

Rubi [A] time = 0.21, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1813, 1803, 12, 264}

$$\frac{x^7\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)}{105a^4\left(a+bx^2\right)^{7/2}}+\frac{x^5\left(a(3aC+4bB)+24Ab^2\right)}{15a^3\left(a+bx^2\right)^{7/2}}+\frac{x^3(aB+6Ab)}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (A*x)/(a*(a + b*x^2)^(7/2)) + ((6*A*b + a*B)*x^3)/(3*a^2*(a + b*x^2)^(7/2))
 + ((24*A*b^2 + a*(4*b*B + 3*a*C))*x^5)/(15*a^3*(a + b*x^2)^(7/2)) + ((48*A
*b^3 + a*(8*b^2*B + 6*a*b*C + 15*a^2*D))*x^7)/(105*a^4*(a + b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{x^2\left(6Ab+a\left(B+Cx^2+Dx^4\right)\right)}{\left(a+bx^2\right)^{9/2}}dx}{a}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{x^4\left(4b(6Ab+aB)+3a\left(aC+aDx^2\right)\right)}{\left(a+bx^2\right)^{9/2}}dx}{3a^2}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{\left(2b\left(24Ab^2+\right.\right.}{}}{}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(48Ab^3+a\right.}{}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(48Ab^3+a\right.}{1}$$

Mathematica [A] time = 0.11, size = 98, normalized size = 0.73

$$\frac{a^3\left(105Ax+35Bx^3+21Cx^5+15Dx^7\right)+2a^2bx^3\left(105A+14Bx^2+3Cx^4\right)+8ab^2x^5\left(21A+Bx^2\right)+48Ab^3x^7}{105a^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (48*A*b^3*x^7 + 8*a*b^2*x^5*(21*A + B*x^2) + 2*a^2*b*x^3*(105*A + 14*B*x^2
+ 3*C*x^4) + a^3*(105*A*x + 35*B*x^3 + 21*C*x^5 + 15*D*x^7))/(105*a^4*(a +
b*x^2)^(7/2))
```

fricas [A] time = 0.71, size = 141, normalized size = 1.05

$$\frac{\left(\left(15\,Da^3+6\,Ca^2b+8\,Bab^2+48\,Ab^3\right)x^7+7\left(3\,Ca^3+4\,Ba^2b+24\,Aab^2\right)x^5+105\,Aa^3x+35\left(Ba^3+6\,Aa^2b\right)x\right.}{105\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*((15*D*a^3 + 6*C*a^2*b + 8*B*a*b^2 + 48*A*b^3)*x^7 + 7*(3*C*a^3 + 4*B
*a^2*b + 24*A*a*b^2)*x^5 + 105*A*a^3*x + 35*(B*a^3 + 6*A*a^2*b)*x^3)*sqrt(b
*x^2 + a)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

giac [A] time = 0.53, size = 131, normalized size = 0.98

$$\frac{\left(\left(x^2\left(\frac{\left(15\,Da^3b^3+6\,Ca^2b^4+8\,Bab^5+48\,Ab^6\right)x^2}{a^4b^3}+\frac{7\left(3\,Ca^3b^3+4\,Ba^2b^4+24\,Aab^5\right)}{a^4b^3}\right)+\frac{35\left(Ba^3b^3+6\,Aa^2b^4\right)}{a^4b^3}\right)x^2+\frac{105\,A}{a}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((15*D*a^3*b^3 + 6*C*a^2*b^4 + 8*B*a*b^5 + 48*A*b^6)*x^2/(a^4*b
^3) + 7*(3*C*a^3*b^3 + 4*B*a^2*b^4 + 24*A*a*b^5)/(a^4*b^3)) + 35*(B*a^3*b^3
 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 109, normalized size = 0.81

$$\frac{\left(48A\,b^3x^6+8Ba\,b^2x^6+6a^2bC\,x^6+15Da^3x^6+168Aa\,b^2x^4+28B\,a^2b\,x^4+21a^3C\,x^4+210A\,a^2b\,x^2+35B\,a^3x^2+\right.}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*x*(48*A*b^3*x^6+8*B*a*b^2*x^6+6*C*a^2*b*x^6+15*D*a^3*x^6+168*A*a*b^2*
x^4+28*B*a^2*b*x^4+21*C*a^3*x^4+210*A*a^2*b*x^2+35*B*a^3*x^2+105*A*a^3)/(b*
x^2+a)^(7/2)/a^4
```

maxima [B] time = 1.42, size = 335, normalized size = 2.50

$$-\frac{Dx^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5\,Dax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Cx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{16\,Ax}{35\sqrt{bx^2+a}\,a^4}+\frac{8\,Ax}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Ax}{7\left(bx^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/
4*C*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/
((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^
2 + a)^(7/2)*a) + 1/14*D*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*D*x/(sqrt(b*x^2 +
a)*a*b^3) + 3/56*D*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*D*a^2*x/((b*x^2 + a)
^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)
*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7
/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 + a)*a^3*b
) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5/2)*a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{{\left(b\,x^2+a\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.164 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{2bx^7\left(4b\left(48Ab^2-a(aC+6bB)\right)-3a^3D\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(4b\left(48Ab^2-a(a\right.\right.}{15a^4\left(a+\right.}$$

```
[Out] -A/a/x/(b*x^2+a)^(7/2)-(8*A*b-B*a)*x/a^2/(b*x^2+a)^(7/2)-1/3*(48*A*b^2-a*(6
*B*b+C*a))*x^3/a^3/(b*x^2+a)^(7/2)-1/15*(4*b*(48*A*b^2-a*(6*B*b+C*a))-3*a^3
*D)*x^5/a^4/(b*x^2+a)^(7/2)-2/105*b*(4*b*(48*A*b^2-a*(6*B*b+C*a))-3*a^3*D)*
x^7/a^5/(b*x^2+a)^(7/2)
```

Rubi [A] time = 0.25, antiderivative size = 179, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1803, 1813, 12, 271, 264}

$$-\frac{2bx^7\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^3\left(48Ab^2-a(aC+6b\right.}{3a^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((192*A*b^3
- 4*a*b*(6*b*B + a*C) - 3*a^3*D)*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - (2*b*(19
2*A*b^3 - 4*a*b*(6*b*B + a*C) - 3*a^3*D)*x^7)/(105*a^5*(a + b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx=-\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{8Ab-a\left(B+Cx^2+Dx^4\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a}$$

$$=-\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^2\left(6b(8Ab-aB)+a\left(-aC-aDx^2\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a^2}$$

$$=-\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{\left(4b\left(48Ab^2-6abB\right.\right.}{\left(a+b\right.}}{3}$$

$$=-\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{\left(192Ab^3-4ab\right.}{}$$

$$=-\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{\left(192Ab^3-4ab\right.}{15a^4}$$

$$=-\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{\left(192Ab^3-4ab\right.}{15a^4}$$

Mathematica [A] time = 0.18, size = 133, normalized size = 0.72

$$\frac{-7a^4\left(15A-15Bx^2-5Cx^4-3Dx^6\right)+2a^3bx^2\left(-420A+105Bx^2+14Cx^4+3Dx^6\right)+8a^2b^2x^4\left(-210A+21Bx^2+\right.}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 21
*B*x^2 + C*x^4) - 7*a^4*(15*A - 15*B*x^2 - 5*C*x^4 - 3*D*x^6) + 2*a^3*b*x^2
*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6))/(105*a^5*x*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.94, size = 182, normalized size = 0.98

$$\frac{\left(2\left(3\,Da^3b+4\,Ca^2b^2+24\,Bab^3-192\,Ab^4\right)x^8+7\left(3\,Da^4+4\,Ca^3b+24\,Ba^2b^2-192\,Aab^3\right)x^6-105\,Aa^4+35\left(C\right.\right.}{105\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*D*a^3*b + 4*C*a^2*b^2 + 24*B*a*b^3 - 192*A*b^4)*x^8 + 7*(3*D*a^
4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35*(C*a^4 + 6
```

```
*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a)
/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)
```

giac [A] time = 0.58, size = 211, normalized size = 1.14

$$\frac{\left(\left(x^2\left(\frac{(6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7)x^2}{a^{14}b^3}+\frac{7\left(3\,Da^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-132\,Aa^{10}b^6\right)}{a^{14}b^3}\right)+\frac{35\left(Ca^{13}b^3+6\,Ba^{12}b^4-30\,Aa^{11}b^5\right)}{a^{14}b^3}\right.\right.}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 - 279*A*a^9*b^7)*
x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11*b^5 - 132*A*a^1
0*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a^11*b^5)/(a^14*b
^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b*x^2 + a)^(7/2)
+ 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

maple [A] time = 0.01, size = 157, normalized size = 0.85

$$-\frac{384A\,b^4x^8-48Ba\,b^3x^8-8C\,a^2b^2x^8-6Da^3b\,x^8+1344Aa\,b^3x^6-168B\,a^2b^2x^6-28C\,a^3b\,x^6-21Da^4x^6+168}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(384*A*b^4*x^8-48*B*a*b^3*x^8-8*C*a^2*b^2*x^8-6*D*a^3*b*x^8+1344*A*a
*b^3*x^6-168*B*a^2*b^2*x^6-28*C*a^3*b*x^6-21*D*a^4*x^6+1680*A*a^2*b^2*x^4-2
10*B*a^3*b*x^4-35*C*a^4*x^4+840*A*a^3*b*x^2-105*B*a^4*x^2+105*A*a^4)/(b*x^2
+a)^(7/2)/x/a^5
```

maxima [A] time = 1.47, size = 313, normalized size = 1.69

$$-\frac{Dx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{16\,Bx}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Bx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Bx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Bx}{7\left(bx^2+a\right)^{\frac{7}{2}}a}+\frac{3\,Dx}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{}{35\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*D*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B
*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b
*x^2 + a)^(7/2)*a) + 3/140*D*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*D*x/(sqrt(b*x
^2 + a)*a^2*b^2) + 1/35*D*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*D*a*x/((b*x^2
+ a)^(7/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a
)*a^3*b) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2
)*a*b) - 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2
)*a^4) - 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)
*a^2) - A/((b*x^2 + a)^(7/2)*a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^2\,\left(b\,x^2+a\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(9/2)),x)
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

3.165 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=242

$$\frac{x\left(80Ab^2-3a(8bB-aC)\right)}{3a^3\left(a+bx^2\right)^{7/2}}+\frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}}+\frac{8b^2x^7\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{7/2}}+\frac{4bx^5\left(160Ab^3\right.}{}$$

```
[Out] -1/3*A/a/x^3/(b*x^2+a)^(7/2)+1/3*(10*A*b-3*B*a)/a^2/x/(b*x^2+a)^(7/2)+1/3*(
80*A*b^2-3*a*(8*B*b-C*a))*x/a^3/(b*x^2+a)^(7/2)+1/3*(160*A*b^3-a*(48*B*b^2-
6*C*a*b-D*a^2))*x^3/a^4/(b*x^2+a)^(7/2)+4/15*b*(160*A*b^3-a*(48*B*b^2-6*C*a
*b-D*a^2))*x^5/a^5/(b*x^2+a)^(7/2)+8/105*b^2*(160*A*b^3-a*(48*B*b^2-6*C*a*b
-D*a^2))*x^7/a^6/(b*x^2+a)^(7/2)
```

Rubi [A] time = 0.32, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1803, 1813, 12, 271, 264}

$$\frac{8b^2x^7\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{7/2}}+\frac{4bx^5\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{15a^5\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(160Ab^3-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(3*a*x^3*(a + b*x^2)^(7/2)) + (10*A*b - 3*a*B)/(3*a^2*x*(a + b*x^2)^(7/2
)) + ((80*A*b^2 - 3*a*(8*b*B - a*C))*x)/(3*a^3*(a + b*x^2)^(7/2)) + ((160*A
*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^3)/(3*a^4*(a + b*x^2)^(7/2)) + (4*
b*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^5)/(15*a^5*(a + b*x^2)^(7/
2)) + (8*b^2*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^7)/(105*a^6*(a
+ b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{10Ab-3a\left(B+Cx^2+Dx^4\right)}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{3a}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{8b(10Ab-3aB)-a\left(-3aC-3aDx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^2}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{(6b(80Ab \cdots}{\cdots}}{\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(160Ab^3 \cdots\right.}{\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(160Ab^3 \cdots\right.}{\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(160Ab^3 \cdots\right.}{\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(160Ab^3 \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.13, size = 165, normalized size = 0.68

$$\frac{-35a^5\left(A+3Bx^2-3Cx^4-Dx^6\right)+14a^4bx^2\left(25A-60Bx^2+15Cx^4+2Dx^6\right)+8a^3b^2x^4\left(350A-210Bx^2+21Cx \cdots\right.}{105a^6x^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (1280*A*b^5*x^10 + 128*a*b^4*x^8*(35*A - 3*B*x^2) + 16*a^2*b^3*x^6*(350*A -
 84*B*x^2 + 3*C*x^4) - 35*a^5*(A + 3*B*x^2 - 3*C*x^4 - D*x^6) + 8*a^3*b^2*x
^4*(350*A - 210*B*x^2 + 21*C*x^4 + D*x^6) + 14*a^4*b*x^2*(25*A - 60*B*x^2 +
 15*C*x^4 + 2*D*x^6))/(105*a^6*x^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.00, size = 225, normalized size = 0.93

$$\frac{\left(8\left(Da^3b^2+6\,Ca^2b^3-48\,Bab^4+160\,Ab^5\right)x^{10}+28\left(Da^4b+6\,Ca^3b^2-48\,Ba^2b^3+160\,Aab^4\right)x^8+35\left(Da^5+6\,C \cdots\right.}{105\left(a^6b^4x^{11}+4\,a^7b^3x^9+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(D*a^3*b^2 + 6*C*a^2*b^3 - 48*B*a*b^4 + 160*A*b^5)*x^10 + 28*(D*a^
4*b + 6*C*a^3*b^2 - 48*B*a^2*b^3 + 160*A*a*b^4)*x^8 + 35*(D*a^5 + 6*C*a^4*b
 - 48*B*a^3*b^2 + 160*A*a^2*b^3)*x^6 - 35*A*a^5 + 35*(3*C*a^5 - 24*B*a^4*b
+ 80*A*a^3*b^2)*x^4 - 35*(3*B*a^5 - 10*A*a^4*b)*x^2)*sqrt(b*x^2 + a)/(a^6*b
^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

giac [A] time = 0.56, size = 349, normalized size = 1.44

$$\frac{\left(\left(x^2\left(\frac{\left(8\,Da^{15}b^5+48\,Ca^{14}b^6-279\,Ba^{13}b^7+790\,Aa^{12}b^8\right)x^2}{a^{18}b^3}+\frac{7\left(4\,Da^{16}b^4+24\,Ca^{15}b^5-132\,Ba^{14}b^6+365\,Aa^{13}b^7\right)}{a^{18}b^3}\right)+\frac{35\left(Da^{17}b^3+6\,Ca^{16}b^4-30\,Ba\cdots\right.}{a^{18}b^3}\cdots}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((8*D*a^15*b^5 + 48*C*a^14*b^6 - 279*B*a^13*b^7 + 790*A*a^12*b^
8)*x^2/(a^18*b^3) + 7*(4*D*a^16*b^4 + 24*C*a^15*b^5 - 132*B*a^14*b^6 + 365*
A*a^13*b^7)/(a^18*b^3)) + 35*(D*a^17*b^3 + 6*C*a^16*b^4 - 30*B*a^15*b^5 + 8
0*A*a^14*b^6)/(a^18*b^3))*x^2 + 105*(C*a^17*b^3 - 4*B*a^16*b^4 + 10*A*a^15*
b^5)/(a^18*b^3))*x/(b*x^2 + a)^(7/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*B*a*sqrt(b) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A
*a*b^(3/2) + 3*B*a^3*sqrt(b) - 14*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2
+ a))^2 - a)^3*a^5)
```

maple [A] time = 0.01, size = 205, normalized size = 0.85

$$-\frac{-1280A\,b^5x^{10}+384Ba\,b^4x^{10}-48C\,a^2b^3x^{10}-8Da^3b^2x^{10}-4480Aa\,b^4x^8+1344B\,a^2b^3x^8-168C\,a^3b^2x^8-2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-1280*A*b^5*x^10+384*B*a*b^4*x^10-48*C*a^2*b^3*x^10-8*D*a^3*b^2*x^1
0-4480*A*a*b^4*x^8+1344*B*a^2*b^3*x^8-168*C*a^3*b^2*x^8-28*D*a^4*b*x^8-5600
*A*a^2*b^3*x^6+1680*B*a^3*b^2*x^6-210*C*a^4*b*x^6-35*D*a^5*x^6-2800*A*a^3*b
^2*x^4+840*B*a^4*b*x^4-105*C*a^5*x^4-350*A*a^4*b*x^2+105*B*a^5*x^2+35*A*a^5
)/(b*x^2+a)^(7/2)/x^3/a^6
```

maxima [A] time = 1.47, size = 337, normalized size = 1.39

$$\frac{16\,Cx}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Cx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Cx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Cx}{7\left(bx^2+a\right)^{\frac{7}{2}}a}-\frac{Dx}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,Dx}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{\cdots}{105\left(b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*C
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 1/7*D*x/((b*x^
2 + a)^(7/2)*b) + 8/105*D*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*D*x/((b*x^2 + a
)^(3/2)*a^2*b) + 1/35*D*x/((b*x^2 + a)^(5/2)*a*b) - 128/35*B*b*x/(sqrt(b*x^
2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*B*b*x/((b*x^2 + a
```

```
)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*A*b^2*x/(sqrt(b*x
^2 + a)*a^6) + 128/21*A*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*A*b^2*x/((b*x^
2 + a)^(5/2)*a^4) + 80/21*A*b^2*x/((b*x^2 + a)^(7/2)*a^3) - B/((b*x^2 + a)^
(7/2)*a*x) + 10/3*A*b/((b*x^2 + a)^(7/2)*a^2*x) - 1/3*A/((b*x^2 + a)^(7/2)*
a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,x^2 + C\,x^4 + x^6\,D}{x^4\,{\left(b\,x^2 + a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(9/2)),x)
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

3.166 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}}+\frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}}-\frac{16x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{105a^7\sqrt{a+bx^2}}-\frac{8x\left(192Ab^3-a\right.}{105}$$

```
[Out] -1/5*A/a/x^5/(b*x^2+a)^(7/2)+1/15*(12*A*b-5*B*a)/a^2/x^3/(b*x^2+a)^(7/2)+1/
3*(-24*A*b^2+a*(10*B*b-3*C*a))/a^3/x/(b*x^2+a)^(7/2)-1/21*(192*A*b^3-a*(80*
B*b^2-24*C*a*b+3*D*a^2))*x/a^4/(b*x^2+a)^(7/2)-2/35*(192*A*b^3-a*(80*B*b^2-
24*C*a*b+3*D*a^2))*x/a^5/(b*x^2+a)^(5/2)-8/105*(192*A*b^3-a*(80*B*b^2-24*C*
a*b+3*D*a^2))*x/a^6/(b*x^2+a)^(3/2)-16/105*(192*A*b^3-a*(80*B*b^2-24*C*a*b+
3*D*a^2))*x/a^7/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 275, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1803, 12, 192, 191}

$$-\frac{16x\left(-3a^3D-8ab(10bB-3aC)+192Ab^3\right)}{105a^7\sqrt{a+bx^2}}-\frac{8x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{3/2}}-\frac{2x\left(-3a^3D-8ab(1\right.}{35a^5\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(5*a*x^5*(a + b*x^2)^(7/2)) + (12*A*b - 5*a*B)/(15*a^2*x^3*(a + b*x^2)^(
7/2)) - (24*A*b^2 - a*(10*b*B - 3*a*C))/(3*a^3*x*(a + b*x^2)^(7/2)) - ((192
*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(21*a^4*(a + b*x^2)^(7/2)) -
(2*(192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(35*a^5*(a + b*x^2)^(5
/2)) - (8*(192*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(105*a^6*(a +
b*x^2)^(3/2)) - (16*(192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(105*
a^7*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
```

```
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{12Ab-5a\left(B+Cx^2+Dx^4\right)}{x^4\left(a+bx^2\right)^{9/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{10b(12Ab-5aB)-3a\left(-5aC-5aDx^2\right)}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{15a^2}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{8b(120Ab^2 \cdots}{\cdots}}{\cdots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3 - \cdots\right.}{\cdots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3 - \cdots\right.}{\cdots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3 - \cdots\right.}{\cdots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3 - \cdots\right.}{\cdots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3 - \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.16, size = 202, normalized size = 0.72

$$\frac{-7a^6\left(3A+5x^2\left(B+3Cx^2-3Dx^4\right)\right)+14a^5bx^2\left(6A+25Bx^2-60Cx^4+15Dx^6\right)+56a^4b^2x^4\left(-15A+50Bx^2-3\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-3072*A*b^6*x^12 + 256*a*b^5*x^10*(-42*A + 5*B*x^2) - 128*a^2*b^4*x^8*(105
*A - 35*B*x^2 + 3*C*x^4) + 16*a^3*b^3*x^6*(-420*A + 350*B*x^2 - 84*C*x^4 +
3*D*x^6) + 56*a^4*b^2*x^4*(-15*A + 50*B*x^2 - 30*C*x^4 + 3*D*x^6) + 14*a^5*
b*x^2*(6*A + 25*B*x^2 - 60*C*x^4 + 15*D*x^6) - 7*a^6*(3*A + 5*x^2*(B + 3*C*
x^2 - 3*D*x^4)))/(105*a^7*x^5*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.21, size = 270, normalized size = 0.96

$$\frac{\left(16\left(3\,Da^3b^3-24\,Ca^2b^4+80\,Bab^5-192\,Ab^6\right)x^{12}+56\left(3\,Da^4b^2-24\,Ca^3b^3+80\,Ba^2b^4-192\,Aab^5\right)x^{10}+70\left(3\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(16*(3*D*a^3*b^3 - 24*C*a^2*b^4 + 80*B*a*b^5 - 192*A*b^6)*x^12 + 56*(
3*D*a^4*b^2 - 24*C*a^3*b^3 + 80*B*a^2*b^4 - 192*A*a*b^5)*x^10 + 70*(3*D*a^5
*b - 24*C*a^4*b^2 + 80*B*a^3*b^3 - 192*A*a^2*b^4)*x^8 - 21*A*a^6 + 35*(3*D*
a^6 - 24*C*a^5*b + 80*B*a^4*b^2 - 192*A*a^3*b^3)*x^6 - 35*(3*C*a^6 - 10*B*a
^5*b + 24*A*a^4*b^2)*x^4 - 7*(5*B*a^6 - 12*A*a^5*b)*x^2)*sqrt(b*x^2 + a)/(a
^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)
```

giac [B] time = 0.63, size = 592, normalized size = 2.11

$$\frac{\left(\left(x^{2}\left(\frac{\left(48\,Da^{18}b^{6}-279\,Ca^{17}b^{7}+790\,Ba^{16}b^{8}-1686\,Aa^{15}b^{9}\right)x^{2}}{a^{22}b^{3}}+\frac{7\left(24\,Da^{19}b^{5}-132\,Ca^{18}b^{6}+365\,Ba^{17}b^{7}-768\,Aa^{16}b^{8}\right)}{a^{22}b^{3}}\right)+\frac{35\left(6\,Da^{20}b^{4}-30\,Ca^{19}b\right.}{\ldots}}{105\left(bx^{2}+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((48*D*a^18*b^6 - 279*C*a^17*b^7 + 790*B*a^16*b^8 - 1686*A*a^15
*b^9)*x^2/(a^22*b^3) + 7*(24*D*a^19*b^5 - 132*C*a^18*b^6 + 365*B*a^17*b^7 -
 768*A*a^16*b^8)/(a^22*b^3)) + 35*(6*D*a^20*b^4 - 30*C*a^19*b^5 + 80*B*a^18
*b^6 - 165*A*a^17*b^7)/(a^22*b^3))*x^2 + 105*(D*a^21*b^3 - 4*C*a^20*b^4 + 1
0*B*a^19*b^5 - 20*A*a^18*b^6)/(a^22*b^3))*x/(b*x^2 + a)^(7/2) + 2/15*(15*(s
qrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*sqrt(b) - 60*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*B*a*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 60*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b) + 270*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*B*a^2*b^(3/2) - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/2) + 90*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*sqrt(b) - 430*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a^3*b^(3/2) + 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2)
- 60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^5*sqrt(b) + 290*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*B*a^4*b^(3/2) - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(
5/2) + 15*C*a^6*sqrt(b) - 70*B*a^5*b^(3/2) + 198*A*a^4*b^(5/2))/(((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

maple [A] time = 0.01, size = 253, normalized size = 0.90

$$-\frac{3072A\,b^{6}x^{12}-1280Ba\,b^{5}x^{12}+384C\,a^{2}b^{4}x^{12}-48Da^{3}b^{3}x^{12}+10752Aa\,b^{5}x^{10}-4480B\,a^{2}b^{4}x^{10}+1344C\,a^{3}b^{3}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(3072*A*b^6*x^12-1280*B*a*b^5*x^12+384*C*a^2*b^4*x^12-48*D*a^3*b^3*x
^12+10752*A*a*b^5*x^10-4480*B*a^2*b^4*x^10+1344*C*a^3*b^3*x^10-168*D*a^4*b^
2*x^10+13440*A*a^2*b^4*x^8-5600*B*a^3*b^3*x^8+1680*C*a^4*b^2*x^8-210*D*a^5*
b*x^8+6720*A*a^3*b^3*x^6-2800*B*a^4*b^2*x^6+840*C*a^5*b*x^6-105*D*a^6*x^6+8
40*A*a^4*b^2*x^4-350*B*a^5*b*x^4+105*C*a^6*x^4-84*A*a^5*b*x^2+35*B*a^6*x^2+
21*A*a^6)/(b*x^2+a)^(7/2)/x^5/a^7
```

maxima [A] time = 1.55, size = 398, normalized size = 1.42

$$\frac{16\,Dx}{35\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{8\,Dx}{35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{6\,Dx}{35\left(bx^{2}+a\right)^{\frac{5}{2}}a^{2}}+\frac{Dx}{7\left(bx^{2}+a\right)^{\frac{7}{2}}a}-\frac{128\,Cbx}{35\,\sqrt{bx^{2}+a}\,a^{5}}-\frac{64\,Cbx}{35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{4}}-\frac{48\ldots}{35\left(bx\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*D*x/(sqrt(b*x^2 + a)*a^4) + 8/35*D*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*D
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*D*x/((b*x^2 + a)^(7/2)*a) - 128/35*C*b*x/(
sqrt(b*x^2 + a)*a^5) - 64/35*C*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*C*b*x/((
b*x^2 + a)^(5/2)*a^3) - 8/7*C*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*B*b^2*x/
(sqrt(b*x^2 + a)*a^6) + 128/21*B*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*B*b^2
*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*B*b^2*x/((b*x^2 + a)^(7/2)*a^3) - 1024/3
5*A*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*A*b^3*x/((b*x^2 + a)^(3/2)*a^6) -
384/35*A*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*A*b^3*x/((b*x^2 + a)^(7/2)*a^
4) - C/((b*x^2 + a)^(7/2)*a*x) + 10/3*B*b/((b*x^2 + a)^(7/2)*a^2*x) - 8*A*b
^2/((b*x^2 + a)^(7/2)*a^3*x) - 1/3*B/((b*x^2 + a)^(7/2)*a*x^3) + 4/5*A*b/((
b*x^2 + a)^(7/2)*a^2*x^3) - 1/5*A/((b*x^2 + a)^(7/2)*a*x^5)
```

mupad [B] time = 2.40, size = 405, normalized size = 1.44

$$\frac{\frac{61\,A\,b}{35\,a^3}+\frac{78\,A\,b^2\,x^2}{35\,a^4}}{x^3\,\left(b\,x^2+a\right)^{5/2}}+\frac{\frac{128\,B\,b}{21\,a^5}+\frac{256\,B\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}}+\frac{x\,D}{\left(b\,x^2+a\right)^{9/2}}-\frac{\frac{B}{3\,a^2}+\frac{19\,B\,b\,x^2}{21\,a^3}}{x^3\,\left(b\,x^2+a\right)^{5/2}}-\frac{\frac{C}{a^4}+\frac{128\,C\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{512\,A\,b^2}{35\,a^6}+\frac{1024\,A\,b^3\,x^2}{35\,a^7}}{x\,\sqrt{b\,x^2+a}}-\frac{A\,\sqrt{b\,x^2}}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((61*A*b)/(35*a^3) + (78*A*b^2*x^2)/(35*a^4))/(x^3*(a + b*x^2)^(5/2)) + ((1
28*B*b)/(21*a^5) + (256*B*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) + (x*D)/
(a + b*x^2)^(9/2) - (B/(3*a^2) + (19*B*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5
/2)) - (C/a^4 + (128*C*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - ((512*A*b^2
)/(35*a^6) + (1024*A*b^3*x^2)/(35*a^7))/(x*(a + b*x^2)^(1/2)) - (A*(a + b*x
^2)^(1/2))/(5*a^5*x^5) + (18*b^2*x^5*D)/(5*a^2*(a + b*x^2)^(9/2)) + (72*b^3
*x^7*D)/(35*a^3*(a + b*x^2)^(9/2)) + (16*b^4*x^9*D)/(35*a^4*(a + b*x^2)^(9/
2)) - (A*b)/(7*a^2*x^3*(a + b*x^2)^(7/2)) - (32*B*b)/(21*a^4*x*(a + b*x^2)^
(3/2)) + (B*b^2*x)/(7*a^3*(a + b*x^2)^(7/2)) + (27*A*b^2)/(7*a^5*x*(a + b*x
^2)^(3/2)) + (3*b*x^3*D)/(a*(a + b*x^2)^(9/2)) - (29*C*b*x)/(35*a^4*(a + b*
x^2)^(3/2)) - (13*C*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (C*b*x)/(7*a^2*(a + b
*x^2)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.167 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=334

$$-\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}}+\frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}}+\frac{128bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^8\sqrt{a+bx^2}}+\frac{64bx\left(48Ab^3-\right.}{10}$$

```
[Out] -1/7*A/a/x^7/(b*x^2+a)^(7/2)+1/5*(2*A*b-B*a)/a^2/x^5/(b*x^2+a)^(7/2)+1/15*(
-24*A*b^2+a*(12*B*b-5*C*a))/a^3/x^3/(b*x^2+a)^(7/2)+1/3*(48*A*b^3-a*(24*B*b
^2-10*C*a*b+3*D*a^2))/a^4/x/(b*x^2+a)^(7/2)+8/21*b*(48*A*b^3-a*(24*B*b^2-10
*C*a*b+3*D*a^2))*x/a^5/(b*x^2+a)^(7/2)+16/35*b*(48*A*b^3-a*(24*B*b^2-10*C*a
*b+3*D*a^2))*x/a^6/(b*x^2+a)^(5/2)+64/105*b*(48*A*b^3-a*(24*B*b^2-10*C*a*b+
3*D*a^2))*x/a^7/(b*x^2+a)^(3/2)+128/105*b*(48*A*b^3-a*(24*B*b^2-10*C*a*b+3*
D*a^2))*x/a^8/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1803, 12, 271, 192, 191}

$$\frac{128bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^8\sqrt{a+bx^2}}+\frac{64bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^7\left(a+bx^2\right)^{3/2}}+\frac{16bx\left(48Ab^3-a\right.}{35a^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(7*a*x^7*(a + b*x^2)^(7/2)) + (2*A*b - a*B)/(5*a^2*x^5*(a + b*x^2)^(7/2)
) - (24*A*b^2 - a*(12*b*B - 5*a*C))/(15*a^3*x^3*(a + b*x^2)^(7/2)) + (48*A*
b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))/(3*a^4*x*(a + b*x^2)^(7/2)) + (8*b
*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(21*a^5*(a + b*x^2)^(7/2
)) + (16*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(35*a^6*(a + b
*x^2)^(5/2)) + (64*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(105
*a^7*(a + b*x^2)^(3/2)) + (128*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2
*D))*x)/(105*a^8*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^8\left(a + bx^2\right)^{9/2}}\,dx = -\frac{A}{7ax^7\left(a + bx^2\right)^{7/2}} - \frac{\int \frac{14Ab - 7a\left(B + Cx^2 + Dx^4\right)}{x^6\left(a + bx^2\right)^{9/2}}\,dx}{7a}$$

$$= -\frac{A}{7ax^7\left(a + bx^2\right)^{7/2}} + \frac{2Ab - aB}{5a^2x^5\left(a + bx^2\right)^{7/2}} + \frac{\int \frac{12b(14Ab - 7aB) - 5a\left(-7aC - 7aDx^2\right)}{x^4\left(a + bx^2\right)^{9/2}}\,dx}{35a^2}$$

$$= -\frac{A}{7ax^7\left(a + bx^2\right)^{7/2}} + \frac{2Ab - aB}{5a^2x^5\left(a + bx^2\right)^{7/2}} - \frac{24Ab^2 - a(12bB - 5aC)}{15a^3x^3\left(a + bx^2\right)^{7/2}} - \frac{\int \frac{10b\left(168Ab^2 \cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{A}{7ax^7\left(a + bx^2\right)^{7/2}} + \frac{2Ab - aB}{5a^2x^5\left(a + bx^2\right)^{7/2}} - \frac{24Ab^2 - a(12bB - 5aC)}{15a^3x^3\left(a + bx^2\right)^{7/2}} - \frac{\left(48Ab^3 - a \cdots\right.}{\cdots}$$

$$= -\frac{A}{7ax^7\left(a + bx^2\right)^{7/2}} + \frac{2Ab - aB}{5a^2x^5\left(a + bx^2\right)^{7/2}} - \frac{24Ab^2 - a(12bB - 5aC)}{15a^3x^3\left(a + bx^2\right)^{7/2}} + \frac{48Ab^3 - a\,(\cdots}{3\cdots}$$

$$= -\frac{A}{7ax^7\left(a + bx^2\right)^{7/2}} + \frac{2Ab - aB}{5a^2x^5\left(a + bx^2\right)^{7/2}} - \frac{24Ab^2 - a(12bB - 5aC)}{15a^3x^3\left(a + bx^2\right)^{7/2}} + \frac{48Ab^3 - a\,(\cdots}{3\cdots}$$

$$= -\frac{A}{7ax^7\left(a + bx^2\right)^{7/2}} + \frac{2Ab - aB}{5a^2x^5\left(a + bx^2\right)^{7/2}} - \frac{24Ab^2 - a(12bB - 5aC)}{15a^3x^3\left(a + bx^2\right)^{7/2}} + \frac{48Ab^3 - a\,(\cdots}{3\cdots}$$

$$= -\frac{A}{7ax^7\left(a + bx^2\right)^{7/2}} + \frac{2Ab - aB}{5a^2x^5\left(a + bx^2\right)^{7/2}} - \frac{24Ab^2 - a(12bB - 5aC)}{15a^3x^3\left(a + bx^2\right)^{7/2}} + \frac{48Ab^3 - a\,(\cdots}{3\cdots}$$

$$= -\frac{A}{7ax^7\left(a + bx^2\right)^{7/2}} + \frac{2Ab - aB}{5a^2x^5\left(a + bx^2\right)^{7/2}} - \frac{24Ab^2 - a(12bB - 5aC)}{15a^3x^3\left(a + bx^2\right)^{7/2}} + \frac{48Ab^3 - a\,(\cdots}{3\cdots}$$

Mathematica [A] time = 0.16, size = 234, normalized size = 0.70

$$\frac{-a^7\left(15A + 21Bx^2 + 35x^4\left(C + 3Dx^2\right)\right) + 14a^6bx^2\left(3A + 6Bx^2 + 25Cx^4 - 60Dx^6\right) - 56a^5b^2x^4\left(3A + 15Bx^2 - 50\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

```
[Out] (6144*A*b^7*x^14 - 3072*a*b^6*x^12*(-7*A + B*x^2) + 256*a^2*b^5*x^10*(105*A
 - 42*B*x^2 + 5*C*x^4) + 14*a^6*b*x^2*(3*A + 6*B*x^2 + 25*C*x^4 - 60*D*x^6)
 + 112*a^4*b^3*x^6*(15*A - 60*B*x^2 + 50*C*x^4 - 12*D*x^6) + 128*a^3*b^4*x^
8*(105*A - 105*B*x^2 + 35*C*x^4 - 3*D*x^6) - 56*a^5*b^2*x^4*(3*A + 15*B*x^2
 - 50*C*x^4 + 30*D*x^6) - a^7*(15*A + 21*B*x^2 + 35*x^4*(C + 3*D*x^2)))/(10
5*a^8*x^7*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.90, size = 311, normalized size = 0.93

$$-\frac{\left(128\left(3\,Da^3b^4-10\,Ca^2b^5+24\,Bab^6-48\,Ab^7\right)x^{14}+448\left(3\,Da^4b^3-10\,Ca^3b^4+24\,Ba^2b^5-48\,Aab^6\right)x^{12}+5\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(128*(3*D*a^3*b^4 - 10*C*a^2*b^5 + 24*B*a*b^6 - 48*A*b^7)*x^14 + 448
*(3*D*a^4*b^3 - 10*C*a^3*b^4 + 24*B*a^2*b^5 - 48*A*a*b^6)*x^12 + 560*(3*D*a
^5*b^2 - 10*C*a^4*b^3 + 24*B*a^3*b^4 - 48*A*a^2*b^5)*x^10 + 280*(3*D*a^6*b
- 10*C*a^5*b^2 + 24*B*a^4*b^3 - 48*A*a^3*b^4)*x^8 + 15*A*a^7 + 35*(3*D*a^7
- 10*C*a^6*b + 24*B*a^5*b^2 - 48*A*a^4*b^3)*x^6 + 7*(5*C*a^7 - 12*B*a^6*b +
 24*A*a^5*b^2)*x^4 + 21*(B*a^7 - 2*A*a^6*b)*x^2)*sqrt(b*x^2 + a)/(a^8*b^4*x
^15 + 4*a^9*b^3*x^13 + 6*a^10*b^2*x^11 + 4*a^11*b*x^9 + a^12*x^7)
```

giac [B] time = 0.71, size = 938, normalized size = 2.81

$$-\frac{\left(\left(x^2\left(\frac{\left(279\,Da^{21}b^7-790\,Ca^{20}b^8+1686\,Ba^{19}b^9-3072\,Aa^{18}b^{10}\right)x^2}{a^{26}b^3}+\frac{7\left(132\,Da^{22}b^6-365\,Ca^{21}b^7+768\,Ba^{20}b^8-1386\,Aa^{19}b^9\right)}{a^{26}b^3}\right)+\frac{35\left(30\,Da^{23}b^5\cdots\right.}{}\right.\right.}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((279*D*a^21*b^7 - 790*C*a^20*b^8 + 1686*B*a^19*b^9 - 3072*A*a
^18*b^10)*x^2/(a^26*b^3) + 7*(132*D*a^22*b^6 - 365*C*a^21*b^7 + 768*B*a^20*
b^8 - 1386*A*a^19*b^9)/(a^26*b^3)) + 35*(30*D*a^23*b^5 - 80*C*a^22*b^6 + 16
5*B*a^21*b^7 - 294*A*a^20*b^8)/(a^26*b^3))*x^2 + 105*(4*D*a^24*b^4 - 10*C*a
^23*b^5 + 20*B*a^22*b^6 - 35*A*a^21*b^7)/(a^26*b^3))*x/(b*x^2 + a)^(7/2) +
2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^3*sqrt(b) - 420*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*C*a^2*b^(3/2) + 1050*(sqrt(b)*x - sqrt(b*x^2 + a))^12
*B*a*b^(5/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) - 630*(sqrt(
b)*x - sqrt(b*x^2 + a))^10*D*a^4*sqrt(b) + 2730*(sqrt(b)*x - sqrt(b*x^2 + a
))^10*C*a^3*b^(3/2) - 7140*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) +
 14700*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(7/2) + 1575*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*D*a^5*sqrt(b) - 7210*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^4*
b^(3/2) + 19950*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2) - 42840*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*D*a^6*sqrt(b) + 9940*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^5*b^(3/2) - 2
8560*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 64680*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*A*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^7*
sqrt(b) - 7560*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^6*b^(3/2) + 21966*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 49812*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*A*a^4*b^(7/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^8*sqrt(b) + 30
10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^7*b^(3/2) - 8652*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*B*a^6*b^(5/2) + 19404*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^
(7/2) + 105*D*a^9*sqrt(b) - 490*C*a^8*b^(3/2) + 1386*B*a^7*b^(5/2) - 3072*A
*a^6*b^(7/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7*a^7)
```

maple [A] time = 0.01, size = 301, normalized size = 0.90

$$-\frac{-6144A\,b^7x^{14}+3072Ba\,b^6x^{14}-1280C\,a^2b^5x^{14}+384Da^3b^4x^{14}-21504Aa\,b^6x^{12}+10752B\,a^2b^5x^{12}-4480C\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-6144*A*b^7*x^14+3072*B*a*b^6*x^14-1280*C*a^2*b^5*x^14+384*D*a^3*b^
4*x^14-21504*A*a*b^6*x^12+10752*B*a^2*b^5*x^12-4480*C*a^3*b^4*x^12+1344*D*a
^4*b^3*x^12-26880*A*a^2*b^5*x^10+13440*B*a^3*b^4*x^10-5600*C*a^4*b^3*x^10+1
680*D*a^5*b^2*x^10-13440*A*a^3*b^4*x^8+6720*B*a^4*b^3*x^8-2800*C*a^5*b^2*x^
8+840*D*a^6*b*x^8-1680*A*a^4*b^3*x^6+840*B*a^5*b^2*x^6-350*C*a^6*b*x^6+105*
D*a^7*x^6+168*A*a^5*b^2*x^4-84*B*a^6*b*x^4+35*C*a^7*x^4-42*A*a^6*b*x^2+21*B
*a^7*x^2+15*A*a^7)/(b*x^2+a)^(7/2)/x^7/a^8
```

maxima [A] time = 1.52, size = 489, normalized size = 1.46

$$-\frac{128\,Dbx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Dbx}{35\,\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{48\,Dbx}{35\,\left(bx^2+a\right)^{\frac{5}{2}}a^3}-\frac{8\,Dbx}{7\,\left(bx^2+a\right)^{\frac{7}{2}}a^2}+\frac{256\,Cb^2x}{21\,\sqrt{bx^2+a}\,a^6}+\frac{128\,Cb^2x}{21\,\left(bx^2+a\right)^{\frac{3}{2}}a^5}+\frac{32\,C}{7\,\left(bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -128/35*D*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*D*b*x/((b*x^2 + a)^(3/2)*a^4) -
 48/35*D*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*D*b*x/((b*x^2 + a)^(7/2)*a^2) +
256/21*C*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*C*b^2*x/((b*x^2 + a)^(3/2)*a^
5) + 32/7*C*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*C*b^2*x/((b*x^2 + a)^(7/2
)*a^3) - 1024/35*B*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*B*b^3*x/((b*x^2 + a
)^(3/2)*a^6) - 384/35*B*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*B*b^3*x/((b*x^
2 + a)^(7/2)*a^4) + 2048/35*A*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*A*b^4*x
/((b*x^2 + a)^(3/2)*a^7) + 768/35*A*b^4*x/((b*x^2 + a)^(5/2)*a^6) + 128/7*A
*b^4*x/((b*x^2 + a)^(7/2)*a^5) - D/((b*x^2 + a)^(7/2)*a*x) + 10/3*C*b/((b*x
^2 + a)^(7/2)*a^2*x) - 8*B*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*A*b^3/((b*x^2
 + a)^(7/2)*a^4*x) - 1/3*C/((b*x^2 + a)^(7/2)*a*x^3) + 4/5*B*b/((b*x^2 + a)
^(7/2)*a^2*x^3) - 8/5*A*b^2/((b*x^2 + a)^(7/2)*a^3*x^3) - 1/5*B/((b*x^2 + a
)^(7/2)*a*x^5) + 2/5*A*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 1/7*A/((b*x^2 + a)^(
7/2)*a*x^7)
```

mupad [B] time = 2.84, size = 421, normalized size = 1.26

$$\frac{\frac{61\,B\,b}{35\,a^3}+\frac{78\,B\,b^2\,x^2}{35\,a^4}}{x^3\,\left(b\,x^2+a\right)^{5/2}}+\frac{\frac{128\,C\,b}{21\,a^5}+\frac{256\,C\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{C}{3\,a^2}+\frac{19\,C\,b\,x^2}{21\,a^3}}{x^3\,\left(b\,x^2+a\right)^{5/2}}-\frac{\frac{167\,A\,b^2}{35\,a^4}+\frac{191\,A\,b^3\,x^2}{35\,a^5}}{x^3\,\left(b\,x^2+a\right)^{5/2}}+\frac{\frac{1024\,A\,b^3}{35\,a^7}+\frac{2048\,A\,b^4\,x^2}{35\,a^8}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{512\,B\,b^2}{35\,a^6}+\frac{1024\,B\,b}{35\,a}}{x\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((61*B*b)/(35*a^3) + (78*B*b^2*x^2)/(35*a^4))/(x^3*(a + b*x^2)^(5/2)) + ((1
28*C*b)/(21*a^5) + (256*C*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) - (C/(3*
a^2) + (19*C*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5/2)) - ((167*A*b^2)/(35*a^
4) + (191*A*b^3*x^2)/(35*a^5))/(x^3*(a + b*x^2)^(5/2)) + ((1024*A*b^3)/(35*
a^7) + (2048*A*b^4*x^2)/(35*a^8))/(x*(a + b*x^2)^(1/2)) - ((512*B*b^2)/(35*
a^6) + (1024*B*b^3*x^2)/(35*a^7))/(x*(a + b*x^2)^(1/2)) - (A*(a + b*x^2)^(1
/2))/(7*a^5*x^7) - (B*(a + b*x^2)^(1/2))/(5*a^5*x^5) - ((a/(b*x^2) + 1)^(9/
2)*D*hypergeom([9/2, 5], 6, -a/(b*x^2)))/(10*x*(a + b*x^2)^(9/2)) + (34*A*b
```

```
*(a + b*x^2)^(1/2))/(35*a^6*x^5) - (B*b)/(7*a^2*x^3*(a + b*x^2)^(7/2)) - (3
2*C*b)/(21*a^4*x*(a + b*x^2)^(3/2)) + (C*b^2*x)/(7*a^3*(a + b*x^2)^(7/2)) -
 (58*A*b^3)/(7*a^6*x*(a + b*x^2)^(3/2)) + (A*b^2)/(7*a^3*x^3*(a + b*x^2)^(7
/2)) + (27*B*b^2)/(7*a^5*x*(a + b*x^2)^(3/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

3.168 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=392

$$-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{256b^2x\left(128Ab^3-3a\left(5a^2D-12abC+24b^2B\right)\right)}{315a^9\sqrt{a+bx^2}}-\frac{128b^2x\left(128Ab\right.}{}$$

```
[Out] -1/9*A/a/x^9/(b*x^2+a)^(7/2)+1/63*(16*A*b-9*B*a)/a^2/x^7/(b*x^2+a)^(7/2)+1/
45*(-32*A*b^2+9*a*(2*B*b-C*a))/a^3/x^5/(b*x^2+a)^(7/2)+1/45*(128*A*b^3-3*a*
(24*B*b^2-12*C*a*b+5*D*a^2))/a^4/x^3/(b*x^2+a)^(7/2)-2/9*b*(128*A*b^3-3*a*(
24*B*b^2-12*C*a*b+5*D*a^2))/a^5/x/(b*x^2+a)^(7/2)-16/63*b^2*(128*A*b^3-3*a*
(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^6/(b*x^2+a)^(7/2)-32/105*b^2*(128*A*b^3-3*
a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^7/(b*x^2+a)^(5/2)-128/315*b^2*(128*A*b^3
-3*a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^8/(b*x^2+a)^(3/2)-256/315*b^2*(128*A*
b^3-3*a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^9/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 380, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1803, 12, 271, 192, 191}

$$-\frac{256b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{315a^9\sqrt{a+bx^2}}-\frac{128b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{315a^8\left(a+bx^2\right)^{3/2}}-\frac{32b^2x\left(-15a^3\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(9*a*x^9*(a + b*x^2)^(7/2)) + (16*A*b - 9*a*B)/(63*a^2*x^7*(a + b*x^2)^(
7/2)) - (32*A*b^2 - 9*a*(2*b*B - a*C))/(45*a^3*x^5*(a + b*x^2)^(7/2)) + (12
8*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)/(45*a^4*x^3*(a + b*x^2)^(7/2)) -
 (2*b*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D))/(9*a^5*x*(a + b*x^2)^(
7/2)) - (16*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*x)/(63*a^6*(a
 + b*x^2)^(7/2)) - (32*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*x)
/(105*a^7*(a + b*x^2)^(5/2)) - (128*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) -
 15*a^3*D)*x)/(315*a^8*(a + b*x^2)^(3/2)) - (256*b^2*(128*A*b^3 - 36*a*b*(2
*b*B - a*C) - 15*a^3*D)*x)/(315*a^9*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
```

```
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{16Ab-9a\left(B+Cx^2+Dx^4\right)}{x^8\left(a+bx^2\right)^{9/2}}\,dx}{9a}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{14b(16Ab-9aB)-7a\left(-9aC-9aDx^2\right)}{x^6\left(a+bx^2\right)^{9/2}}\,dx}{63a^2}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{12b(224\cdots}{\cdots}}{\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} - \frac{\left(128Ab^3\cdots\right.}{\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3\cdots}{\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3\cdots}{\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3\cdots}{\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3\cdots}{\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3\cdots}{\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3\cdots}{\cdots}$$

Mathematica [A] time = 0.18, size = 270, normalized size = 0.69

$$\frac{-a^8\left(35A+45Bx^2+63Cx^4+105Dx^6\right)+2a^7bx^2\left(40A+21\left(3Bx^2+6Cx^4+25Dx^6\right)\right)-56a^6b^2x^4\left(4A+9Bx^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-32768*A*b^8*x^16 + 2048*a*b^7*x^14*(-56*A + 9*B*x^2) - 1024*a^2*b^6*x^12*
(140*A - 63*B*x^2 + 9*C*x^4) - 56*a^6*b^2*x^4*(4*A + 9*B*x^2 + 45*C*x^4 - 1
50*D*x^6) + 4480*a^4*b^4*x^8*(-2*A + 9*B*x^2 - 9*C*x^4 + 3*D*x^6) + 256*a^3
*b^5*x^10*(-280*A + 315*B*x^2 - 126*C*x^4 + 15*D*x^6) - a^8*(35*A + 45*B*x^
2 + 63*C*x^4 + 105*D*x^6) + 112*a^5*b^3*x^6*(8*A + 45*B*x^2 - 180*C*x^4 + 1
50*D*x^6) + 2*a^7*b*x^2*(40*A + 21*(3*B*x^2 + 6*C*x^4 + 25*D*x^6)))/(315*a^
9*x^9*(a + b*x^2)^(7/2))
```

fricas [A] time = 2.31, size = 354, normalized size = 0.90

$$\frac{\left(256\left(15\,Da^3b^5-36\,Ca^2b^6+72\,Bab^7-128\,Ab^8\right)x^{16}+896\left(15\,Da^4b^4-36\,Ca^3b^5+72\,Ba^2b^6-128\,Aab^7\right)x^{14}+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/315*(256*(15*D*a^3*b^5 - 36*C*a^2*b^6 + 72*B*a*b^7 - 128*A*b^8)*x^16 + 89
6*(15*D*a^4*b^4 - 36*C*a^3*b^5 + 72*B*a^2*b^6 - 128*A*a*b^7)*x^14 + 1120*(1
5*D*a^5*b^3 - 36*C*a^4*b^4 + 72*B*a^3*b^5 - 128*A*a^2*b^6)*x^12 + 560*(15*D
*a^6*b^2 - 36*C*a^5*b^3 + 72*B*a^4*b^4 - 128*A*a^3*b^5)*x^10 - 35*A*a^8 + 7
0*(15*D*a^7*b - 36*C*a^6*b^2 + 72*B*a^5*b^3 - 128*A*a^4*b^4)*x^8 - 7*(15*D*
a^8 - 36*C*a^7*b + 72*B*a^6*b^2 - 128*A*a^5*b^3)*x^6 - 7*(9*C*a^8 - 18*B*a^
7*b + 32*A*a^6*b^2)*x^4 - 5*(9*B*a^8 - 16*A*a^7*b)*x^2)*sqrt(b*x^2 + a)/(a^
9*b^4*x^17 + 4*a^10*b^3*x^15 + 6*a^11*b^2*x^13 + 4*a^12*b*x^11 + a^13*x^9)
```

giac [B] time = 0.74, size = 1162, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((790*D*a^24*b^8 - 1686*C*a^23*b^9 + 3072*B*a^22*b^10 - 5053*A*
a^21*b^11)*x^2/(a^30*b^3) + 7*(365*D*a^25*b^7 - 768*C*a^24*b^8 + 1386*B*a^2
3*b^9 - 2264*A*a^22*b^10)/(a^30*b^3)) + 35*(80*D*a^26*b^6 - 165*C*a^25*b^7
+ 294*B*a^24*b^8 - 476*A*a^23*b^9)/(a^30*b^3))*x^2 + 105*(10*D*a^27*b^5 - 2
0*C*a^26*b^6 + 35*B*a^25*b^7 - 56*A*a^24*b^8)/(a^30*b^3))*x/(b*x^2 + a)^(7/
2) - 2/315*(1260*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a^3*b^(3/2) - 3150*(sqr
t(b)*x - sqrt(b*x^2 + a))^16*C*a^2*b^(5/2) + 6300*(sqrt(b)*x - sqrt(b*x^2 +
 a))^16*B*a*b^(7/2) - 11025*(sqrt(b)*x - sqrt(b*x^2 + a))^16*A*b^(9/2) - 10
710*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^4*b^(3/2) + 27720*(sqrt(b)*x - sqr
t(b*x^2 + a))^14*C*a^3*b^(5/2) - 56700*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a
^2*b^(7/2) + 100800*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*a*b^(9/2) + 39270*(s
qrt(b)*x - sqrt(b*x^2 + a))^12*D*a^5*b^(3/2) - 105840*(sqrt(b)*x - sqrt(b*x
^2 + a))^12*C*a^4*b^(5/2) + 223020*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*b
^(7/2) - 405300*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(9/2) - 81270*(sqr
t(b)*x - sqrt(b*x^2 + a))^10*D*a^6*b^(3/2) + 226800*(sqrt(b)*x - sqrt(b*x^2
 + a))^10*C*a^5*b^(5/2) - 495180*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^4*b^(
7/2) + 927360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^3*b^(9/2) + 103950*(sqrt
```

```
(b)*x - sqrt(b*x^2 + a))^8*D*a^7*b^(3/2) - 297108*(sqrt(b)*x - sqrt(b*x^2 +
 a))^8*C*a^6*b^(5/2) + 666036*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^5*b^(7/2)
 - 1291374*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(9/2) - 84210*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*D*a^8*b^(3/2) + 243432*(sqrt(b)*x - sqrt(b*x^2 + a))^
6*C*a^7*b^(5/2) - 551124*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^6*b^(7/2) + 10
73856*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^5*b^(9/2) + 42210*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*D*a^9*b^(3/2) - 121968*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a
^8*b^(5/2) + 275076*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^7*b^(7/2) - 533124*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^6*b^(9/2) - 11970*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*D*a^10*b^(3/2) + 34272*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^9*b^(
5/2) - 76644*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^8*b^(7/2) + 147456*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*A*a^7*b^(9/2) + 1470*D*a^11*b^(3/2) - 4158*C*a^10*
b^(5/2) + 9216*B*a^9*b^(7/2) - 17609*A*a^8*b^(9/2))/(((sqrt(b)*x - sqrt(b*x
^2 + a))^2 - a)^9*a^8)
```

maple [A] time = 0.01, size = 349, normalized size = 0.89

$$-\frac{32768A\,b^{8}x^{16}-18432Ba\,b^{7}x^{16}+9216C\,a^{2}b^{6}x^{16}-3840Da^{3}b^{5}x^{16}+114688Aa\,b^{7}x^{14}-64512B\,a^{2}b^{6}x^{14}+32\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/315*(32768*A*b^8*x^16-18432*B*a*b^7*x^16+9216*C*a^2*b^6*x^16-3840*D*a^3*
b^5*x^16+114688*A*a*b^7*x^14-64512*B*a^2*b^6*x^14+32256*C*a^3*b^5*x^14-1344
0*D*a^4*b^4*x^14+143360*A*a^2*b^6*x^12-80640*B*a^3*b^5*x^12+40320*C*a^4*b^4
*x^12-16800*D*a^5*b^3*x^12+71680*A*a^3*b^5*x^10-40320*B*a^4*b^4*x^10+20160*
C*a^5*b^3*x^10-8400*D*a^6*b^2*x^10+8960*A*a^4*b^4*x^8-5040*B*a^5*b^3*x^8+25
20*C*a^6*b^2*x^8-1050*D*a^7*b*x^8-896*A*a^5*b^3*x^6+504*B*a^6*b^2*x^6-252*C
*a^7*b*x^6+105*D*a^8*x^6+224*A*a^6*b^2*x^4-126*B*a^7*b*x^4+63*C*a^8*x^4-80*
A*a^7*b*x^2+45*B*a^8*x^2+35*A*a^8)/(b*x^2+a)^(7/2)/x^9/a^9
```

maxima [A] time = 1.57, size = 579, normalized size = 1.48

$$\frac{256\,Db^{2}x}{21\,\sqrt{bx^{2}+a}\,a^{6}}+\frac{128\,Db^{2}x}{21\,\left(bx^{2}+a\right)^{\frac{3}{2}}a^{5}}+\frac{32\,Db^{2}x}{7\,\left(bx^{2}+a\right)^{\frac{5}{2}}a^{4}}+\frac{80\,Db^{2}x}{21\,\left(bx^{2}+a\right)^{\frac{7}{2}}a^{3}}-\frac{1024\,Cb^{3}x}{35\,\sqrt{bx^{2}+a}\,a^{7}}-\frac{512\,Cb^{3}x}{35\,\left(bx^{2}+a\right)^{\frac{3}{2}}a^{6}}-\frac{38\cdots}{35\,(b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 256/21*D*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*D*b^2*x/((b*x^2 + a)^(3/2)*a^
5) + 32/7*D*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*D*b^2*x/((b*x^2 + a)^(7/2
)*a^3) - 1024/35*C*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*C*b^3*x/((b*x^2 + a
)^(3/2)*a^6) - 384/35*C*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*C*b^3*x/((b*x^
2 + a)^(7/2)*a^4) + 2048/35*B*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*B*b^4*x
/((b*x^2 + a)^(3/2)*a^7) + 768/35*B*b^4*x/((b*x^2 + a)^(5/2)*a^6) + 128/7*B
*b^4*x/((b*x^2 + a)^(7/2)*a^5) - 32768/315*A*b^5*x/(sqrt(b*x^2 + a)*a^9) -
16384/315*A*b^5*x/((b*x^2 + a)^(3/2)*a^8) - 4096/105*A*b^5*x/((b*x^2 + a)^(
5/2)*a^7) - 2048/63*A*b^5*x/((b*x^2 + a)^(7/2)*a^6) + 10/3*D*b/((b*x^2 + a)
^(7/2)*a^2*x) - 8*C*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*B*b^3/((b*x^2 + a)^(
7/2)*a^4*x) - 256/9*A*b^4/((b*x^2 + a)^(7/2)*a^5*x) - 1/3*D/((b*x^2 + a)^(7
/2)*a*x^3) + 4/5*C*b/((b*x^2 + a)^(7/2)*a^2*x^3) - 8/5*B*b^2/((b*x^2 + a)^(
7/2)*a^3*x^3) + 128/45*A*b^3/((b*x^2 + a)^(7/2)*a^4*x^3) - 1/5*C/((b*x^2 +
a)^(7/2)*a*x^5) + 2/5*B*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 32/45*A*b^2/((b*x^2
 + a)^(7/2)*a^3*x^5) - 1/7*B/((b*x^2 + a)^(7/2)*a*x^7) + 16/63*A*b/((b*x^2
+ a)^(7/2)*a^2*x^7) - 1/9*A/((b*x^2 + a)^(7/2)*a*x^9)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^{10}\,{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(9/2)),x)
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(9/2)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

3.169 $\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$\frac{\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{\left(a+bx^2\right)^{5/2}\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(-5a^3f+4a^2\right.}{3b^6}$$

```
[Out] -1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*(b*x^2+a)^(3/2)/b^6+1/5*(-10*
a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*(b*x^2+a)^(5/2)/b^6+1/7*(10*a^2*f-4*a*b*e+
b^2*d)*(b*x^2+a)^(7/2)/b^6+1/9*(-5*a*f+b*e)*(b*x^2+a)^(9/2)/b^6+1/11*f*(b*x
^2+a)^(11/2)/b^6+a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^6
```

Rubi [A] time = 0.22, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1811, 1799, 1850}

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^2be\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1811

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx = \int \frac{x\left(cx^4+dx^6+ex^8+fx^{10}\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{cx^2+dx^3+ex^4+fx^5}{\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^2\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^5\sqrt{a+bx}} + \frac{a\left(-2b^3c+3ab^2d-4a^2be+5a^3\right.}{b^5}\right.\right.$$
$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^6} - \frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)\left(a+b\right.}{3b^6}$$

Mathematica [A] time = 0.19, size = 158, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-1280a^5f+128a^4b\left(11e+5fx^2\right)-16a^3b^2\left(99d+44ex^2+30fx^4\right)+8a^2b^3\left(231c+99dx^2+66ex^4+5\right.\right.}{3465b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(99
*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f*x
^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*(69
3*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

fricas [A] time = 0.55, size = 177, normalized size = 0.83

$$\frac{\left(315\,b^5fx^{10}+35\left(11\,b^5e-10\,ab^4f\right)x^8+5\left(99\,b^5d-88\,ab^4e+80\,a^2b^3f\right)x^6+1848\,a^2b^3c-1584\,a^3b^2d+1408\,a^4b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b^5*d - 88*
a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*b*
e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d + 176*a^2*b^3*e - 160*a^3*b^2*f
)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x^2)*
sqrt(b*x^2 + a)/b^6
```

giac [A] time = 0.45, size = 264, normalized size = 1.23

$$\frac{\left(a^2b^3c-a^3b^2d-a^5f+a^4be\right)\sqrt{bx^2+a}}{b^6} + \frac{693\left(bx^2+a\right)^{\frac{5}{2}}b^3c-2310\left(bx^2+a\right)^{\frac{3}{2}}ab^3c+495\left(bx^2+a\right)^{\frac{7}{2}}b^2d-2079\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*sqrt(b*x^2 + a)/b^6 + 1/3465*(693
*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 495*(b*x^2 + a)
^(7/2)*b^2*d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*
b^2*d + 315*(b*x^2 + a)^(11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2
 + a)^(7/2)*a^2*f - 6930*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a
^4*f + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 + a)^(7/2)*a*b*e + 4158*(b*x
^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/2)*a^3*b*e)/b^6
```

maple [A] time = 0.01, size = 193, normalized size = 0.90

$$-\frac{\sqrt{b\,x^2+a}\,\left(-315 f\,x^{10} b^5+350 a\,b^4 f\,x^8-385 b^5 e\,x^8-400 a^2 b^3 f\,x^6+440 a\,b^4 e\,x^6-495 b^5 d\,x^6+480 a^3 b^2 f\,x^4 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3465*(b*x^2+a)^(1/2)*(-315*b^5*f*x^10+350*a*b^4*f*x^8-385*b^5*e*x^8-400*
a^2*b^3*f*x^6+440*a*b^4*e*x^6-495*b^5*d*x^6+480*a^3*b^2*f*x^4-528*a^2*b^3*e
*x^4+594*a*b^4*d*x^4-693*b^5*c*x^4-640*a^4*b*f*x^2+704*a^3*b^2*e*x^2-792*a^
2*b^3*d*x^2+924*a*b^4*c*x^2+1280*a^5*f-1408*a^4*b*e+1584*a^3*b^2*d-1848*a^2
*b^3*c)/b^6
```

maxima [A] time = 1.43, size = 347, normalized size = 1.62

$$\frac{\sqrt{bx^2+a}\,fx^{10}}{11\,b}+\frac{\sqrt{bx^2+a}\,ex^8}{9\,b}-\frac{10\,\sqrt{bx^2+a}\,afx^8}{99\,b^2}+\frac{\sqrt{bx^2+a}\,dx^6}{7\,b}-\frac{8\,\sqrt{bx^2+a}\,aex^6}{63\,b^2}+\frac{80\,\sqrt{bx^2+a}\,a^2fx^6}{693\,b^3}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/11*sqrt(b*x^2 + a)*f*x^10/b + 1/9*sqrt(b*x^2 + a)*e*x^8/b - 10/99*sqrt(b*
x^2 + a)*a*f*x^8/b^2 + 1/7*sqrt(b*x^2 + a)*d*x^6/b - 8/63*sqrt(b*x^2 + a)*a
*e*x^6/b^2 + 80/693*sqrt(b*x^2 + a)*a^2*f*x^6/b^3 + 1/5*sqrt(b*x^2 + a)*c*x
^4/b - 6/35*sqrt(b*x^2 + a)*a*d*x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*e*x^4/
b^3 - 32/231*sqrt(b*x^2 + a)*a^3*f*x^4/b^4 - 4/15*sqrt(b*x^2 + a)*a*c*x^2/b
^2 + 8/35*sqrt(b*x^2 + a)*a^2*d*x^2/b^3 - 64/315*sqrt(b*x^2 + a)*a^3*e*x^2/
b^4 + 128/693*sqrt(b*x^2 + a)*a^4*f*x^2/b^5 + 8/15*sqrt(b*x^2 + a)*a^2*c/b^
3 - 16/35*sqrt(b*x^2 + a)*a^3*d/b^4 + 128/315*sqrt(b*x^2 + a)*a^4*e/b^5 - 2
56/693*sqrt(b*x^2 + a)*a^5*f/b^6
```

mupad [B] time = 1.20, size = 186, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{x^6\,\left(400\,f\,a^2\,b^3-440\,e\,a\,b^4+495\,d\,b^5\right)}{3465\,b^6}-\frac{1280\,f\,a^5-1408\,e\,a^4\,b+1584\,d\,a^3\,b^2-1848\,c\,a^2\,b^3}{3465\,b^6}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5 + d*x^7 + e*x^9 + f*x^11)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^6*(495*b^5*d + 400*a^2*b^3*f - 440*a*b^4*e))/(3465*b^
6) - (1280*a^5*f - 1848*a^2*b^3*c + 1584*a^3*b^2*d - 1408*a^4*b*e)/(3465*b^
6) + (x^4*(693*b^5*c + 528*a^2*b^3*e - 480*a^3*b^2*f - 594*a*b^4*d))/(3465*
b^6) + (f*x^10)/(11*b) + (x^8*(385*b^5*e - 350*a*b^4*f))/(3465*b^6) - (4*a*
x^2*(231*b^3*c - 160*a^3*f - 198*a*b^2*d + 176*a^2*b*e))/(3465*b^5))
```

sympy [A] time = 9.37, size = 442, normalized size = 2.07

$$\begin{cases} -\frac{256a^5f\sqrt{a+bx^2}}{693b^6}+\frac{128a^4e\sqrt{a+bx^2}}{315b^5}+\frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5}-\frac{16a^3d\sqrt{a+bx^2}}{35b^4}-\frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4}-\frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4}+\frac{8a^2c\sqrt{a+bx^2}}{15b^3}+\cdots \\ \frac{\frac{cx^6}{6}+\frac{dx^8}{8}+\frac{ex^{10}}{10}+\frac{fx^{12}}{12}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**11+e*x**9+d*x**7+c*x**5)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b*
x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*d*
sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4) - 3
2*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b
**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b
*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2
*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*e
*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99*b**2) +
 c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e*x**8*sqr
t(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)), ((c*x**6/
6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

3.170 $\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=167

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{\left(a+bx^2\right)^{3/2}\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}-\frac{a\sqrt{a+bx^2}\left(a^3(-f)+a^2be-a\right.}{b^5}$$

```
[Out] 1/3*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*(b*x^2+a)^(3/2)/b^5+1/5*(6*a^2*f-3
*a*b*e+b^2*d)*(b*x^2+a)^(5/2)/b^5+1/7*(-4*a*f+b*e)*(b*x^2+a)^(7/2)/b^5+1/9*
f*(b*x^2+a)^(9/2)/b^5-a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^5
```

Rubi [A] time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1811, 1799, 1850}

$$\frac{\left(a+bx^2\right)^{3/2}\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{3b^5}-\frac{a\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^5}+\frac{\left(a+bx^2\right)^{5/2}\left(6a^2f-3a\right.}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1811

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx &= \int \frac{x\left(cx^2+dx^4+ex^6+fx^8\right)}{\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{cx+dx^2+ex^3+fx^4}{\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^4\sqrt{a+bx}}+\frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)\sqrt{a}}{b^4}\right.\right.\\
&= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^5}+\frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)\left(a+b\right.}{3b^5}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 122, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(128a^4f-16a^3b\left(9e+4fx^2\right)+24a^2b^2\left(7d+3ex^2+2fx^4\right)-2ab^3\left(105c+42dx^2+27ex^4+20fx^6\right)+b\ldots\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d +
3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b^4
*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)
```

fricas [A] time = 0.67, size = 134, normalized size = 0.80

$$\frac{\left(35\,b^4fx^8+5\left(9\,b^4e-8\,ab^3f\right)x^6-210\,ab^3c+168\,a^2b^2d-144\,a^3be+128\,a^4f+3\left(21\,b^4d-18\,ab^3e+16\,a^2b^2f\right)x^4\ldots\right.}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c + 168*a^2*b
^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3*e + 16*a^2*b^2*f)*x
^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e - 64*a^3*b*f)*x^2)*sqrt(b*x^2 +
 a)/b^5
```

giac [A] time = 0.50, size = 197, normalized size = 1.18

$$-\frac{\left(ab^3c-a^2b^2d-a^4f+a^3be\right)\sqrt{bx^2+a}}{b^5}+\frac{105\left(bx^2+a\right)^{\frac{3}{2}}b^3c+63\left(bx^2+a\right)^{\frac{5}{2}}b^2d-210\left(bx^2+a\right)^{\frac{3}{2}}ab^2d+35\left(bx^2+\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*sqrt(b*x^2 + a)/b^5 + 1/315*(105*(
b*x^2 + a)^(3/2)*b^3*c + 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)
*a*b^2*d + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)*a*f + 378*(b*x^2
+ a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f + 45*(b*x^2 + a)^(7/2)*b*e -
 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*x^2 + a)^(3/2)*a^2*b*e)/b^5
```

maple [A] time = 0.01, size = 145, normalized size = 0.87

$$\frac{\sqrt{b\,x^2+a}\,\left(35f\,x^8b^4-40a\,b^3f\,x^6+45b^4e\,x^6+48a^2b^2f\,x^4-54a\,b^3e\,x^4+63b^4d\,x^4-64a^3bf\,x^2+72a^2b^2e\,x^2-84\ldots\right.}{315b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(1/2)*(35*b^4*f*x^8-40*a*b^3*f*x^6+45*b^4*e*x^6+48*a^2*b^2*
f*x^4-54*a*b^3*e*x^4+63*b^4*d*x^4-64*a^3*b*f*x^2+72*a^2*b^2*e*x^2-84*a*b^3*
d*x^2+105*b^4*c*x^2+128*a^4*f-144*a^3*b*e+168*a^2*b^2*d-210*a*b^3*c)/b^5
```

maxima [A] time = 1.37, size = 263, normalized size = 1.57

$$\frac{\sqrt{bx^2+a}\,fx^8}{9\,b}+\frac{\sqrt{bx^2+a}\,ex^6}{7\,b}-\frac{8\,\sqrt{bx^2+a}\,afx^6}{63\,b^2}+\frac{\sqrt{bx^2+a}\,dx^4}{5\,b}-\frac{6\,\sqrt{bx^2+a}\,aex^4}{35\,b^2}+\frac{16\,\sqrt{bx^2+a}\,a^2fx^4}{105\,b^3}+\frac{\sqrt{bx^2+a}\,\ldots}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(b*x^2 + a)*f*x^8/b + 1/7*sqrt(b*x^2 + a)*e*x^6/b - 8/63*sqrt(b*x^2
 + a)*a*f*x^6/b^2 + 1/5*sqrt(b*x^2 + a)*d*x^4/b - 6/35*sqrt(b*x^2 + a)*a*e*
x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*f*x^4/b^3 + 1/3*sqrt(b*x^2 + a)*c*x^2/
b - 4/15*sqrt(b*x^2 + a)*a*d*x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*e*x^2/b^3 -
 64/315*sqrt(b*x^2 + a)*a^3*f*x^2/b^4 - 2/3*sqrt(b*x^2 + a)*a*c/b^2 + 8/15*
sqrt(b*x^2 + a)*a^2*d/b^3 - 16/35*sqrt(b*x^2 + a)*a^3*e/b^4 + 128/315*sqrt(
b*x^2 + a)*a^4*f/b^5
```

mupad [B] time = 1.14, size = 146, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{128\,f\,a^4-144\,e\,a^3\,b+168\,d\,a^2\,b^2-210\,c\,a\,b^3}{315\,b^5}+\frac{x^4\,\left(48\,f\,a^2\,b^2-54\,e\,a\,b^3+63\,d\,b^4\right)}{315\,b^5}+\frac{f\,x^8}{9\,b}+\frac{x^6\,\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^3 + d*x^5 + e*x^7 + f*x^9)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((128*a^4*f + 168*a^2*b^2*d - 210*a*b^3*c - 144*a^3*b*e)/
(315*b^5) + (x^4*(63*b^4*d + 48*a^2*b^2*f - 54*a*b^3*e))/(315*b^5) + (f*x^8
)/(9*b) + (x^6*(45*b^4*e - 40*a*b^3*f))/(315*b^5) + (x^2*(105*b^4*c + 72*a^
2*b^2*e - 84*a*b^3*d - 64*a^3*b*f))/(315*b^5))
```

sympy [A] time = 5.41, size = 340, normalized size = 2.04

$$\begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5} - \frac{16a^3e\sqrt{a+bx^2}}{35b^4} - \frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4} + \frac{8a^2d\sqrt{a+bx^2}}{15b^3} + \frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3} - \frac{2ac\sqrt{a+bx^2}}{3b^2} - \frac{4adx^2\sqrt{a\cdot}}{15b^2} \\ \frac{\frac{cx^4}{4}+\frac{dx^6}{6}+\frac{ex^8}{8}+\frac{fx^{10}}{10}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**7+d*x**5+c*x**3)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x*
*2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*
f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*
d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) -
 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*
x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*sqrt(a
 + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x**10/10)
/sqrt(a), True))
```

3.171 $\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=121

$$\frac{\left(a+bx^2\right)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}(be-3af)}{5b^4}+\frac{f\left(a+bx^2\right)^7}{7b^4}$$

```
[Out] 1/3*(3*a^2*f-2*a*b*e+b^2*d)*(b*x^2+a)^(3/2)/b^4+1/5*(-3*a*f+b*e)*(b*x^2+a)^
(5/2)/b^4+1/7*f*(b*x^2+a)^(7/2)/b^4+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a
)^(1/2)/b^4
```

Rubi [A] time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1811, 1799, 1850}

$$\frac{\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}+\frac{\left(a+bx^2\right)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\left(a+bx^2\right)^{5/2}(be-3af)}{5b^4}+\frac{f\left(a+bx^2\right)^7}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1811

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}} \, dx &= \int \frac{x\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}}+\frac{\left(b^2d-2abe+3a^2f\right)\sqrt{a+bx}}{b^3}+\frac{(be-3af}{}\right.\right. \\
&= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^4}+\frac{\left(b^2d-2abe+3a^2f\right)\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{(be-3a}{}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 89, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-48a^3f+8a^2b\left(7e+3fx^2\right)-2ab^2\left(35d+14ex^2+9fx^4\right)+b^3\left(105c+35dx^2+21ex^4+15fx^6\right)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14*
e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4)
```

fricas [A] time = 0.67, size = 94, normalized size = 0.78

$$\frac{\left(15\,b^3fx^6+3\left(7\,b^3e-6\,ab^2f\right)x^4+105\,b^3c-70\,ab^2d+56\,a^2be-48\,a^3f+\left(35\,b^3d-28\,ab^2e+24\,a^2bf\right)x^2\right)\sqrt{b}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 70*a*b^2*d
+ 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x^2)*sqrt(b*
x^2 + a)/b^4
```

giac [A] time = 0.40, size = 130, normalized size = 1.07

$$\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\sqrt{bx^2+a}}{b^4}+\frac{35\left(bx^2+a\right)^{\frac{3}{2}}b^2d+15\left(bx^2+a\right)^{\frac{7}{2}}f-63\left(bx^2+a\right)^{\frac{5}{2}}af+105\left(bx^2+a\right)^{\frac{3}{2}}a}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*sqrt(b*x^2 + a)/b^4 + 1/105*(35*(b*x^2
+ a)^(3/2)*b^2*d + 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*
(b*x^2 + a)^(3/2)*a^2*f + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a
*b*e)/b^4
```

maple [A] time = 0.01, size = 99, normalized size = 0.82

$$-\frac{\sqrt{b\,x^2+a}\,\left(-15f\,x^6b^3+18a\,b^2f\,x^4-21b^3e\,x^4-24a^2bf\,x^2+28a\,b^2e\,x^2-35b^3d\,x^2+48a^3f-56a^2be+70a\,b\right.}{105b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(-15*b^3*f*x^6+18*a*b^2*f*x^4-21*b^3*e*x^4-24*a^2*b*
f*x^2+28*a*b^2*e*x^2-35*b^3*d*x^2+48*a^3*f-56*a^2*b*e+70*a*b^2*d-105*b^3*c)
/b^4
```

maxima [A] time = 1.36, size = 180, normalized size = 1.49

$$\frac{\sqrt{bx^2+a}\,fx^6}{7\,b}+\frac{\sqrt{bx^2+a}\,ex^4}{5\,b}-\frac{6\,\sqrt{bx^2+a}\,afx^4}{35\,b^2}+\frac{\sqrt{bx^2+a}\,dx^2}{3\,b}-\frac{4\,\sqrt{bx^2+a}\,aex^2}{15\,b^2}+\frac{8\,\sqrt{bx^2+a}\,a^2fx^2}{35\,b^3}+\frac{\sqrt{bx^2+}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*f*x^6/b + 1/5*sqrt(b*x^2 + a)*e*x^4/b - 6/35*sqrt(b*x^2
 + a)*a*f*x^4/b^2 + 1/3*sqrt(b*x^2 + a)*d*x^2/b - 4/15*sqrt(b*x^2 + a)*a*e*
```

```
x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*f*x^2/b^3 + sqrt(b*x^2 + a)*c/b - 2/3*sq
rt(b*x^2 + a)*a*d/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*e/b^3 - 16/35*sqrt(b*x^2 +
 a)*a^3*f/b^4
```

mupad [B] time = 1.08, size = 103, normalized size = 0.85

$$\sqrt{b\,x^2+a}\,\left(\frac{-48\,f\,a^3+56\,e\,a^2\,b-70\,d\,a\,b^2+105\,c\,b^3}{105\,b^4}+\frac{f\,x^6}{7\,b}+\frac{x^2\,\left(24\,f\,a^2\,b-28\,e\,a\,b^2+35\,d\,b^3\right)}{105\,b^4}+\frac{x^4\,\left(21\,b^3\,e-\right.}{105}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x + d*x^3 + e*x^5 + f*x^7)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((105*b^3*c - 48*a^3*f - 70*a*b^2*d + 56*a^2*b*e)/(105*b^
4) + (f*x^6)/(7*b) + (x^2*(35*b^3*d - 28*a*b^2*e + 24*a^2*b*f))/(105*b^4) +
 (x^4*(21*b^3*e - 18*a*b^2*f))/(105*b^4))
```

sympy [A] time = 3.41, size = 238, normalized size = 1.97

$$\begin{cases} -\frac{16a^3f\sqrt{a+bx^2}}{35b^4}+\frac{8a^2e\sqrt{a+bx^2}}{15b^3}+\frac{8a^2fx^2\sqrt{a+bx^2}}{35b^3}-\frac{2ad\sqrt{a+bx^2}}{3b^2}-\frac{4aex^2\sqrt{a+bx^2}}{15b^2}-\frac{6afx^4\sqrt{a+bx^2}}{35b^2}+\frac{c\sqrt{a+bx^2}}{b}+\frac{dx^2\sqrt{a+bx^2}}{3b}+\frac{ex^4\sqrt{}}{5} \\ \frac{\frac{cx^2}{2}+\frac{dx^4}{4}+\frac{ex^6}{6}+\frac{fx^8}{8}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**7+e*x**5+d*x**3+c*x)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**2
)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b*x
**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(a +
 b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) +
 e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)),
((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

3.172 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=261

$$\frac{x^3\left(a\left(162a^3F-71a^2bD+15ab^2C+6b^3B\right)+8Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)+4Ab^4\right)}{35a^2b^4\left(a+bx^2\right)^{5/2}}+\frac{x}{-}$$

```
[Out] 1/7*(A*b^4-a*(B*b^3-C*a*b^2+D*a^2*b-F*a^3))*x^3/a/b^4/(b*x^2+a)^(7/2)+1/35*
(4*A*b^4+a*(3*B*b^3-10*C*a*b^2+17*D*a^2*b-24*F*a^3))*x^3/a^2/b^4/(b*x^2+a)^
(5/2)+1/105*(8*A*b^4+a*(6*B*b^3+15*C*a*b^2-71*D*a^2*b+162*F*a^3))*x^3/a^3/b
^4/(b*x^2+a)^(3/2)+1/2*(2*D*b-9*F*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
11/2)-(D*b-4*F*a)*x/b^5/(b*x^2+a)^(1/2)+1/2*F*x*(b*x^2+a)^(1/2)/b^5
```

Rubi [A] time = 0.72, antiderivative size = 257, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {1804, 1800, 1585, 1263, 1584, 455, 388, 217, 206}

$$\frac{x^3\left(a\left(-71a^2bD+162a^3F+15ab^2C+6b^3B\right)+8Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(17a^2bD-24a^3F-10ab^2C+3b^3B\right)+4Ab^4\right)}{35a^2b^4\left(a+bx^2\right)^{5/2}}+\frac{x}{-}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x^3)/(7*(a + b*x^2)^(7/2))
 + ((4*A*b^4 + a*(3*b^3*B - 10*a*b^2*C + 17*a^2*b*D - 24*a^3*F))*x^3)/(35*a
^2*b^4*(a + b*x^2)^(5/2)) + ((8*A*b^4 + a*(6*b^3*B + 15*a*b^2*C - 71*a^2*b*
D + 162*a^3*F))*x^3)/(105*a^3*b^4*(a + b*x^2)^(3/2)) - ((b*D - 4*a*F)*x)/(b
^5*Sqrt[a + b*x^2]) + (F*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*D - 9*a*F)*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
```

```
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1800

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/c, Int[(c*x)^(m + 1)*PolynomialQuotient[Pq, x, x]*(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && EqQ[Coeff[Pq, x, 0], 0]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6+Fx^8\right)}{\left(a+bx^2\right)^{9/2}}dx = \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x\left(-\left(4Ab+\frac{3a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^3}\right)x-\frac{7a\left(b^2C-ab\right.}{b}}{\left(a+bx^2\right)^{7/2}}}{7ab}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(-4Ab-\frac{3a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^3}-\frac{7a\left(b^2C-abD\right.}{b^2}}{\left(a+bx^2\right)^{7/2}}}{7ab}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$

Mathematica [A] time = 0.59, size = 221, normalized size = 0.85

$$\frac{105a^{7/2}\left(a+bx^2\right)^3\sqrt{\frac{bx^2}{a}+1}\,(2bD-9aF)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b}\,x\left(945a^7F-210a^6b\left(D-15Fx^2\right)+14a^5b^2x^2\left(261\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[b]*x*(945*a^7*F + 16*A*b^7*x^6 + 4*a*b^6*x^4*(14*A + 3*B*x^2) - 210*a
^6*b*(D - 15*F*x^2) + a^3*b^4*x^6*(-352*D + 105*F*x^2) + 14*a^5*b^2*x^2*(-5
```

```
0*D + 261*F*x^2) + 4*a^4*b^3*x^4*(-203*D + 396*F*x^2) + 2*a^2*b^5*x^2*(35*A
 + 21*B*x^2 + 15*C*x^4)) + 105*a^(7/2)*(2*b*D - 9*a*F)*(a + b*x^2)^3*Sqrt[1
 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(210*a^3*b^(11/2)*(a + b*x^2)^(
7/2))
```

fricas [A] time = 0.92, size = 705, normalized size = 2.70

$$\left[-\frac{105\left(9\,Fa^{8}-2\,Da^{7}b+\left(9\,Fa^{4}b^{4}-2\,Da^{3}b^{5}\right)x^{8}+4\left(9\,Fa^{5}b^{3}-2\,Da^{4}b^{4}\right)x^{6}+6\left(9\,Fa^{6}b^{2}-2\,Da^{5}b^{3}\right)x^{4}+4\left(9\,Fa^{7}b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fri
cas")
```

```
[Out] [-1/420*(105*(9*F*a^8 - 2*D*a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9*
F*a^5*b^3 - 2*D*a^4*b^4)*x^6 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F*a
^7*b - 2*D*a^6*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) - 2*(105*F*a^3*b^5*x^9 + 2*(792*F*a^4*b^4 - 176*D*a^3*b^5 + 15*C*a^2*
b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7 + 14*(261*F*a^5*b^3 - 58*D*a^4*b^4 + 3*B*a^2
*b^6 + 4*A*a*b^7)*x^5 + 70*(45*F*a^6*b^2 - 10*D*a^5*b^3 + A*a^2*b^6)*x^3 +
105*(9*F*a^7*b - 2*D*a^6*b^2)*x)*sqrt(b*x^2 + a))/(a^3*b^10*x^8 + 4*a^4*b^9
*x^6 + 6*a^5*b^8*x^4 + 4*a^6*b^7*x^2 + a^7*b^6), 1/210*(105*(9*F*a^8 - 2*D*
a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9*F*a^5*b^3 - 2*D*a^4*b^4)*x^6
 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F*a^7*b - 2*D*a^6*b^2)*x^2)*sqr
t(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (105*F*a^3*b^5*x^9 + 2*(792*F*a^
4*b^4 - 176*D*a^3*b^5 + 15*C*a^2*b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7 + 14*(261*F
*a^5*b^3 - 58*D*a^4*b^4 + 3*B*a^2*b^6 + 4*A*a*b^7)*x^5 + 70*(45*F*a^6*b^2 -
 10*D*a^5*b^3 + A*a^2*b^6)*x^3 + 105*(9*F*a^7*b - 2*D*a^6*b^2)*x)*sqrt(b*x^
2 + a))/(a^3*b^10*x^8 + 4*a^4*b^9*x^6 + 6*a^5*b^8*x^4 + 4*a^6*b^7*x^2 + a^7
*b^6)]
```

giac [A] time = 0.56, size = 224, normalized size = 0.86

$$\frac{\left(\left(\left(\left(\frac{105\,Fx^{2}}{b}+\frac{2\left(792\,Fa^{4}b^{7}-176\,Da^{3}b^{8}+15\,Ca^{2}b^{9}+6\,Bab^{10}+8\,Ab^{11}\right)}{a^{3}b^{9}}\right)x^{2}+\frac{14\left(261\,Fa^{5}b^{6}-58\,Da^{4}b^{7}+3\,Ba^{2}b^{9}+4\,Aab^{10}\right)}{a^{3}b^{9}}\right)x^{2}+\frac{70\left(45\,Fa^{6}b^{5}-10\right.}{a^{3}b}\right.\right.}{210\left(bx^{2}+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="gia
c")
```

```
[Out] 1/210*((((105*F*x^2/b + 2*(792*F*a^4*b^7 - 176*D*a^3*b^8 + 15*C*a^2*b^9 + 6
*B*a*b^10 + 8*A*b^11)/(a^3*b^9))*x^2 + 14*(261*F*a^5*b^6 - 58*D*a^4*b^7 + 3
*B*a^2*b^9 + 4*A*a*b^10)/(a^3*b^9))*x^2 + 70*(45*F*a^6*b^5 - 10*D*a^5*b^6 +
 A*a^2*b^9)/(a^3*b^9))*x^2 + 105*(9*F*a^7*b^4 - 2*D*a^6*b^5)/(a^3*b^9))*x/(
b*x^2 + a)^(7/2) + 1/2*(9*F*a - 2*D*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)
))/b^(11/2)
```

maple [B] time = 0.01, size = 478, normalized size = 1.83

$$\frac{F\,x^{9}}{2\left(b\,x^{2}+a\right)^{\frac{7}{2}}b}-\frac{Dx^{7}}{7\left(b\,x^{2}+a\right)^{\frac{7}{2}}b}+\frac{9Fa\,x^{7}}{14\left(b\,x^{2}+a\right)^{\frac{7}{2}}b^{2}}-\frac{C\,x^{5}}{2\left(b\,x^{2}+a\right)^{\frac{7}{2}}b}-\frac{Dx^{5}}{5\left(b\,x^{2}+a\right)^{\frac{5}{2}}b^{2}}+\frac{9Fa\,x^{5}}{10\left(b\,x^{2}+a\right)^{\frac{5}{2}}b^{3}}-\frac{B\,x^{3}}{4\left(b\,x^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 2/35/(b*x^2+a)^(1/2)*B/a^2/b^2*x-1/(b*x^2+a)^(1/2)*D/b^4*x-9/2*F*a/b^(11/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*F*x^9/b/(b*x^2+a)^(7/2)-1/7/(b*x^2+a)^(7
/2)*D/b*x^7-1/5/(b*x^2+a)^(5/2)*D/b^2*x^5-1/3/(b*x^2+a)^(3/2)*D/b^3*x^3-1/2
/(b*x^2+a)^(7/2)*C/b*x^5+1/14/(b*x^2+a)^(3/2)*C/b^3*x-1/4/(b*x^2+a)^(7/2)*B
/b*x^3+3/140/(b*x^2+a)^(5/2)*B/b^2*x-1/7/(b*x^2+a)^(7/2)*A/b*x+8/105/(b*x^2
+a)^(1/2)*A/a^3/b*x+4/105/(b*x^2+a)^(3/2)*A/a^2/b*x+3/56/(b*x^2+a)^(5/2)*C*
a/b^3*x+1/7/(b*x^2+a)^(1/2)*C/a/b^3*x-3/28/(b*x^2+a)^(7/2)*B*a/b^2*x+1/35/(
b*x^2+a)^(3/2)*B/a/b^2*x+1/35/(b*x^2+a)^(5/2)*A/a/b*x+9/14*F*a/b^2*x^7/(b*x
^2+a)^(7/2)-15/56/(b*x^2+a)^(7/2)*C*a^2/b^3*x+9/2*F*a/b^5*x/(b*x^2+a)^(1/2)
+9/10*F*a/b^3*x^5/(b*x^2+a)^(5/2)+3/2*F*a/b^4*x^3/(b*x^2+a)^(3/2)-5/8/(b*x^
2+a)^(7/2)*C*a/b^2*x^3+D/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [B] time = 1.75, size = 826, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="max
ima")
```

```
[Out] 1/2*F*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/
((b*x^2 + a)^(7/2)*b^4))*D*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^
4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b
*x^2 + a)^(7/2)*b^4))*F*a*x/b + 3/10*F*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) +
20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/1
5*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*
a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) + 3/2*F*a*
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*D*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 9/2*F*a^
2*x^3/((b*x^2 + a)^(5/2)*b^4) - D*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*C*a*x
^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) - 417/70*F*a*x
/(sqrt(b*x^2 + a)*b^5) - 51/70*F*a^2*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*F*a
^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*D*x/(sqrt(b*x^2 + a)*b^4) + 17/105*D
*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*D*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14
*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x
/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*B*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/(
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/((b
*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^2
+ a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) - 9/2*F*a*arcsinh(b*x/
sqrt(a*b))/b^(11/2) + D*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(A+B\,x^2+C\,x^4+F\,x^8+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.173 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=214

$$\frac{x\left(Ab^4-a^4F\right)}{ab^4\left(a+bx^2\right)^{7/2}}+\frac{x^5\left(a\left(-58a^3F+3ab^2C+4b^3B\right)+24Ab^4\right)}{15a^3b^2\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(-10a^4F+ab^3B+6Ab^4\right)}{3a^2b^3\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(a\left(-176a^3F+15a\right.\right.}{105}$$

```
[Out] (A*b^4-F*a^4)*x/a/b^4/(b*x^2+a)^(7/2)+1/3*(6*A*b^4+B*a*b^3-10*F*a^4)*x^3/a^
2/b^3/(b*x^2+a)^(7/2)+1/15*(24*A*b^4+a*(4*B*b^3+3*C*a*b^2-58*F*a^3))*x^5/a^
3/b^2/(b*x^2+a)^(7/2)+1/105*(48*A*b^4+a*(8*B*b^3+6*C*a*b^2+15*D*a^2*b-176*F
*a^3))*x^7/a^4/b/(b*x^2+a)^(7/2)+F*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/
2)
```

Rubi [A] time = 0.41, antiderivative size = 250, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1814, 1157, 385, 217, 206}

$$\frac{x\left(\frac{15a^2bD-176a^3F+6ab^2C+8b^3B}{b^4}+\frac{48A}{a}\right)}{105a^3\sqrt{a+bx^2}}+\frac{x\left(a\left(-45a^2bD+122a^3F+3ab^2C+4b^3B\right)+24Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x\left(\frac{15a^2bD-22a^3F-8ab^2C+}{b^4}\right.}{35a\left(a+bx^2\right)^{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x)/(7*(a + b*x^2)^(7/2)) +
 (((6*A)/a + (b^3*B - 8*a*b^2*C + 15*a^2*b*D - 22*a^3*F)/b^4)*x)/(35*a*(a +
 b*x^2)^(5/2)) + ((24*A*b^4 + a*(4*b^3*B + 3*a*b^2*C - 45*a^2*b*D + 122*a^3
*F))*x)/(105*a^3*b^4*(a + b*x^2)^(3/2)) + (((48*A)/a + (8*b^3*B + 6*a*b^2*C
 + 15*a^2*b*D - 176*a^3*F)/b^4)*x)/(105*a^3*Sqrt[a + b*x^2]) + (F*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
```

```
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{\left(a+bx^2\right)^{9/2}}dx = \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-6A-\frac{a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^4}-\frac{7a\left(b^2C-abD+a^2F\right)x^2}{b^3}-\frac{7a(b\cdots}{\cdots}}{\left(a+bx^2\right)^{7/2}}}{7a}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \int \frac{24Ab^4+4\cdots}{\cdots}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4\cdots\right.}{\cdots}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4\cdots\right.}{\cdots}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4\cdots\right.}{\cdots}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.51, size = 197, normalized size = 0.92

$$\frac{x\left(-105a^7F-350a^6bFx^2-406a^5b^2Fx^4-176a^4b^3Fx^6+a^3b^4\left(105A+35Bx^2+21Cx^4+15Dx^6\right)+2a^2b^5x^2\left(1\cdots\right.\right.}{105a^4b^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(-105*a^7*F - 350*a^6*b*F*x^2 - 406*a^5*b^2*F*x^4 + 48*A*b^7*x^6 - 176*a
^4*b^3*F*x^6 + 8*a*b^6*x^4*(21*A + B*x^2) + 2*a^2*b^5*x^2*(105*A + 14*B*x^2
 + 3*C*x^4) + a^3*b^4*(105*A + 35*B*x^2 + 21*C*x^4 + 15*D*x^6)))/(105*a^4*b
^4*(a + b*x^2)^(7/2)) + (Sqrt[a]*F*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/
Sqrt[a]])/(b^(9/2)*Sqrt[a + b*x^2])
```

fricas [A] time = 1.17, size = 567, normalized size = 2.65

$$\left[\frac{105\left(Fa^4b^4x^8 + 4\,Fa^5b^3x^6 + 6\,Fa^6b^2x^4 + 4\,Fa^7bx^2 + Fa^8\right)\sqrt{b}\,\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right) - 2\left(\left(176\,Fa^4b^4 \cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas"
)
```

```
[Out] [1/210*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*a^7*b*
x^2 + F*a^8)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*((
176*F*a^4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 + 7*
(58*F*a^5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*a^6*
b^2 - B*a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt(b*x^
2 + a))/(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2 + a^8*
b^5), -1/105*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*
a^7*b*x^2 + F*a^8)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((176*F*a^
4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 + 7*(58*F*a^
5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*a^6*b^2 - B*
a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt(b*x^2 + a))/
(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2 + a^8*b^5)]
```

giac [A] time = 0.59, size = 204, normalized size = 0.95

$$-\frac{\left(\left(x^2\left(\frac{\left(176\,Fa^4b^6-15\,Da^3b^7-6\,Ca^2b^8-8\,Bab^9-48\,Ab^{10}\right)x^2}{a^4b^7} + \frac{7\left(58\,Fa^5b^5-3\,Ca^3b^7-4\,Ba^2b^8-24\,Aab^9\right)}{a^4b^7}\right) + \frac{35\left(10\,Fa^6b^4-Ba^3b^7-6\,Aa^2b^8\right)}{a^4b^7}\right)x^2 + \cdots}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((176*F*a^4*b^6 - 15*D*a^3*b^7 - 6*C*a^2*b^8 - 8*B*a*b^9 - 48*
A*b^10)*x^2/(a^4*b^7) + 7*(58*F*a^5*b^5 - 3*C*a^3*b^7 - 4*B*a^2*b^8 - 24*A*
a*b^9)/(a^4*b^7)) + 35*(10*F*a^6*b^4 - B*a^3*b^7 - 6*A*a^2*b^8)/(a^4*b^7))*
x^2 + 105*(F*a^7*b^3 - A*a^3*b^7)/(a^4*b^7))*x/(b*x^2 + a)^(7/2) - F*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

maple [B] time = 0.01, size = 427, normalized size = 2.00

$$-\frac{F\,x^7}{7\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{Dx^5}{2\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{F\,x^5}{5\left(b\,x^2+a\right)^{\frac{5}{2}}b^2}-\frac{C\,x^3}{4\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{5Da\,x^3}{8\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}+\frac{Ax}{7\left(b\,x^2+a\right)^{\frac{7}{2}}a}-\frac{Bx}{7\left(b\,x^2+a \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*F*x^7/b/(b*x^2+a)^(7/2)-1/5*F/b^2*x^5/(b*x^2+a)^(5/2)-1/3*F/b^3*x^3/(b
*x^2+a)^(3/2)-F/b^4*x/(b*x^2+a)^(1/2)+F/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2
))-1/2*D*x^5/b/(b*x^2+a)^(7/2)-5/8*D*a/b^2*x^3/(b*x^2+a)^(7/2)-15/56*D*a^2/
b^3*x/(b*x^2+a)^(7/2)+3/56*D*a/b^3*x/(b*x^2+a)^(5/2)+1/14*D/b^3*x/(b*x^2+a)
^(3/2)+1/7*D/a/b^3*x/(b*x^2+a)^(1/2)-1/4*C*x^3/b/(b*x^2+a)^(7/2)-3/28*C*a/b
^2*x/(b*x^2+a)^(7/2)+3/140*C/b^2*x/(b*x^2+a)^(5/2)+1/35*C/a/b^2*x/(b*x^2+a)
^(3/2)+2/35*C/a^2/b^2*x/(b*x^2+a)^(1/2)-1/7*B/b*x/(b*x^2+a)^(7/2)+1/35*B/a/
b*x/(b*x^2+a)^(5/2)+4/105*B*x/a^2/b/(b*x^2+a)^(3/2)+8/105*B*x/a^3/b/(b*x^2+
a)^(1/2)+1/7*A*x/a/(b*x^2+a)^(7/2)+6/35*A/a^2*x/(b*x^2+a)^(5/2)+8/35*A/a^3*
x/(b*x^2+a)^(3/2)+16/35*A/a^4*x/(b*x^2+a)^(1/2)
```

maxima [B] time = 1.74, size = 597, normalized size = 2.79

$$-\frac{1}{35}\left(\frac{35\,x^{6}}{\left(bx^{2}+a\right)^{\frac{7}{2}}b}+\frac{70\,ax^{4}}{\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}+\frac{56\,a^{2}x^{2}}{\left(bx^{2}+a\right)^{\frac{7}{2}}b^{3}}+\frac{16\,a^{3}}{\left(bx^{2}+a\right)^{\frac{7}{2}}b^{4}}\right)Fx-\frac{Fx\left(\frac{15\,x^{4}}{\left(bx^{2}+a\right)^{\frac{5}{2}}b}+\frac{20\,ax^{2}}{\left(bx^{2}+a\right)^{\frac{5}{2}}b^{2}}+\frac{8\,a^{2}}{\left(bx^{2}+a\right)^{\frac{5}{2}}b^{3}}\right.}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima"
)
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*F*x - 1/
15*F*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8
*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*F*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - F*a*x^3/
((b*x^2 + a)^(5/2)*b^3) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*C*x^3/(
(b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 +
 a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7
/2)*a) + 139/105*F*x/(sqrt(b*x^2 + a)*b^4) + 17/105*F*a*x/((b*x^2 + a)^(3/2
)*b^4) - 29/35*F*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*D*x/((b*x^2 + a)^(3/2
)*b^3) + 1/7*D*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*D*a*x/((b*x^2 + a)^(5/2)*b^
3) - 15/56*D*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b
^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^
2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8
/105*B*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/
35*B*x/((b*x^2 + a)^(5/2)*a*b) + F*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x^2+C\,x^4+F\,x^8+x^6\,D}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.174 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(192Ab^3-a\left(3a^2D+4abC+24b^2B\right)\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^7\left(384Ab^4-a\left(15a^3F\right.\right.}{105a^5}$$

```
[Out] -A/a/x/(b*x^2+a)^(7/2)-(8*A*b-B*a)*x/a^2/(b*x^2+a)^(7/2)-1/3*(48*A*b^2-a*(6
*B*b+C*a))*x^3/a^3/(b*x^2+a)^(7/2)-1/15*(192*A*b^3-a*(24*B*b^2+4*C*a*b+3*D*
a^2))*x^5/a^4/(b*x^2+a)^(7/2)-1/105*(384*A*b^4-a*(48*B*b^3+8*C*a*b^2+6*D*a^
2*b+15*F*a^3))*x^7/a^5/(b*x^2+a)^(7/2)
```

Rubi [A] time = 0.34, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {1803, 1813, 12, 264}

$$-\frac{x^7\left(384Ab^4-a\left(6a^2bD+15a^3F+8ab^2C+48b^3B\right)\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(192Ab^3-a\left(3a^2D+4abC+24b^2B\right)\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^3\left(48Ab^2-a\right.}{3a^3\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((192*A*b^3
- a*(24*b^2*B + 4*a*b*C + 3*a^2*D))*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - ((384
*A*b^4 - a*(48*b^3*B + 8*a*b^2*C + 6*a^2*b*D + 15*a^3*F))*x^7)/(105*a^5*(a
+ b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{8Ab-a\left(B+Cx^2+Dx^4+Fx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(6b(8Ab-aB)+a\left(-aC-aDx^2-aFx^4\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a^2}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \int \frac{x^4(\cdots}{\cdots}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{(192\cdots}{\cdots}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{(192\cdots}{\cdots}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{(192\cdots}{\cdots}$$

Mathematica [A] time = 0.19, size = 138, normalized size = 0.72

$$\frac{a^4\left(-105A+105Bx^2+35Cx^4+21Dx^6+15Fx^8\right)+2a^3bx^2\left(-420A+105Bx^2+14Cx^4+3Dx^6\right)+8a^2b^2x^4\left(-\cdots\right.}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 21
*B*x^2 + C*x^4) + 2*a^3*b*x^2*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6) + a
^4*(-105*A + 105*B*x^2 + 35*C*x^4 + 21*D*x^6 + 15*F*x^8))/(105*a^5*x*(a + b
*x^2)^(7/2))
```

fricas [A] time = 1.02, size = 187, normalized size = 0.97

$$\frac{\left(\left(15\,Fa^4+6\,Da^3b+8\,Ca^2b^2+48\,Bab^3-384\,Ab^4\right)x^8+7\left(3\,Da^4+4\,Ca^3b+24\,Ba^2b^2-192\,Aab^3\right)x^6-105\,A\cdots\right.}{105\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fri
cas")
```

```
[Out] 1/105*((15*F*a^4 + 6*D*a^3*b + 8*C*a^2*b^2 + 48*B*a*b^3 - 384*A*b^4)*x^8 +
7*(3*D*a^4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35*(
C*a^4 + 6*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b
*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*
x)
```

giac [A] time = 0.60, size = 220, normalized size = 1.14

$$\frac{\left(\left(x^2\left(\frac{\left(15\,Fa^{13}b^3+6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7\right)x^2}{a^{14}b^3}+\frac{7\left(3\,Da^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-132\,Aa^{10}b^6\right)}{a^{14}b^3}\right)+\frac{35\left(Ca^{13}b^3+6\,Ba^{12}b^4-30\,A\right.}{a^{14}b^3}\right.\right.}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="gia
c")
```

```
[Out] 1/105*((x^2*((15*F*a^13*b^3 + 6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 -
 279*A*a^9*b^7)*x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11
*b^5 - 132*A*a^10*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a
^11*b^5)/(a^14*b^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b
*x^2 + a)^(7/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

maple [A] time = 0.01, size = 166, normalized size = 0.86

$$-\frac{384A\,b^4x^8-48Ba\,b^3x^8-8C\,a^2b^2x^8-6Da^3b\,x^8-15F\,a^4x^8+1344Aa\,b^3x^6-168B\,a^2b^2x^6-28C\,a^3b\,x^6-21Da^4}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(384*A*b^4*x^8-48*B*a*b^3*x^8-8*C*a^2*b^2*x^8-6*D*a^3*b*x^8-15*F*a^4
*x^8+1344*A*a*b^3*x^6-168*B*a^2*b^2*x^6-28*C*a^3*b*x^6-21*D*a^4*x^6+1680*A*
a^2*b^2*x^4-210*B*a^3*b*x^4-35*C*a^4*x^4+840*A*a^3*b*x^2-105*B*a^4*x^2+105*
A*a^4)/(b*x^2+a)^(7/2)/x/a^5
```

maxima [B] time = 1.54, size = 421, normalized size = 2.18

$$-\frac{Fx^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5\,Fax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Dx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{16\,Bx}{35\sqrt{bx^2+a}\,a^4}+\frac{8\,Bx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Bx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Bx}{7\left(bx^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="max
ima")
```

```
[Out] -1/2*F*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*F*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/
4*D*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*x/
((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b*x^
2 + a)^(7/2)*a) + 1/14*F*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*F*x/(sqrt(b*x^2 +
a)*a*b^3) + 3/56*F*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*F*a^2*x/((b*x^2 + a)
^(7/2)*b^3) + 3/140*D*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*D*x/(sqrt(b*x^2 + a)
*a^2*b^2) + 1/35*D*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*D*a*x/((b*x^2 + a)^(7
/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b
) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b)
- 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2)*a^4)
- 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)*a^2) -
 A/((b*x^2 + a)^(7/2)*a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x^2+C\,x^4+F\,x^8+x^6\,D}{x^2\,{\left(b\,x^2+a\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)),x)
[Out] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```